Complete Diallel Crosses plan from a BIBD with series v= b = 4λ+3, r = k = 2λ+1, λ

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***Abstract:* In this paper we have developed a simple method for the construction of Complete Diallel Crosses(CDC) plan through BIBD with parameter v= b = 4λ+3, r = k = 2λ+1, λ. Analysis of plans is carried out. We have also obtained the efficiency factor of complete diallel crosses plans comparing randomized block design and found that as large value of *v*(line) is increasing thenthe efficiency factor tends to one. Method of construction and analysis of CDC plan is further supported by suitable example.**

***Keywords* – BIBD, GCA effect, CDC plan.**

I. INTRODUCTION

Diallel crossing is a very useful technique for conducting plant and animal breeding experiments, especially for estimating the combining ability effects of lines. Diallel crosses in which all the possible distinct crosses in pairs among the available lines are taken is called complete diallel crosses.

The concept of diallel crosses plan was introduced by Kempthrone (1956), Gilbert (1958), Hinkelmann and Stern (1960), Kempthrone and Curnow (1961), Agarwal and Das (1990), Divecha and Ghosh (1994) and Dey and Midha (1996) they have discussed several method of construction of complete diallel crosses plan.

Balanced Incomplete Block Design introduced by Yates (1936a) and developed by Fisher and Yates (1938), Bose (1939) which were extensively tabled in Fisher and Yates (1963) and Cochran and Cox(1957). All of which are developed for use in agricultural and biological experiments. The concept of diallel crosses plan was introduced by Kempthrone (1956), Gilbert (1958), Hinkelmann and Stern (1960) and Kempthrone and Curnow (1961). Ghosh and Divecha (1997) and Ghosh and Biswas (2001).

In this research we have developed a method to have the crosses column wise. Here we have considered a BIBD having parameter v = b = 4λ+3, r = k = 2λ+1, λ. Next developed a series of CDC plan by using this series BIBD. We also carried out its analysis.

To Construct CDC Plan, we take first line present in the first column and cross it with the first line of the same column present in second row. Again take the second line of the second column of first block and cross it with the second line of second block of the same column. Continue this process for all *k* lines of the first block and second blocks. In this way we get k crosses kept them in one block. Repeat this procedure of crossing for *k* line of 1^{st}block with 2^{nd}, 3^{rd}, ..., b^{th} block. Similarly taking crosses of *k* lines of second block with the lines of 3^{rd},4^{th},... , b^{th} blocks. Continue this process for all the possible blocks. In section 2 we have developed method of Construction of CDC plan. In section 3 we have developed method of analysis of CDC plan where crosses are taken row-wise. This analysis is further supported by an example and we obtained efficiency factor in section 4.

II. METHOD OF CONSTRUCTION

We can construct CDC Plan using a BIBD with parameter v = b = 4λ+3, r = k = 2λ+1, λ. This BIBD has *k* columns and *v* rows. Consider first column. Take first line of the first column present in the first row and cross this line with the remaining line present in the (b-1) = (v-1) rows for the same column. This way we get (v-1) crosses. Next, consider second row for the same column and cross this line with remaining (b-2) lines in the same column. Repeat this process for all the rows. Under this process of crosses we get (v-1) + (v-2) +…..+ 1 crosses for one column that is number of crosses for first column is equal to $\frac{(v-1)(v-1+1)}{2} = \frac{v(v-1)}{2}$ $= \frac{b(b-1)}{2}$.

Since we have *k* columns and hence we repeat the same process for all the *k* columns that is, crossing with the lines present in the *v* rows. In this process we get $\frac{b(b-1)}{2}$. *k* crosses. Since for this series of BIBD, b = v and hence total number of crosses under this method are $\frac{vk(v-1)}{2}$. The method of construction of CDC plan using the given series of parameter of BIBD is shown in Theorem 1.

Theorem 1: Complete diallel crosses plan can always be constructed using the BIBD with parameters v = b = 4λ+3, r = k = 2λ+1, λ by taking crosses column wise with v_1= v lines arranged in $b_1 = \frac{b(b-1)}{2} = \frac{v(v-1)}{2}$ blocks. Each block having *k* crosses and with total number of crosses as $\frac{vk(v-1)}{2}$.

