# More Path Unions Invariance under Cordial Graphs

**Mukund V.Bapat**[1]

Research Scholar

***Abstract*: In this paper we show that path unions obtained from bowtie, paw, house, temple and tail($C_3$,$P_3$) are cordial graphs.**

***Keywords*: cordial graph, house, bowtie, temple,tail graph, path union, labeling**

Subject classification: 05C78.

2. Introduction:

The graphs we consider are simple ,finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6],A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5].f:V(G)→{0,1} be a function.From this label of any edge (uv) is given by |f(u)-f(v)|.Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by atmost one.Then the function f is called as cordial labeling.Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of $C_3$) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel $W_n$ is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J.Gallian[8].The graph that has cordiallabeling is called as cordial graph.

For the same given graph G there are many path union $P_m(G)$ structures possible. It depends on which point on G is used to fuse with vertex on $P_m$. If this point is changed and path union is designed then it may be a different (up to isomorphism) structure.We have shown that for G = bull on $C_3$,bull on $C_4$,$C_3^+$,$C_4^+$-e the different path union $P_m(G)$ are cordial[4].It is called as invariance under cordial labeling. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a and that number of vertices labeled with 1 are b. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are y. In this paper we show that path union of bowtie,paw,house,temple and tail ($C_3$,$P_3$) with it's different non-isomorphic structures are cordial.

3. Preliminaries :In tail graph we have a path $P_m$ attached at any vertex of graph G.For a (p,q) graph G wehaveantenna graph denoted by ante(G,$P_m$) or tail(G,$P_m$).It has p+m-1 vertices and q+ m-1 edges. In this paper we consider G= $C_3$ and $P_m$(m=3).We design path-union on ante($C_3$,$P_3$) and discuss for cordiality. The graph that follows cordiallabeling is called as cordial graph.

4.Definitions:

4.1 Fusion of vertices. Let $u \neq v$ be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.

4.2 Path union : $P_m(G)$ is obtained by taking a path on m points and m copies of G are taken.At each vertex of path a copy each of G is fused. The point of fusion on G is same and fixed for all copies of G.

5. Theorems Proved:

5.1 Path union of bow-tie is cordial. Proof:There are three structures possible on path union .These depends on which vertex on bow-tie is used to fuse with vertex of $P_m=(v_1,v_2,\ldots v_m)$.From the figure below it follows that path union taken on x,y or z will be structurally different.(non- isomorphic).Thus there are three structures possible on :In structure1 the vertex on bow-tie used is x in structure 2 the vertex used is y and in structure 3 the vertexused is z.The structure 3 is same as double path union on $C_4$ which we have shown to be cordial in [4]. Below we give type A and Type B labeling which are used to built the two structures. In both structures we use type Ato fuse with $v_i$of $P_m$ if $i\equiv1,0$(mod 4) and B is used when $i\equiv2,3$(mod 4). In structure 1 vertex 'r' on type A and vertex's' on type B is fused with $v_i$. In structure 2 vertex a on type A and vertex b on type B is fused with vi as explained above.

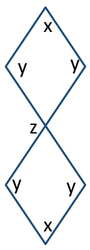


Fig 5.1 copy of bowtie

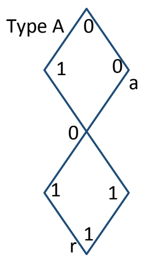


Fig 5.2: $v_f(0,1)$ =(3,4),$e_f(0,1)$=(4,4)

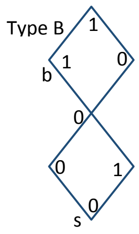


Fig 5.3: $v_f(0,1)$ =(4,3),$e_f(0,1)$=(4,4)

For both structures the label numbers observed are : $v_f(0,1)$ =(7x,7x) when m is even number given by 2x.x= 1,2,… $v_f(0,1)$=(3+14x,4+14x) when mis odd number given by m =1+4x ,(m≡1(mod 4))x=0,1,2… We have $v_f(0,1)$ = ( 11+14x,10+14x)for m≡3(mod 4) i.e. When m is of the type 4x+3 . Thus The graph is cordial. #.

5.2 Theorem. Path union of paw is cordial.( paw is actually flag of $C_3$ ) Proof:There are three structures possible on pathunion.It depends on the vertex on paw used to fuse with vertex on $P_m=(v_1,v_2,..v_m)$.We can take pathunion on vertex x , y or zdepending on which structure 1,structure2 or structure3 is formed respectively.

