NEW METHODS OF INTEGRATION BY PARTS

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ABSTRACT

Integration is a reversible order of differentiation in the present paper sum interesting methods have been obtained on integrating multivariable. I introduce a new methods of integrating by parts. This method is very useful to student easily understanding.

Key Words

Differentiation, Integration, Exponential, Logarithms, Trigometric

$\int e^x \, sinb\, x \, dx$ by using Integration by parts.

Sol

$u = \sin bx$ $\qquad dv = \int e^{ax}$

$du = cos\, bx \,.\, bdx$ $\qquad v = \frac{e^{ax}}{a}$

$$\int udv \quad = \quad uv - \int udv$$

$$= \quad sinbx\, \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a}\, b\, asbx\, dx$$

$$= \quad \frac{1}{a} e^{ax} sinbx - \frac{b}{a} \int e^{ax} cosbx$$

$$= \quad \frac{1}{a} e^{ax} sinbx - \frac{b}{a} I. \quad \text{--- (1)}$$

Put $u = cosbx$

$du = -bsinbx\, dx$

$\int dv = \int e^{ax}$

$v = \frac{e^{ax}}{a}$

$$\therefore I_1 \quad = \quad uv - \int vdu$$

$$= \quad cosbx\, \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} (-b) sinbx \;\; dx$$

$$= \quad cosbx\, \frac{e^{ax}}{a} + \frac{b}{a} \int e^{ax} sinbx\, dx \quad \text{--- (2)}$$

Sub (2) in (1)

$$I \quad = \quad \frac{1}{a} e^{ax} sinbx - \frac{b}{a} \left[e^{ax} \frac{cosbx}{a} + \frac{b}{a} \int e^{ax} sinbx\, dx \right]$$

$$= \quad \frac{1}{a} e^{ax} sinbx - \frac{b}{a} e^{ax} cosbx - \frac{b^2}{a^2} \int e^{ax} sinbx\, dx$$

$$= \quad \frac{1}{a} e^{ax} sinbx - \frac{b^2}{a^2} e^{ax} cosbx - \frac{b^2}{a^2} [I_2] \quad \text{--- (3)}$$

$$\therefore I_2 \quad = \quad \int e^{ax} sinbx\, dx$$

$$= \quad \frac{e^{ax}}{a} sinx - \int \frac{e^{ax}}{a} bcosbx \;\; dx$$

$$= \frac{e^{ax}}{a} sinbx - \frac{b^2}{a^2} \int e^{ax} \cos bx$$

$$= \int e^{ax}\ sinbxdx \left(1 \frac{b^2}{a^2}\right) = \frac{e^{ax}}{a} sinbx \frac{be^{ax}}{a^2}\ cosbx$$

$$= e^{ax}\ sinbxdx \left(\frac{a^2+b^2}{a^2}\right) = \frac{e^{ax}}{a} \left(\frac{sinbx}{a} - \frac{b}{a^2} cosbx\right)$$

$$= \int e^{ax}\ sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$$

Proof by New Method of Integration by parts

Evaluate

$\int e^{ax}\ sinb\ x\ dx$ by using Integration by parts. (which is the order is same)

Sol

Differentiation	Integration
$+ \quad e^{ax}$	$sinbx$
$-ae^{ax}$	$-\frac{cosbx}{b}$
$+ \quad a^2e^{ax}$	$-\frac{sinbx}{b^2}$

Using Bernoulli's formula

$$\int udv = uv' - u_1v'' + u_2v''' \ldots.$$

$$\int e^{ax}\ sinb\ x\ dx = \frac{e^{ax}cosbx}{-b} + \frac{ae^{ax}sinbx}{b^2} - \frac{a^2}{b^2}\int e^{ax}\ sinb\ x\ dx$$

$$\int e^{ax}\ sinbxdx + \frac{a^2}{b^2}\int e^{ax}\ sinbxdx = \frac{e^{ax}cosbx}{-b} + \frac{ae^{ax}sinbx}{b^2}$$

$$\int e^{ax}\ sinbxdx \left(1 + \frac{a^2}{b^2}\right) = \frac{ae^{ax}sinbx - be^{ax}cosbx}{b^2}$$

$$\int e^{ax}\ sinbxdx \left(\frac{a^2+b^2}{b^2}\right) = e^{ax}\left(\frac{asinbx - bcosbx}{b^2}\right)$$

$$\int e^{ax}\ sinbxdx = \frac{b^2}{a^2+b^2}e^{ax}\left(\frac{asinbx - bcosb ⍰}{b^2}\right)$$

$$\int e^{ax}\ sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$$

Proof by previous method

Evaluate

$\int e^{ax}\ cosbx\ dx$ by using Integration by parts.