Proof: Consider a BIBD with parameter v = b = 4λ+3, r = k = 2λ+1, λ. Since BIBD is symmetric and hence v = b. Consider first column. Take first line of the first column present in the first row and cross this line with the remaining line present in the (b-1) = (v-1) rows for the same column. This way we get (v-1) crosses. Next, consider second row for the same column and cross this line with remaining (b-2) lines in the same column. Repeat this process for all the rows. Under this process of crosses we get (v-1) + (v-2) +…..+ 1 crosses for one column that is, number of crosses for first column is equal to $\frac{(v-1)(v-1+1)}{2} = \frac{v(v-1)}{2} = \frac{b(b-1)}{2}$.

Since we have *k* columns and hence we repeat the same process for all the *k*columns that is crossing with the lines present in the *v* rows. In this process we get $\frac{b(b-1)}{2}$ .kcrosses. Since for this series of BIBD, b = v and hence total number of crosses under this method are $\frac{vk(v-1)}{2}$.It is verified that one line is crossed with those lines available in remaining (b-1) blocks.

So that line occurs (b-1) times in that column. Again each line occur in *r* blocks, So each line is repeated r(b-1) = r(v-1) times. In each column each line occurs only once that is, number of distinct lines occurs once. Further each line is repeated *r* times, so each cross occurs r times.

Since, Let N be the incidence matrix of CDC Plan. The integer 0,1,2 are the element of the incidence matrix N. Next we obtain the frequency of 0,1,2 occurring in the given incidence matrix. This is discussed in section 2.1, 2.2 and 2.3.

2.1 Frequency of zero in incidence matrix

Since frequency of zero occur in those blockswhere self-line does not occur, that is, (i x i) is absent. Since we have *b* blocks, out of which *r* blocks contain same line and hence (b-r) blocks does not contain that line. Now we are making a cross in two blocks, so possible number of blocks are $\binom{b-r}{2}$. Thus zero occurs $\binom{b-r}{2}$ times.

2.2 Frequency of one occur in incidence matrix

We have total number of *b* blocks. After crossing the lines of one block with the lines of each other block, the possible number of blocks in diallel crosses plan is $\binom{b}{2}$. Since two occurs $\frac{r(r-1)}{2}$ times and zero occur $\binom{b-r}{2}$ times. So one occurs $\binom{b}{2} - \frac{r(r-1)}{2} - \binom{b-r}{2}$ times. Or,

$$= \frac{b(b-1)}{2} - \frac{r(r-1)}{2} - \frac{(b-r)(b-r-1)}{2} = r(b-r)$$

2.3 Frequency of two in incidence matrix

Since each line occurs in *r* blocks and we are taking cross of lines of first block with the lines of remaining (b – 1) blocks corresponding to any blocks in remaining (r – 1) blocks where the same lines occurs. Hence frequency '2 ' occurs (r – 1)+(r – 2)+. . . .+(r – (r – 1)) times. Or,

= r(r – 1) – (1+2+. . . +(r – 1))

$$= r\,(r-1) - \frac{r(r-1)}{2} = \frac{r^2 - r}{2} = \frac{r(r-1)}{2}$$

That is, element 2 is occurs $\frac{r(r-1)}{2}$ times for each line in the incidence matrix N.

Now, Incidence matrix N can be define as,

N=

v/b	1	2	. . .	b
1	0	2	. . .	1
2	1	0	. . .	2
.	.	.	. . .	.
V	2	1	. . .	0

Next, the concurrence matrix NN′ can be obtain as NN′

$$NN' = \begin{bmatrix} x & y & \cdots & . & y \\ y & x & \cdots & . & y \\ : & : & \vdots & : & : \\ . & . & \ddots & . & . \\ y & y & \ddots & . & x \end{bmatrix}$$

Where x and y are diagonal and off diagonal elements of an incidence matrix.

Now, the value of x and y are obtained as following.

2.4 The Diagonal elements of NN′ matrix

The Diagonal elements of NN′ matrix is expressed as following.

Since frequency two occurs $\frac{r(r-1)}{2}$ times, one occurs r(b – r) times and zero occurs $\binom{b-r}{2}$ times in the incidence matrix N. So the diagonal element of NN′ is,

$$\frac{r(r-1)}{2}(2)^2 + r(b-r)(1)^2 + \binom{b-r}{2}(0)^2 = r\,(b + r - 2)$$

So r (b + r – 2) is a diagonal elements in NN′ matrix.