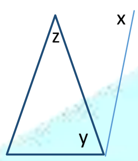


Fig 5.4 : paw graph

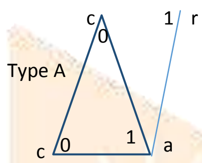


Fig 5.5 $v_f(0,1)$ = ( 2,2),$e_f(0,1)$ = ( 2,2)

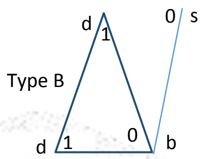


Fig 5.6$v_f(0,1)$ = ( 2,2),$e_f(0,1)$ = ( 2,2)

Toobtain structure 1 vertex 'r' on type A and vertex 's' on type B is used to fuse with vertex on $P_m$. For structure 2 vertex 'a' on type A and vertex 'b' on type B is used to fuse with vertex on $P_m$. For structure 3 vertex 'c' on type A and vertex'd' on type B is used to fuse with vertex on $P_m$. All structures are obtained by fusing vertex on type A with $v_i$ when i ≡ 1,4 (mod 4) and type B is used when i≡ 2,3 (mod 4).The observed label numbers are ( for all three structures): For edges :$e_f(0,1)$=(5x+2,5x+2) when m is of type 2x+1, x= 0,1,2,..$e_f(0,1)$=(5x+4,5x+5) when m is of type 2x, x= 0,1,2,… On vertices we have $v_f(0,1)$ = (2m,2m).Thus the graph is cordial. #. 5.3Theorem Pathunion on house graph is cordial. Proof:There are three non-isomorphic structures possible. For that vertex 'a' or 'b' or vertex 'c' on house graph (fig 5.7) is used to fuse with vertex on path $P_m$ respectively to obtain structure 1,structure 2, structure 3.

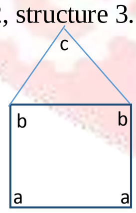


Fig 5.7 house

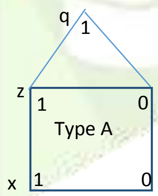


Fig5.8 $v_f(0,1)$=(2,3),$e_f(0,1)$=(3,3)

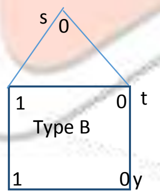


Fig5.9 $v_f(0,1)$=(3,2),$e_f(0,1)$=(3,3)

Define a function f:V(G) →{0,1} as follows. Under f we define two types of labels Type A and type B.These are cordial but differ in label number of vertex 'q' on type A and ' s' on type B. In structure 1 vertex 'q' on type A and 's' on type B is fused with vertex of path $P_m=(v_1,v_2,..v=)$.In structure 2 vertex 'z' on type A and 't' on type B is fused with vertex of path $P_m=(v_1,v_2,..v=)$.In structure 3 vertex 'x' on type A and 'y' on type B is fused with vertex of path $P_m=(v_1,v_2,..v=)$.In all the three structures type A is used to fuse at vertex $v_i$ if i≡1,0(mod 4) ,Type B if i≡2,3 (mod 3).

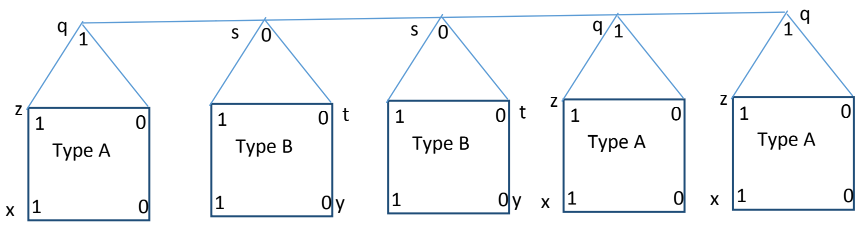


Fig5.10 : labeled copy of $P_5$(house),structure 3 :$v_f(0,1)$=(12,13),$e_f(0,1)$=(17,17)

For all structures label number distribution is : $v_f(0,1) = (5x,5x)$ when m is even number given by m= 2x, x=1,2,.., $v_f(0,1) = (5x+2,5x+3)$ when m is odd number given by m= 2x+1, x= 0,1,2,..,And $v_f(0,1) = (5x+8,5x+7)$ when m is even number given by m= 2x+3. On edges we have $e_f(0,1)=(3+7x,3+7x)$ when m is of type m =2x+1,x=0,1,2,3.. When m is of type 2x we have $e_f(0,1)=(6+14(x-1),7+14(x-1))$, x = 1,2,3.. Thus the graph is cordial. #

5.3 Theorem: Path union on temple graph G = $P_m$(temple) is cordial. Proof. Define f:V(G)→{0,1} to obtain type A and Type B labeling as follows.These are used to obtain labeled copy of path-union.

