Formulation of Mathematical Model for Resistive Torque in Human Powered Energized Fertilizer Mixer using Dimensional Analysis and Multiple Regressions

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Abstract : **In the light of heavy demand of electricity, consistently developing environmental pollution and substantial demand of power, the country with monstrous population living in towns having scarcity of power have driven the world to think genuinely for research in the areas which focuses and utilizes the human power as a source of energy. A couple of authors of this paper have officially discovered a pedal worked human powered flywheel motor (HPFM) as a vitality hotspot for applications. In this paper an endeavor is made to create and tentatively validate human powered nursery fertilizer mixing machine to mix nursery compost in proper degree which is then used as a fertilizer in small size farming. This paper reports the development of model through dimensional and regression analysis for response variable resistive torque in the phenomena of mixing of various ingredients of fertilizers. Dimensional analysis is utilized to make the dependent and independent variables dimensionless to get dimensionless equation. Afterward, by applying multiple regression analysis to this dimensionless equation, index values are obtained. The mathematical model for resistive torque is then formulated using these obtained index values.**

Index Terms **- Nursery Fertilizer Mixer, Human Powered Flywheel Motor (HPFM), Process unit, Energy unit, Experimental data based Model, Mathematical model, Dimensional Analysis, Buckingham's π theorem, Regression analysis, Sensitivity**

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1. Introduction

A nursery is a place where in small plants are grown and created to a usable length for transplanting, for use as stocks for budding and grafting, or for sale. Fertilizers are the nutrient, combined in the soils or carried out to plant tissues as an approach to supply plant supplement require for the advancement of the plant. The three key minerals plant should get from the soil are nitrogen, phosphorus and potassium. Flowers accumulate carbon, hydrogen, and oxygen from air and water. The remainder of the additives is taken from the correct combination of soil, cow dung and sand [10].

Different strategies are there for consolidating the media together. Utilization of scoop for the intention is normal for physically mixing the media together, notwithstanding, it's extremely laborites activity. A decent framework for mixing ingredients together is utilizing a mechanized mixer that required electric energy to run it, yet its extreme cost and lack or non availability of power in the interior part of the nation is the huge obstructions in utilizing the motorized mixer by the poor farmer of the country. Subsequently, it felt crucial that there is a critical need to develop the structure to make utilization of Human Powered stimulated Flywheel Motor as an option for an alternative source of energy for different farming applications which is also environment friendly.The present research paper report development of a human powered Nursery fertilizer mixing machine to mix fertilizer's different ingredients viz sand , soil , compost and water in require proportion and formulation of experimental work executed for establishing mathematical model in mixing phenomena for the response variable resistive torque using methodology of experimentation proposed by Hilbert Schank Jr. [7]

2. Working of Nursery Fertilizer Mixer energized by HPFM

Human Power empowered nursery fertilizer mixer is a machinet that blends different elements of compost like cow-dung, soil, sand and water in proper extent. This mix is then used as fertilizer for cultivating. A machine has been made which will play out this mixing activity not by electric power but using human power as a source of energy. Human power stimulated machine includes three sub structures specifically (I) Energy unit which contain a flywheel and speed accelerating arrangement (ii) torque enhancement gear pair and (iii) a process unit which is a real machine. Because of utilization of human source, the system devices may face energy fluctuation all through its deliver. To minimize this upward thrust and fall effect of the energy, the concept of HPFM is introduced in the course of its operation

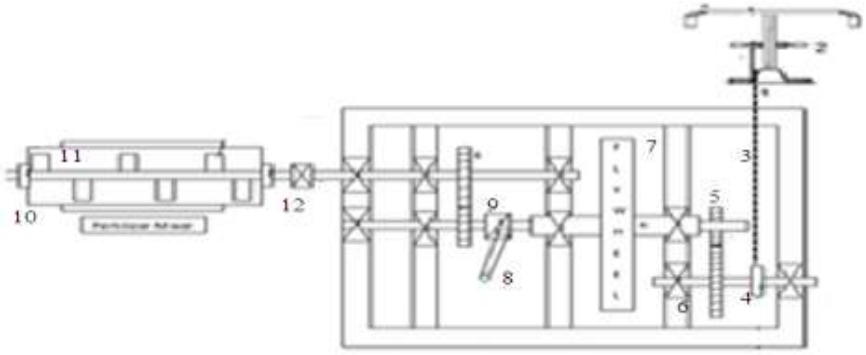

Figure 2.1: Schematic of Nursery Fertilizer Mixing by HPFM
(1 - Seat, 2 - Pedal, 3 - Chain , 4- Freewheel, 5 –Speed rising gear pair , 6-Bearing ,7 - Flywheel, 8 – Clutch, 9-coupling,10-mixing drum, 11 –Mixing blade, 12 –Torque amplification Gear)

The various ingredients of the compost viz Soil, Sand, Cow dung and water in required extent by weight is conceded in the mixer drum the through the opening provided. Then operator put energy in flywheel by pedalling the bicycle mechanism. At the point when flywheel achieves wanted speed, accelerating is halted and it is associated with the process unit through torque intensification gear pair by engaging clutch and the energy put in flywheel is made available at the expected rate to mixer for mixing ingredients. The fabricated unit of human powered stimulated Nursery Fertilizer Mixer is appeared in the figure1.1above.

