Fibonacci Prime Labeling Of Graphs

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***Abstract*: In this paper, we introduced Fibonacci prime labeling of graphs. A Fibonacci prime labeling of a graph $G = (V(G), E(G))$ with $|V(G)| = n$ is an injective function $g: V(G) \to \{f_2, f_3, \dots . f_{n+1}\}$,where f_n is the n^{th} Fibonacci number ,that induces a function $g^*: E(G) \to N$ defined by $g^*(uv) = gcd\{g(u), g(v)\} = 1 \ \forall \ uv \in E(G)$.If $g^*(uv) = 1 \forall \ uv \in E(G)$, we say that the graph G admits a Fibonacci prime labeling and is called a Fibonacci prime graph . In this paper we prove that path, cycle, friendship graph, fan graph, star graph, dragon graph and an umbrella graph are Fibonacci prime graphs.**

***Keywords*: Fibonacci prime labeling, Fibonacci prime graph, path, cycle, friendship graph, fan graph, star graph, dragon graph, umbrella graph.**

I.INTRODUCTION

In this paper, only finite simple undirected connected graphs are considered. The graph G has vertex set V =V (G) and edge set E = E (G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H[3] has proved that the path P_n on n vertices is a prime graph.Deretsky et al [2] have proved that the cycle C_n on n vertices is a prime graph .Around 1980 Roger Entringer conjectured that all trees have prime labeling which has not been settled so far.

In [7] S.K. Vaidhya and K.K. Kanmani have proved that the graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle C_n admit prime labeling. In [5] Meena and Vaithilingam have proved the Prime labeling for some helm related graphs.

Definition 1.1

The Fibonacci number f_n is defined recursively by the equations $f_1 = 1$; $f_2 = 1$; $f_{n+1} = f_n + f_{n-1}$ $(n \geq 2)$. Then g.c.d $(f_n, f_{n+1}) = 1$, for all $n \geq 1$.

Definition 1.2

A prime labeling of a graph G is an injective function $f: V(G) \to \{1,2, \dots . . |V(G)|\}$ such that for every pair of adjacent vertices u and v, $gcd\{f(u), f(v)\} = 1$.A graph which admits a prime labeling is called a prime graph .

Definition 1.3

A fan graph P_n^* is obtained by joining all vertices of a path $P_n, n \geq 2$ to a further vertex, called the centre.

Definition 1.4

The friendship graph F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Definition 1.5

The Dragon graph $D_n(m)$ is the graph obtained by joining a cycle C_n to a path P_m with a bridge.

Definition 1.6

An umbrella graph $U_{(m,n)}$ is the graph obtained by joining a path P_n with the central vertex of a fan P_m^* .

II MAIN RESULTS

Definition 2.1

A Fibonacci prime labeling of a graph $G = (V, E)$ with $|V(G)| = n$ is an injective function

$g: V(G) \to \{f_2, f_3, \ldots \ldots . f_{n+1}\}$,where f_n is the n^{th} Fibonacci number, that induces a function $g^* : E(G) \to N$ defined by $g^*(uv) = g.c.d\{g(u), g(v) = 1 \ \forall \ uv \in E(G)$.

The graph which admits a fibonacci prime labeling is called Fibonacci prime graph.

Theorem 2.2

Path P_n is a Fibonacci prime graph.

Proof:

Let G be a path P_n with n vertices. Then |V (G)|= n .

Denote the vertices of P_n as $v_1, v_2, \ldots \ldots \ldots \ldots \ldots . v_n$ in that order .

Define $g: V(G) \to \{f_2, f_3, \ldots \ldots \ldots \ldots \ldots f_{n+1}\}$ as

$$g(v_i) = f_{i+1}, \qquad 1 \le i \le n \ \forall \ v_i \in V(G).$$

The induced function $g^* : E(G) \to N$ is defined by

$$g^*(uv) = gcd\{g(u), g(v), \forall uv \in E(G).$$

Now gcd $\{f(v_i), f(v_{i+1})\} = gcd\{f_{i+1}, f_{i+2}\} = 1, \quad 1 \le i \le n-1 \ \ \forall v_i v_{i+1} \in E(G)$.

Thus G admits a Fibonacci prime labeling.

Hence G is a Fibonacci prime graph.