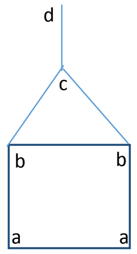

Fig 5.10 flag house or Temple graph

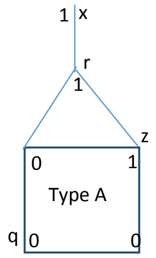

Fig 5.11 $v_f(0,1)=(3,3),e_f(0,1)=(4,3)$

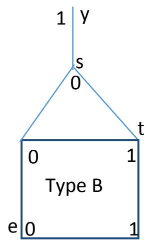

Fig 5.12 $v_f(0,1)=(3,3),e_f(0,1)=(3,4)$

Path union is defined by taking m copies of temple graph and fusing a copy each at vertex of Pm = ( $v_1$, $v_2$, $v_3$.. $v_m$). There are four non-isomorphic structures possible depending on the vertex on temple a,b, c or d used to form path union( refer fig 5.10).To obtain structure 1 vertex 'x' on type A and vertex 'y' on type B is used to fuse with vertex $v_i$ on $P_m$.To obtain structure 2 vertex 'r' on type A and vertex 's' on type B is used to fuse with vertex $v_i$ on $P_m$.To obtain structure 3 vertex 'z' on type A and vertex 't' on type B is used to fuse with vertex $v_i$ on $P_m$.To obtain structure 4 vertex 'q' on type A and vertex 'e' on type B is used to fuse with vertex $v_i$ on $P_m$. For all structures A is fused at $v_1$ of Pm and at all other vertices of $P_m$ copies temple used are type B. The label number distribution is as follows: For structure 1 ,structure 3 and structure 4 we have $e_f(0,1) = (4+4(m-1),3+4(m-1))$. For structure 2 we have $e_f(0,1) = (7+4(m-2),8+4(m-2))$ form >2 and if m=1 we have $e_f(0,1) = (4,3)$ and if m=2 then $e_f(0,1) =(7,8)$.All structures have same vertexlabels given by $v_f(0,1) =(3m,3m)$. Thus the graphis cordial. #

5.4 Theorem : G= tail($C_3$,$p_3$) Then path union of G is cordial.(all four structures)

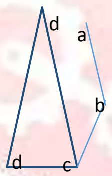

Fig 5.12 $v_f(0,1)=(3,2)$, $e_f(0,1)=(3,2)$ Tail($C_3$,$P_3$)

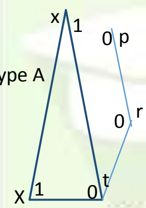

Fig 5.13 $v_f(0,1)=(3,2)$, $e_f(0,1)=(3,2)$

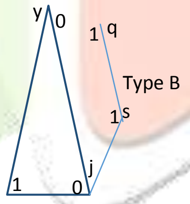

Fig 5.14 $v_f(0,1)=(2,3)$, $e_f(0,1)=(2,3)$

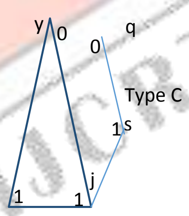

Fig 5.15 $v_f(0,1)=(2,3)$, $e_f(0,1)=(2,3)$

To obtain path union on G we start with a path $P_m$ and m copies of G .A particular vertex on G is fused with vertex of path $P_m$= ($v_1$, $v_2$,.. $v_m$). Depending on if we use vertex d, a,b,c on G ,see fig 5.12 ,we get structure 1,structure 2, structure 3 and structure 4 respectively. Define a function f:V(G) →{0,1} as follows. Under f we define three types of labelings type A, type B and Type C ,see fig 5.13, 5.14, 5.15 above. All are cordial copies and differ in edge label numbers or certain vertex labels. For structure 1 we fuse Type A at vertex ' x' on it with vertex $v_i$ of $P_m$ when i≡1,4(mod 4) and type B at vertex 'y' on it when i≡2,3 (mod 4). For structure 2 we fuse Type A at vertex ' t' on it with vertex $v_i$ of $P_m$ when i≡2,3(mod 4) and type B at vertex 'j' on it when i≡1,0 (mod 4). For structure 3 we fuse Type A at vertex ' r' on it with vertex $v_i$ of $P_m$ when i≡2,3(mod 4) and type B at vertex 's' on it when i≡1,0 (mod 4). For structure 4 we fuse Type A at vertex ' p' on it with vertex $v_i$ of $P_m$ when i≡2,3(mod 4) and type B at vertex 'q' on it when i≡1,0 (mod 4). For all structures the number distribution is as follows : For m = 2x , x= 1,2,..we have $v_f(0,1)=(5x,5x)$ and $e_f(0,1) = (12x +5,12x+6)$ when m≡2(mod 4) given by m = 4x+2 and when m=4x we have $e_f(0,1)=(12x-1,12x)$. When m≡1(mod 4) write m= 4x+1 ,x=0,1,2..we have $v_f(0,1)=(10x+3,10x+2)$ and $e_f(0,1) = (3+12x,2+12x)$When m≡3(mod 4) write m= 4x+3 ,x=0,1,2..we have $v_f(0,1)=(10x+7,10x+8)$ and $e_f(0,1) = (8+12x,9+12x)$. It follows that the family of graph is cordial. #

Conclusions: We have discussed path-union of certain graphs and have shown that they are cordial. Doing so we have considered all possible structures on path-union and have shown that all of them are cordial. This is also called as invariance under cordiallabeling.

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