2.5 The Off- diagonal elements of NN′ matrix

The Off - diagonal elements of NN′ matrix is expressed as following.

Since in the NN′ matrix, product of (2x2) occur $\binom{\lambda}{2}$ times, products of (1x2) and (2x1) occurs λ(r – λ) times and product of (1x1) occur r(r – λ) times, hence the sum and products of any two columns of N matrix is, given by

$$\binom{\lambda}{2}(2\times 2) + \lambda(r-\lambda)(2\times 1) + \lambda(r-\lambda)(1\times 2) + r(r-\lambda)(1\times 1) = \binom{\lambda}{2}(4) + (r-\lambda)\,(4\lambda + r)$$

This is the value of off diagonal elements.
Hence, the resulting plan is complete diallel crosses with
$v_1 = v$, $b_1 = \binom{b}{2} = \binom{v}{2}$, $r_1 = r(b-1) = r(v-1)$, $k_1 = k$ where each pair is repeated *r* times. This proves the theorem.
To support this result we provide an example given below.
Example 2.1 Consider a BIBD with parameter v = 7, b = 7, r = 3, k = 3, λ = 1.
Blocks of the existing BIBD are
1 234567
2 3 45671
4 5 67123
Now, taking crosses row-wise, we have 63 crosses which are shown in Table 2.1

Table 2.1

B_1 :	1x2	2x3	4x5
B_2 :	1x3	2x4	4x6
B_3 :	1x4	2x5	4x7
B_4 :	1x5	2x6	1x4
B_5 :	1x6	2x7	2x4
B_6 :	1x7	1x2	3x4
B_7 :	2x3	3x4	5x6
B_8 :	2x4	3x5	5x7
B_9 :	2x5	3x6	1x5
B_{10} :	2x6	3x7	2x5
B_{11} :	2x7	1x3	3x5

B_{12} :	3x4	4x5	6x7
B_{13} :	3x5	4x6	1x6
B_{14} :	3x6	4x7	2x6
B_{15} :	3x7	1x4	3x6
B_{16} :	4x5	5x6	1x7
B_{17} :	4x6	5x7	2x7
B_{18} :	4x7	1x5	3x7
B_{19} :	5x6	6x7	1x2
B_{20} :	5x7	1x6	1x3
B_{21} :	6x7	1x7	2x3

Hence, the total numbers of crosses are = $\frac{vk(v-1)}{2} = 63$

Each line occurs $r(v-1) = 18$ times
The incidence matrix is shown in Table 2.2

Table 2.2

v/b	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1	1	1	2	1	2	0	0	1	0	1	0	1	0	1	1	0	1	1	2	1
2	2	1	1	1	2	1	1	1	1	2	1	0	0	1	0	0	1	0	1	0	1
3	1	1	0	0	0	1	2	1	1	1	2	1	1	1	2	0	0	1	0	1	1
4	1	2	2	1	1	1	1	1	0	0	0	2	1	1	1	1	1	1	0	0	0
5	1	0	1	1	0	0	1	2	2	1	1	1	1	0	0	2	1	1	1	1	0
6	0	1	0	1	1	0	1	0	1	1	0	1	2	2	1	1	1	0	2	1	1
7	0	0	1	0	1	1	0	1	0	1	1	1	0	1	1	1	2	2	1	1	2

Each cross occurs r times that is, $r = 3$. Diagonal elements $= r(b + r - 2) = 3(7 + 3 - 2) = 24$. Similarly off diagonal elements are $4\binom{\lambda}{2} + (r - \lambda)(4\lambda + r) = 4 \times 0 + 2 \times (4 + 3) = 14$ where $\binom{\lambda}{2} = 0$.
The concurrence matrix is shown in Table 2.3

Table 2.3

$$NN' = \begin{pmatrix} 24 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 24 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 24 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 24 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 24 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 24 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 24 \end{pmatrix}$$