Example 2.3

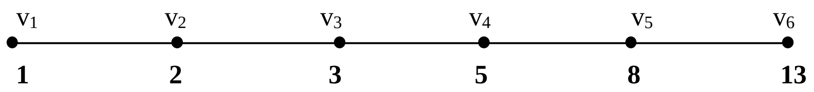

Figure: 1 Fibonacci prime labeling of path P_6

Theorem2.4

Cycle C_n is a Fibonacci prime graph for $n \ge 3$.

Proof:

Let $v_1, v_2, \ldots . . v_n$ be the vertices of the cycle C_n .The edge set of C_n is $E(C_n) = \{ v_i v_{i+1} \mid 1 \le i \le n-1\} \cup \{v_n v_1\}$. Define $g: V(C_n) \to \{f_2, f_3 \ldots \ldots f_{n+1}\}$ as

$$g(v_i) = f_{i+1}, 1 \le i \le n \ .$$

Then the induced function $g^* : E(G) \to N$ is defined by

$g^*(xy) = g.c.d\{g(x), g(y)\} \ \forall \ xy \in E(G)$.

Now,$gcd\{g(v_i), g(v_{i+1})\} = gcd\{f_{i+1}, f_{i+2}\} = 1$,$1 \le i \le n-1$.

and $gcd\{g(v_n), g(v_1)\} = gcd\{(f_{n+1}, f_2\} = gcd\{f_{n+1}, 1\} = 1$

Thus $f^*(xy) = g.c.d\{f(x), f(y)\} = 1, \forall \ xy \in E(G)$.Hence C_n is a Fibonacci prime graph.

Example 2.5

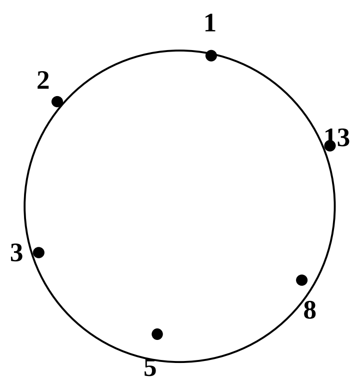

Figure: 2 C_6 is a Fibonacci prime graph

Theorem2.6

The Fan graph $P_n^*, n \geq 2$ is a Fibonacci Prime graph.

Proof :

Let $v_1, v_2, \ldots \ldots v_n, v_{n+1}$ be the vertices of the fan graph P_n^* with centre vertex v_1. Let $G = P_n^*$.

The edge set $E(G) = \{v_1 v_i\ ,\ 2 \leq i \leq n+1\}$ and $\{v_i v_{i+1},\ \ 2 \leq i \leq n-1\}$.

Then $|V(G)| = n+1$ and $|E(G)| = 2n-1$.

Define $g: V(G) \rightarrow \{f_2 \ldots \ldots \ldots \ldots f_{n+2}\}$ by $g(v_i) = f_{i+1},\ \ 1 \leq i\ \leq n+1$.

The induced function $g^*: E(G) \rightarrow N$ is defined by $g^*(uv) = g.c.d\{g(u), g(v)\}, \quad \forall\ uv \in E(G)$

Now, $g.c.d\{g(v_1), g(v_i)\} = g.c.d\{f_2, f_{i+1}\} = g.c.d\ \{1, f_{i+1}\} = 1$ for $2 \leq i \leq n+1$

$g.c.d\{g(v_i), g(v_{i+1})\} = g.c.d\{f_{i+1}, f_{i+2}\} = 1$ for $2 \leq i \leq n$.

Thus $g^*(uv) = g.c.d\{g(u), g(v)\} = 1, \quad \forall\ uv \in E(G)$.

Hence the fan graph P_n^* is a Fibonacci Prime graph.

Example 2.7

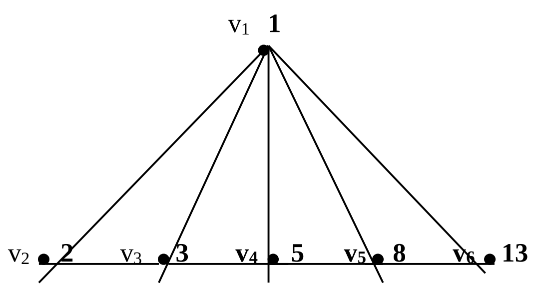

Figure: 3 Fibonacci prime labeling of the fan graph P_5^*

Theorem2.8

Star graph $K_{1,n}$, $n \geq 1$ is a Fibonacci prime graph.