3. Design of Experimentation

While performing experiments, one needs to manage factors, variables or components associated with the phenomena. We intentionally transform at least one process variable (factors) to watch the impact the progressions have on at least one reaction variable that are probably going to change. The main intention of the experimental investigation is to figure out which factors is having high effect on the reaction finding where to set the most persuasive controllable factors with the goal that the reaction is quite often close to the desired optimal value to have the variability in the response small.

As expressed before that in the phenomenon of mixing empowered by HPFM is very complex hence it is exceptionally hard to reproduce mechanics of such a transient mixing phenomena based on logic. Just option left for such complex phenomena is to formulate experimental data based model. Subsequently it is proposed to detail such model in this examination utilizing the approach of experimentation recommended by Hilbert Schenk Jr. [10] for forecasting behaviour of this complex phenomenon.

The Design of Experimentation includes following steps

a) Identifying the independent /dependent variables which influence the phenomenon and setting up the dimensional equations for the phenomena
b) Using dimensional analysis technique , lessening number of factors and henceforth brings about diminishment of number of dimensional equations which are the focused form of mathematical models,
c) Test planning comprising of choosing test envelope, test sequence and plan of experimentation for the set of deduced dimensional equations.
d) Evolving physical design of experimental set up in setting up the test points.
e) Execution of proposed experimental plan.
f) Arrangement for important instrumentation for finding relation of dependent pi terms of the dimensional equation in terms of independent pi terms.

3.1. Identification of Variables

In the present work of human controlled Nursery Fertilizer Mixer, different recognized independent /dependent variables in mixing phenomena are recorded in the table 3.1 beneath

Table 3.1: Variables involved in the phenomena of mixing of nursery fertilizers energized by HPFM.

SN	Variable	Nature (Independent / Dependent)	Nomenclature	M.L.T.
01	Quantity of Sand	Independent	Ws	M
02	Quantity of Soil	Independent	Wso	M
03	Quantity of cow-dung	Independent	Wcd	M
04	Quantity of water	Independent	Ww	M
05	Quantity of Mixture	Independent	W	M
06	Diameter of Blade tip	Independent	Dt	L
07	Blade Pitch	Independent	P	L

08	Length of mixing drum	Independent	L	L
09	Shaft diameter	Independent	d	L
10	Diameter of Drum	Independent	D	L
11	Input energy to the machine	Independent	E	ML^2T^{-2}
12	Acc. due to gravity	Independent	g	LT^{-2}
13	Torque amplification Gear ratio	Independent	G	-
14	Instantaneous Torque on shaft	Dependent	Tr	ML^2T^{-2}
15	Time of Mixing	Dependent	Tp	T

3.2 Dimensional Analysis

Dimensional analysis is an valuable numerical system utilized in reducing variables by forming non-dimensional groups of the variables which are called as pi (π) terms. Deducing the dimensional equation for a phenomenon, diminishes number of independent variables pi terms in the experiment keeping in mind the end goal to influence the entire experimentation to process less time taking having generation of optimum data. Buckingham π Theorem is utilized for dimensional analysis.

It is seen from table 5.1 that there are add up to fifteen factors influencing the phenomenon mixing. The essential physical measurements to express all these fifteen factors are just three i.e. Mass (M), Length (L) and Time (T). Out of these aggregate fifteen factors the initial thirteen factors are the independent variables and last two variables are dependant/ response variables.

3.2.1 Formation of pi (π) terms for all Dependant and Independent variables affecting the phenomenon

The Buckingham's π Theorem is used for the dimensional analysis of proposed machine. The process of dimensional analysis using Buckingham's Π- Theorem ,is followed step by step as explained below.