III. ANALYSIS OF COMPLETE DIALLEL PLAN

Consider the CDC Plan discussed in theorem 2.1 the concurrence matrix of CDC Plan is given by

$$NN' = \begin{bmatrix} x & y & .. & . & y \\ y & x & .. & . & y \\ : & : & :. & : & : \\ . & . & .. & . & . \\ y & y & .. & . & x \end{bmatrix} \quad (3.1)$$

Where, $x = r(b+r-2)$, $y = \binom{\lambda}{2}(4) + (r - \lambda)(4\lambda + r)$

The information matrix C can be defined as,

$$C = G_d - NK^{-1}N' \quad (3.2)$$

For this CDC Plan, block size is same, that is $k_1 = k_2 = \ldots = k_b = k$. So (3.2) is reduced as,

$$C = G_d - \frac{NN'}{K} \quad (3.3)$$

Here, C is regarded as the coefficient matrix of the reduced normal equation for estimating the linear function of general combining ability (GCA) effects, G_d is the matrix with diagonal elements as replication number of lines and off diagonal elements as replication number of crosses, N is the incidence matrix of lines versus block *k* is the diagonal matrix with elements as block sizes.

That is, G_d is defined as

$$G_d = \begin{bmatrix} w_{di} & g_{ii'} \\ g_{ii} & w_{di'} \end{bmatrix} \quad (3.4)$$

Where, w_{di} denotes the diagonal elements as replication number of lines. $g_{ii'}$ denotes the off diagonal elements as replication number of crosses.

Since each line occurs r(b – 1) times and each cross occurs *r* times and hence using (3.4) we have G_d as,

$$G_d = \begin{bmatrix} r(b-1) & r & \cdots\cdots & r \\ r & r(b-1) & \cdots\cdots & r \\ r & r & \cdots\cdots & r \\ \vdots & \vdots & \vdots\,\vdots & \vdots \\ r & r & \cdots\cdots & r(b-1) \end{bmatrix}$$

Now, using (3.1), (3.2), (3.3) and (3.4), coefficient matrix is expressed as

$$C = \begin{pmatrix} r(b-1) & r & \cdots\cdots & r \\ r & r(b-1) & \cdots\cdots & r \\ r & r & \cdots\cdots & r \\ \vdots & \vdots & \vdots\,\vdots & \vdots \\ r & r & \cdots\cdots & r(b-1) \end{pmatrix} - \frac{1}{k}\begin{bmatrix} x & y & \cdot\cdot & \cdot & y \\ y & x & \cdot\cdot & \cdot & y \\ \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ \cdot & \cdot & \ddots & \cdot & \cdot \\ y & y & \ddots & \cdot & x \end{bmatrix} \tag{3.5}$$

Where, x = r (b+r – 2) and y = $\binom{\lambda}{2}(4) + (r-\lambda)(4\lambda + r)$

Next we solve equation (3.5) by taking diagonal and off-diagonal elements separately. Which are following,

3.1 The diagonal elements of C-matrix of CDC plan

The diagonal elements of C-matrix of CDC plan is expressed as

$$r(b-1) - \frac{r(b+r-2)}{k}$$

$$= \frac{rk(b-1) - r(b+r-2)}{k} \tag{3.6}$$

In this series b = v, r = k so (3.6) reduced to

$$\frac{r^2(b-1) - r(r+b-2)}{k} = \frac{r[(r-1)(b-2)]}{k} \tag{3.7}$$

3.2 The Off - diagonal elements of C-matrix of CDC plan

The Off -diagonal elements of C-matrix of CDC plan is expressed as

$$r - \frac{\left[\binom{\lambda}{2}(4) + (r-\lambda)(4\lambda + r)\right]}{k}$$

$$= \frac{rk - \binom{\lambda}{2}(4) - (3\lambda r - \lambda^2 + r^2)}{k} \tag{3.8}$$

For BIBD having series v = b = 4λ+ 3, r = k = 2λ+1 we have r = k, and hence (3.8) becomes,

$$\frac{r^2 - \binom{\lambda}{2}(4) - (3\lambda r - \lambda^2 + r^2)}{k} = \frac{-\lambda[3r - (2(\lambda+1)]}{k} \tag{3.9}$$