Proof:

Let G be the star graph $k_{1,n}$. The vertex set of G is $V(G) = \{v_1, v_2 \ldots \ldots \ldots \ldots v_{n+1}\}$ where v_1 is the centre of the star graph.

Then $|V(G)| = n+1$ and $|E(G)| = n$.

Define $g: V(G) \rightarrow \{f, f_3, \ldots\ldots\ldots f_{n+2}\}$ by

$g(v_i) = f_{i+1}$,$1 \le i \le n+1$.

The induced function $g^*: V(G) \rightarrow N$ is defined by $g^*(uv) = \gcd\{g(u), g(v)\} \forall\ uv \in E(G)$.

Now, $\gcd\{g(v_1), g(v_i)\} = g.c.d\{f_2, f_{i+1}\} = g.c.d\{1, f_{i+1}\} = 1 \forall\ 2 \le i \le n+1$

Thus all the vertices have distinct labels and $g^*(uv) = \gcd\{g(u), g(v)\} = 1 \forall\ uv \in E(G)$

Hence G is a Fibonacci prime graph.

Example 2.9

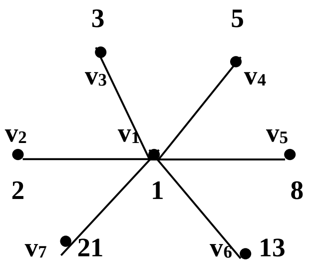

Figure: 4 Fibonacci prime labeling of $K_{1,6}$

Theorem 2.10

The friendship graph $F_n, n \ge 2$ is a Fibonacci prime graph.

Proof

Let G be the friendship graph F_n .

Let $v_1, v_2, \ldots \ldots . v_n, v_{n+1}$ be the vertices of G where v_1 is the centre vertex of G.

The edge set $E(G) = \{v_1v_i | 2 \le i \le 2n+1\} \cup \{v_{2i}v_{2i+1} | 1 \le i \le n\}$.

Then $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

Define a labeling $g: V(G) \rightarrow \{f_2, f_3 \ldots\ldots\ldots\ldots . f_{2n+2}\}$by $g(v_i) = f_{i+1}, 1 \le i \le 2n+1$.

The induced function $g^*: E(G) \rightarrow N$ is defined by $g^*(uv) = gcd\{f(u), f(v)\}, \forall\ uv \in E(G)$.

Now,$gcd\{g(v_1), g(v_i)\} = gcd\{f_2, f_{i+1}\}$

$= gcd\{1, f_{i+1}\}$

$= 1\ for\ 2 \le i \le 2n+1$

$gcd\{f(v_{2i}), f(v_{2i+1})\} = gcd\{f_{2i+1}, f_{2i+2}\}$

$= 1 , 1 \le i \le n.$

Thus $g^*(uv) = gcd\{g(u), g(v)\} = 1 \forall\ uv \in E(G)$.

Hence G admits a Fibonacci prime Labeling. Hence the friendship graph F_n is a Fibonacci prime graph .

Example 2.11

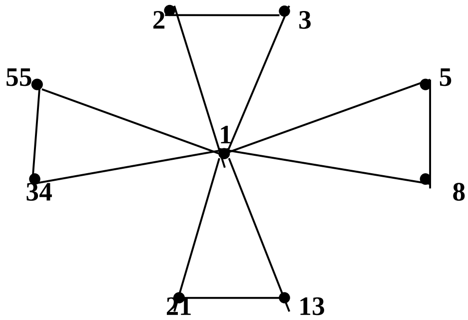

Figure: 5 Fibonacci prime labeling of the fan graph F_4

Theorem 2.12

The dragon graph $D_n(m)$ is a Fibonacci prime graph for $n \geq 3$, $m \geq 1$.

Proof:

Let G be the Dragon graph $D_n(m)$.

Let $u_1, u_2, \ldots\ldots u_n$ be the vertices of the cycle C_n and $v_1, v_2, \ldots\ldots v_{\text{m}}$ be the vertices of the path $P_{m.}$

The edge set $E(G) = \{u_i u_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{u_1 u_n\} \cup \{u_1 v_1\} \cup \{v_i v_{i+1} \ / \ 1 \leq i \leq m-1\}$,where $u_1 v_1$ is the bridge joining C_n with P_m .