As seen from the table , total number of Variable indentified in the phenomena of Nursery fertilizer mixing are 15

Total number of Variable = 15

Number of dependent variable = 02

Number of independent variables, n = 13

Hence we have equation

$Tr = f(Ws, Wcd, Wso, Ww, W, Dt, P, L, d, D, E, g, G)$

Or

$f_1 = f(Ws, Wcd, Wso, Ww, W, Dt, P, L, d, D, E, g, G)$

Considering W , g , D as the repeating variables i.e. m = 3

So as per Buckingham's Π- Theorem ,

No. of Π terms = n – m = 13 - 3 = 10

Hence in the phenomena of mixing here , there will be 10 pi terms

$\Pi_{D1} = f_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \Pi_7, \Pi_8, \Pi_9, \Pi_{10}) = 0$

First Π term:

$\Pi_1 = (W)^{a1}(g)^{b1}(D)^{c1}W_{cd}$

$(M)^0(L)^0(T)^0 = (M)^{a1}(LT^{-2})^{b1}(L)^{c1}M$

The values of a_1, b_1 and c_1 are computed by equating the powers of M, L and T on both sides as given below

For 'M'	For 'L'	For 'T'
$M \rightarrow 0 = a_1 + 1$ $a_1 = -1$	$L \rightarrow 0 = b_1 + c_1$ (From eq. of T, subst. $b_1 = 0$) $0 = 0 + c_1$,Hence $c_1 = 0$	$T \rightarrow 0 = -2b_1$ $0 = -2b_1$, Hence $b_1 = 0$

Substituting the values of a_1, b_1 and c_1 in the eq. of Π_1 term, we have:

$\Pi_1 = (W)^{a1}(g)^{b1}(D)^{c1}W_{cd}$

$\Pi_1 = (W)^{-1}(g)^0(D)^0W_{cd}$

$\Pi_1 = \frac{W_{cd}}{W}$

$\Pi_1 = W_{cd} / W$

In the similar way, all remaining pi (π) terms for independent variables are calculated

Table 3.2: Pi terms for independent variables

pi terms	pi terms equations
π_1	W_s / W
π_2	W_{cd} / W
π_3	W_w /W
π_4	W_{so} / W
π_5	P / D
π_6	L / D
π_7	d / D
π_8	Dt/ D
π_9	E / WgD
π_{10}	W

3.2.2 Formulation of Pi terms for Dependent Variables

In the similar way, the dimensional analysis for dependent variables Resistive Torque **Tr (Π_{01})** is performed by applying again Buckingham's Π- Theorem.

$\Pi_{01} = (W)^{a1}(g)^{b1}(D)^{c1}T_r$

$(M)^0(L)^0(T)^0 = (M)^{a1}(LT^{-2})^{b1}(L)^{c1}(M)(L)^2(T)^{-2}$

The values of a_1, b_1 and c_1 are computed by equating the powers of M, L and T on both sides as given below

For 'M'	**For 'L'**	**For 'T'**
M→0 = a_2+1 **a_1= -1**	L→0 =b_1+ c_1+2 (From eq. of T, subst. b_1= **-1)** 0=-1+ c_1 +2 Hence **c_1 = -1**	T→ 0 = -2b_1 -2 Hence **b_1 = -1**

Substituting the values of a_1, b_1 and c_1 in the eq. of Π_{01} term, we have:

$\Pi_{01} = (W)^{-1}(g)^{-1}(D)^{-1}T_r$

$$\Pi_{01} = \frac{Tr}{WgD}$$

3.3 Reduction of Variables

Whenever no. of factors (n) is large in the phenomena then much diminishment in number of factors isn't accomplished even by applying Buckingham's π hypothesis. It is obvious that, in the event that we take the product of the π terms, it will likewise be dimensionless number and consequently a π term.This property is used to achieve further reduction of the number of variables. Thus few π terms are formed by logically taking the product of few other π terms and final mathematical equations derived .After applying this techniques of reduction , total ten pi terms of independent variables are reduced to four new pi terms . The following table shows the new pi terms of independent variables in reduced form

Table 3.3 Reduced Pi terms for Independent variables

pi terms	pi terms equations	Description
$\boldsymbol{\pi_1}$	$\left(\frac{W_s.W_{so}W_{cd}W_w}{W^4}\right)$	The term related to ingredients of Nursery Fertilizers
$\boldsymbol{\pi_2}$	$\left(\frac{D_t.P.L.d}{D^4}\right)$	The term related to Geometrical parameters of process unit
$\boldsymbol{\pi_3}$	$\left(\frac{E}{W.g.D}\right)$	The term related to energy of flywheel
$\boldsymbol{\pi_4}$	(G)	The term related to Gear Ratio