Sol

$u = cos\ bx \qquad dv = \int e^{ax}$

$du = -sinbx\ .bdx \qquad v = \frac{e^{ax}}{a}$

$$\int udv = uv - \int udv$$

$$= cosbx\ \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} - sinbx\ dx$$

$$= \frac{1}{a}\ e^{ax}\ cosbx + \frac{b}{a}\int e^{ax}sinbx$$

$$= \frac{1}{a}\ e^{ax}\ cosbx + \frac{b}{a}\ I. \qquad \text{--- (1)}$$

Put $u = sinbx$

$du = bcosbx\ dx$

$\int dv = \int e^{ax}$

$v = \frac{e^{ax}}{a}$

$\therefore I_1 \quad = \quad uv - \int vdu$

$= \quad sinbx \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a}(b)cosbx\ dx$

$= \quad sinbx \frac{e^{ax}}{a} - \frac{b}{a}\int e^{ax}\ cosbx\ dx$ --- (2)

Sub (2) in (1)

I $= \quad \frac{1}{a} e^{ax}\ cosbx + \frac{b}{a}\left[e^{ax}\ \frac{sinbx}{a} - \frac{b}{a}\int e^{ax}\ cosbx\ dx\right]$

$= \quad \frac{1}{a} e^{ax}\ cosbx + \frac{b}{a} e^{ax} sinbx - \frac{b^2}{a^2}\int e^{ax}\ sinbx\ dx$

$= \quad \frac{1}{a} e^{ax}\ cosbx - \frac{b^2}{a^2} e^{ax} sinbx - \frac{b^2}{a^2}\ [I_2]$ --- (3)

$\therefore I_2 \quad = \quad \int e^{ax}\ cosbx\ dx$

$= \quad \frac{e^{ax}}{a} cosx + \int \frac{e^{ax}}{a}\ bsinbx\ dx$

$= \quad \frac{e^{ax}}{a} cosbx + \frac{b^2}{a^2}\int e^{ax}\ sin\, bx$

$= \quad \int e^{ax}\ cosbxdx\left(1\frac{b^2}{a^2}\right) = \frac{e^{ax}}{a} cosbx + \frac{be^{ax}}{a^2} sinbx$

$= \quad e^{ax}\ cosbxdx\left(\frac{a^2+b^2}{a^2}\right) = \frac{e^{ax}}{a}\left(\frac{cosbx}{a} + \frac{b}{a^2} sinbx\right)$

$= \quad \int e^{ax}\ cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$

Proof by New Method

Evaluate

$\int e^{ax}\ cosbx\ dx$ by using Integration by parts. (which is the order is same)

Sol

Differentiation	Integration
+ e^{ax}	$cosbx$
- $-ae^{ax}$	$\frac{sinbx}{b}$
+ a^2e^{ax}	$-\frac{cosbx}{b^2}$

using Bernoulli's formula

$\int udv \quad = \quad uv' - u_1v'' + u_2v''' \ldots.$

$\int e^{ax}\ cosbxdx \quad = \quad \frac{e^{ax}sinbx}{b} + \frac{ae^{ax}cosbx}{b^2} - \frac{a^2}{b^2}\int e^{ax}\ cosbxdx$

$\int e^{ax}\ cosbxdx + \frac{a^2}{b^2}\int e^{ax}\ cosbxdx \quad = \quad \frac{e^{ax}sinbx}{b} + \frac{ae^{ax}cosbx}{b^2}$

$\int e^{ax}\ cosbxdx\ \left(1 + \frac{a^2}{b^2}\right) \quad = \quad \frac{ae^{ax}cosbx+be^{ax}sinbx}{b^2}$

$\int e^{ax}\ cos$ 嗤$xdx\ \left(\frac{a^2+b^2}{b^2}\right) \quad = \quad e^{ax}\left(\frac{acosbx+bsinbx}{b^2}\right)$

$\int e^{ax}\ cosbxdx \quad = \quad \frac{b^2}{a^2+b^2} e^{ax}\left(\frac{acosbx+bsinbx}{b^2}\right)$

$\int e^{ax}\ cosbxdx \quad = \quad \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$

Conclusion

This method is very useful to school student, Arts and Science student and Engineering Student. It is very shortcut method. It is method is very useful to Gate Exam and Competitive Exams.

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