Then $|V(G)| = n + m$ and $|E(G)| = n + m$.

Define the mapping $g: V(G) \rightarrow \{f_2, f_3, \ldots.\ f_{n+m+1}\}$ as follows

$$g(u_i) = f_{i+1} \ ,1 \leq i \leq n$$

$$g(v_i) = f_{n+i+1} \ ,1 \leq i \leq m.$$

Then the induced function $g^*: E(G) \rightarrow N$ is defined by $g^*(xy) = g.c.d\{g(x), g(y)\} \ \forall \ xy \in E(G)$.

Now ,$g.c.d\{g(u_i), g(u_{i+1}) = g.c.d\{f_{i+1}, f_{i+2}\} = 1 \ , \ 1 \leq i \leq n-1$.

$g.c.d\{\ g(u_1), g(u_{n)}\} = g.c.d\{f_2, f_{n+1}\} = g.c.d\{1, f_{n+1}\} = 1.$

$g.c.d\ \{g(u_1), g(v_1)\} = g.c.d\{\ 1, f_{n+2}\} = 1.$

$g.c.d\{g(v_i), g(v_{i+1})\} = g.c.d\{g_{n+i+1}, F_{n+i+2})\} = 1\ , 1 \leq i \leq m-1$.

Thus$g^*(xy) = g.c.d\{g(x), g(y)\} = 1\ \forall\ xy \in E(G)$.

Hence the dragon graph is the Fibonacci prime graph.

Example 2.13

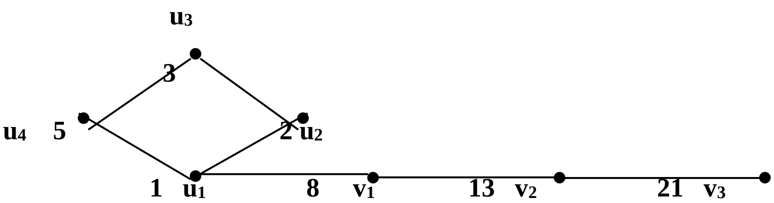

Figure:6 Fibonacci prime labeling of a dragon graph $D_4(3)$

Theorem 2.14

An umbrella graph $U_{m,n}$ is a Fibonacci prime graph for all m and n .

Proof

Let $G = U_{m,n}$. The vertex set of G is $V(G) = \{x_1, x_2 \ldots\ldots\ldots\ldots\ldots.. x_m, y_1, y_2 \ldots\ldots\ldots y_n\}$ .The edge set of G is

$E(G) = \{x_i x_{i+1} \ / \ 1 \le i \le m-1\} \cup \{x_i y_1 \ / \ 1 \le i \le m\} \cup \{y_i y_{i+1} \ / \ 1 \le i \le n-1\}$

Then $|V(G)| = m + n$ *and* $|E(G)| = 2m + n - 2$. Define a function $g: V(G) \to \{f_2, f_3 \ldots\ldots\ldots. f_{m+n+1}\}$ as follows.

$g(y_1) = f_2$

$g(x_i) = f_{i+2}$ for $1 \le i \le m$

$g(y_i) = f_{m+i+1}$ for $2 \le i \le n$

The induced function $g^*: E(G) \to N$ is defined by

$g^*(uv) = gcd\{g(u), \mathrm{g(v)}\} \ \forall \ \mathrm{uv} \in \mathrm{E(G)}$.

Now,$\mathrm{g.c.d}\{\mathrm{g}(\mathrm{x_i}), \mathrm{g}(\mathrm{x_{i+1}})\} = \mathrm{gcd}\{\mathrm{f_{i+2}}, \mathrm{f_{i+3}}\} = 1$ for $1 \le \mathrm{i} \le \mathrm{m} - 1$.

$\mathrm{g.c.d}\{\mathrm{g}(\mathrm{x_i}), \mathrm{g}(\mathrm{y_1})\} = \mathrm{gcd}\{\mathrm{f_{i+2}}, \mathrm{f_2}\} = \mathrm{g.c.d}\{\mathrm{f_{i+2}}, 1\} = 1$ for $1 \le \mathrm{i} \le \mathrm{m}$.