3.4 Dimensional equation for Response variable Resistive Torque (Tr)

Four independent pi terms (i.e. $\pi_1, \pi_2, \pi_3, \pi_4$) and dependent pi terms (π_{01}) have been identified and are available for the formulation of dimensional equation for response variable. Each dependent π term is assumed to be function of the available independent π terms

$$\Pi_{01} = f_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4)$$

Dimensional equation for Response variable, Resistive torque(Tr) in terms of all other independent variables can be expressed as:

$$\Pi_{01} = f_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4)$$

$$\pi_{01} = f_2\left\{\left(\frac{W_s.W_{so}.W_{cd}.W_w}{W^4}\right)\left(\frac{D_t.P.L.d}{D^4}\right)\left(\frac{E}{W.g.D}\right)(G)\right\}$$

$$\pi_{01} = \left(\frac{T_r}{W.g.D}\right) = f_2\left\{\left(\frac{W_s.W_{so}.W_{cd}.W_w}{W^4}\right)\left(\frac{D_t.P.L.d}{D^4}\right)\left(\frac{E}{W.g.D}\right)(G)\right\} \text{----------(1)}$$

3.5. Developing the model for dependent pi term ,Resistive torque , Tr, (Π_{01})

Generalized experimental models for predicting processing time for fertilizer mixing process by human powered flywheel motor has been established as

$$\Pi_{01} = k1 \times (\Pi_1)a1 \times (\Pi_2)b1 \times (\Pi_3)c1 \times (\Pi_4)d1$$

The values of exponential a1, b1, c1 and d1 are established, considering exponential relationship between dependent pi term Tr and Independent Π terms $\Pi_1, \Pi_2, \Pi_3, \Pi_4$ independently taken one at a time, on the basic of data collected through classical experimentation. There are five unknown terms in the above equation. These are curve fitting constant K1 and indices a1, b1, c1, d1. To get the values of these unknown we need minimum five sets of values of ($\Pi_1, \Pi_2, \Pi_3, \Pi_4$).

Taking log on the both sides of equation for Π_{D1}, to get four unknown terms in the equations,

$$\text{Log } \Pi_{D1} = \log k1 + a1 \log \Pi_1 + b1 \log \Pi_2 + c1 \log \Pi_3 + d1 \log \Pi_4 \text{ ----------(3.1)}$$

Let, $Z1 = \log \Pi_{D1}$, $K1 = \log k1$, $A = \log \Pi_1$, $B = \log \Pi_2$, $C = \log \Pi_3$, $D = \log \Pi_4$

Putting the values in equations (1)

Hence same can be written as

$$Z1 = K1 + a1\,A + b1\,B + c1\,C + d1\,D \text{ ----------(3.2)}$$

Equation 3.2 is a regression equation of Z on A, B, C, D in n dimensional co-ordinate system. This represents a regression hyper plane [12]. To determine the regression hyper plane, determine a_1, b_1, c_1, d_1 in equation 3.2 so that,

$$\Sigma Z_1 = nK_1 + a_1*\Sigma A + b_1*\Sigma B + c_1*\Sigma C + d_1*\Sigma D$$
$$\Sigma Z_1*A = K_1*\Sigma A + a_1*\Sigma A*A + b_1*\Sigma B*A + c_1*\Sigma C*A + d_1*\Sigma D*A$$
$$\Sigma Z_1*B = K_1*\Sigma B + a_1*\Sigma A*B + b_1*\Sigma B*B + c_1*\Sigma C*B + d_1*\Sigma D*B$$
$$\Sigma Z_1*C = K_1*\Sigma C + a_1*\Sigma A*C + b_1*\Sigma B*C + c_1*\Sigma C*C + d_1*\Sigma D*C$$
$$\Sigma Z_1*D = K_1*\Sigma D + a_1*\Sigma A*D + b_1*\Sigma B*D + c_1*\Sigma C*D + d_1*\Sigma D*D \text{ ----------(3.3)}$$

In the above set of equations the values of the multipliers of K_1, a_1, b_1, c_1, d_1 are substituted to compute the values of the unknowns (viz. K_1, a_1, b_1, c_1, d_1). The values of the terms on L H S and the multipliers of K1, a_1, b_1, $c_1$andd_1 in the set of equations are calculated. After substituting these values in the equations 7.5 ,one will get a set of 5 equations, which are to be solved simultaneously to get the values of K_1, a_1, b_1, c_1, d_1. The above equations can be verified in the matrix form and further values of K_1, a_1, b_1, c_1, d_1 can be obtained by using matrix analysis.

$$\mathbf{X_1 = inv\ (W) \times P_1} \text{ ----------(3.4)}$$

The matrix method of solving these equations using 'MATLAB' is given below.