$\mathrm{g.c.d}\{\mathrm{g}(\mathrm{y_1}), \mathrm{g}(\mathrm{y_2}) = \mathrm{g.c.d}\{1, \mathrm{f_{n+3}}\} = 1$.

$\mathrm{g.c.d}\{\mathrm{g}(\mathrm{y_i}), \mathrm{g}(\mathrm{y_{i+1}})\} = \mathrm{g.c.d}\{\mathrm{f_{m+i+1}}, \mathrm{f_{m+i+2}}\} = 1$, $2 \le \mathrm{i} \le \mathrm{n} - 1$.

Thus $\mathrm{g}^*(\mathrm{uv}) = \mathrm{gcd}\{\mathrm{g}(\mathrm{u}), \mathrm{g}(\mathrm{v})\} = 1 \forall$ uv ∈ E(G).Hence G admits a Fibonacci prime labeling and hence the umbrella graph $U_{(m,n)}$ is a Fibonacci prime graph.

Example 2.15

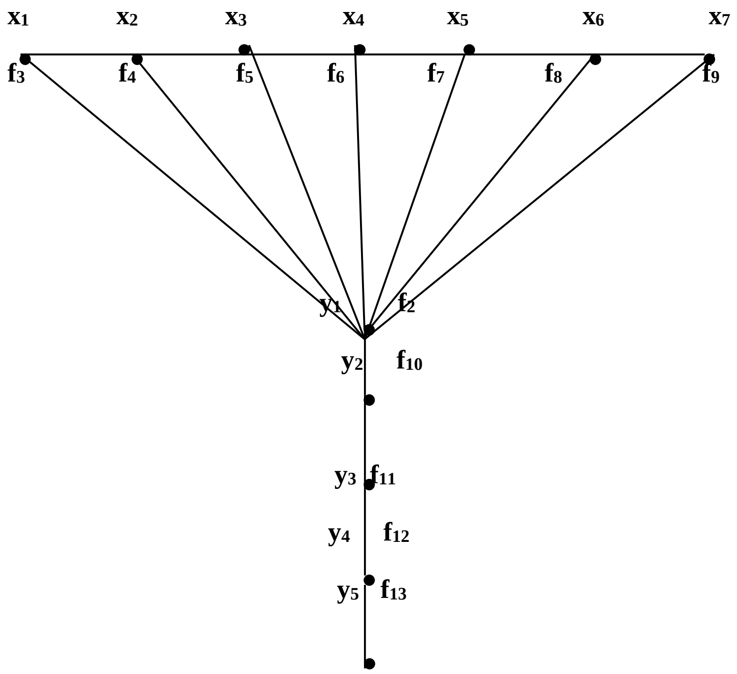

Figure: 8 Fibonacci prime labeling of an Umbrella graph $U_{7,5}$

CONCLUTION

We have introduced a new labeling namely Fibonacci prime labeling of graphs .We prove that path, cycle,star,fan graph ,friendship graph, dragon graph and an umbrella graph are all Fibonacci prime graphs .Extending the study to other families of graph is an open area of research .

References

[1] Bondy .J.A and Murthy U.S.R, "Graph Theory and Application" (North Holland). New York (1976).

[2] Deretsky .T, Lee.S.M and Mitchem .J, "On Vertex Prime Labeling of Graphs in Graph Theory, Combinatorics and Applications", Vol.I (Ed.Alavi .J, Chartrand. G, Oellerman.O, and Schwenk. A) Proceedings of the 6th international conference Theory and Applications of Graphs, Wiley, New York, (1991) 359-369.

[3] Fu, H.L and Huang .K.C. , " On prime Labelings", Discrete Mathematics, 127(1994), 181-186.

[4] Gallian J.A, "A Dynamic survey of Graph labeling", The Electronic Journal of Combinatorics, 18(2011), 147, #DS6.

[5] Meena.S and Vaithilingam.K "Prime Labeling for some Helm Related Graphs", International Journal of innovative research in science, Engineering and Technology, Vol.2, 4 April 2009.

[6] Tout.A, Dabboucy .A.N and Howalla. K, "Prime Labeling of Graphs", Nat.Acad .Sci letters 11 (1982) 365-368

[7] Vaidya S.K and Kanmani K.K "Prime Labeling for some cycle Related Graphs", Journal of Mathematics Research vol.2. No.2.pp 98-104, May 2010.