W = 5 x 5 matrix of the multipliers of K_1, a_1, b_1, c_1, and d_1

P_1 = 5 x 1 matrix of the terms on L H S and

X_1 = 5 x 1 matrix of solutions of values of K_1, a_1, b_1, c_1, and d_1

Then, the matrix obtained is given by,

3.4.1 Model of dependent pi term Resistive torque, T_r:

$$Z_1 \times \begin{bmatrix} 1 \\ A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} n & A & B & C & D \\ A & AA & BA & CA & DA \\ B & AB & BB & CB & DB \\ C & AC & BC & CC & DC \\ D & AD & BD & CD & DD \end{bmatrix} \times \begin{bmatrix} K_1 \\ a \\ b \\ c \\ d \end{bmatrix}$$

$$\mathbf{P_1 = W_1 \times X_1}$$

$$\begin{vmatrix} -705.891 \\ 2250.964 \\ 939.1334 \\ 890.9566 \\ 321.5236 \end{vmatrix} = \begin{vmatrix} 216 & -688.771 & -287.338 & -272.397 & -99.6895 \\ -688.771 & 2199.882 & 916.2497 & 868.6063 & 317.8854 \\ -287.338 & 916.2497 & 383.2178 & 362.3602 & 132.6136 \\ -272.397 & 868.6063 & 362.3602 & 354.4323 & 125.7179 \\ -99.6895 & 317.8854 & 132.6136 & 125.7179 & 49.31454 \end{vmatrix} \times \begin{vmatrix} K \\ a1 \\ b1 \\ c1 \\ d1 \end{vmatrix}$$

$[P_1] = [W_1]\,[X_1]$

Using Mat lab, $X_1 = W_1 \backslash P_1$, after solving X_1 matrix with K_1 and indices a_1, b_1, c_1, d_1 are as follows

K_1	-3.5796
a_1	0.0144
b_1	0.1125
c_1	0.0697
d_1	-1.2897

But K_1 is log value so to convert into normal value taking antilog of K_1

Antilog (**-3.5796**) = 0.000263

Hence the model for dependent term π_{01}

$\pi_{01} = k_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1}$

$$\pi_{01} = \left(\frac{T_{r_{Avg}}}{W.g.D}\right) = K_2 \left\{\left(\frac{W_s.W_{so}.W_{cd}.W_w}{W^4}\right)^{a2} \left(\frac{D_t.P.L.d}{D^4}\right)^{b2} \left(\frac{E}{W.g.D}\right)^{c2} (G)^{d2}\right\}$$

$$\pi_{01} = \left(\frac{T_r}{W.g.D}\right) = 0.000263 \left\{\begin{matrix}\left(\frac{W_s.W_{so}.W_{cd}.W_w}{W^4}\right)^{0.0144} \left(\frac{D_t.P.L.d}{D^4}\right)^{0.1125} \\ \left(\frac{E}{W.g.D}\right)^{0.0697} (G)^{-1.2897}\end{matrix}\right\}$$

$$T_r = 0.000263.(W.g.D) \left\{\begin{matrix}\left(\frac{W_s.W_{so}.W_{cd}.W_w}{W^4}\right)^{0.0144} \left(\frac{D_t.P.L.d}{D^4}\right)^{0.1125} \\ \left(\frac{E}{W.g.D}\right)^{0.0697} (G)^{-1.2897}\end{matrix}\right\}$$

4 Conclusions

In this work, an approach for mathematical model of Process time is talked about. This paper deals with dimensional analysis and multiple regression analysis. It is discovered that there are four groups on which the Process time depends out of which some ordinary gathering have the predominant part in choosing the process time. From condition 4, it is observed that third group ($\pi 3$) assumes critical part in the assessment of the Process time, as the power of the third group is biggest. The generated design data is utilized for the mathematical model formulation for the Nursery Fertilizer mixer invigorated by Human Power Flywheel Motor. This work is completed with specific assumptions. In the event that the information go is other than the range which is utilized in mathematical model formation, then, the curve for the dependent variable deviates for the other data range. This is occurring on the grounds that the model is produced utilizing multiple linear regression analysis. In this manner, exhibited demonstrate is reasonable for the scope of the recognized limits. This paper gives the new heading of work for the researcher to optimize the various responses by generating design data which then be used for the formulation of mathematical model. In future, distinctive scientific model can be defined, such as, productivity model, system efficiency, resistive torque, quality of mixture etc. .Exhibited model can be assessed by utilizing the artificial neural system also. For this, one has to go for the formation neural network based on the generated data for the typical model, here for instance, process time. Reliability of the presented model can be assessed in view of the assessment of the real values of the group terms and the evaluated values of the group terms using the proposed mathematical model. Correlation of these two values can be evaluated, based on which the reliability of the presented model can be predicted.

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