Since, the parameter of a BIBD are v = b = 4λ+3 , r = k = 2λ+1 so,

Using (3.8) and (3.9), and the parameters of BIBD, the C – matrix becomes,

$$C = \frac{\begin{bmatrix} 2\lambda(4\lambda+1)(2\lambda+1) & \cdots & -\lambda(4\lambda+1) \\ \vdots & \ddots & \vdots \\ -\lambda(4\lambda+1) & \cdots & 2\lambda(4\lambda+1)(2\lambda+1) \end{bmatrix}}{k}$$

$$= \frac{\begin{bmatrix} 2\lambda(4\lambda+1)(2\lambda+1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 2\lambda(4\lambda+1)(2\lambda+1) \end{bmatrix}}{k} + \frac{\begin{bmatrix} \lambda(4\lambda+1) & \cdots & \lambda(4\lambda+1) \\ \vdots & \ddots & \vdots \\ \lambda(4\lambda+1) & \cdots & \lambda(4\lambda+1) \end{bmatrix}}{k} - \frac{\begin{bmatrix} \lambda(4\lambda+1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda(4\lambda+1) \end{bmatrix}}{k}$$

$$= \frac{\{[2\lambda(2\lambda+1)(4\lambda+1) + \lambda(4\lambda+1)]I_v - \lambda(4\lambda+1)E_{vv}\}}{k}$$

$$= \frac{\{[\lambda(4\lambda+1)(4\lambda+3)]I_v - \lambda(4\lambda+1)E_{vv}\}}{k}$$

$$= \frac{\lambda(4\lambda+1)(4\lambda+3)}{k}\left[I_v - \frac{1}{v}E_{vv}\right]$$

Finally, C – matrix is shown in the form as

$C = \theta\left[I_v - \frac{1}{v}E_{vv}\right]$

Where the Non – zero eigen – value, θ of the C – matrix of the CDC plan is given by

$$\theta = \frac{\lambda(4\lambda+1)(4\lambda+3)}{k}$$

Since the solution of the estimate of GCA line effect is $\frac{1}{\theta}Q_i$,hence,

$$\hat{g}_i = \frac{k}{\lambda(4\lambda+1)(4\lambda+3)}Q_i \tag{3.10}$$

Similarly,

$$\hat{g}_j = \frac{k}{\lambda(4\lambda+1)(4\lambda+3)}Q_j \tag{3.11}$$

Again the solution of the estimate of the variance of line effect $\hat{g}_i$ and $\hat{g}_j$is given by

$V(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2.$

Hence,

$$V(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2 = \frac{2k}{\lambda(4\lambda+1)(4\lambda+3)}\sigma^2 \tag{3.12}$$

Further sum of square of the line effect g_i is $\Sigma g_i\theta_i = \frac{1}{\theta}\sum Q_i^2$

The ANOVA is shown in Table 3.1

Table 3.1

Source of variation	Degree of freedom	Sum of square	Mean sum of square	F – value
Line	v – 1	$\frac{1}{\theta}\sum Q_i^2 = A$	$\frac{A}{v-1} = \sigma_t^2$	$\frac{\sigma_t^2}{\sigma_e^2}$
Block	b – 1	$\frac{\sum B_j^2}{k} - c.f. = B$	$\frac{B}{b-1} = \sigma_b^2$	$\frac{\sigma_b^2}{\sigma_e^2}$
Error	w–b–v–1	By Subtraction B-A= C	$\frac{C}{w-b-v-1} = \sigma_e^2$	
Total	w – 1	$\Sigma\Sigma Y_{ij}^2 - c.f. = D$		

IV. EFFICIENCY FACTOR

If instead of CDC plan, we adopt a randomized complete block design with *r* blocks, each block has *k* crosses so, total number of crosses are $\frac{vk(v-1)}{2}$,then the C – matrix of the randomized block design is obtained as,

$$C_R = r(v-2)(I_v - v^{-1} I_v I_v') \tag{4.1}$$

Hence the variance of the best linear unbiased estimator of any elementary contrast among GCA effects, in case of RBD is obtained as,

$V(\hat{g}_i - \hat{g}_m) = \frac{2}{\theta}\sigma^2 = \frac{2}{r(v-2)}\sigma_r^2$ where $i \neq m = 1,2,3,\dots,v$

Where *v* is number of lines, *r* is number of times each cross occur in CDC plan and σ_r^2 is error variance. Now, in case of CDC plan discussed here variance of best linear unbiased estimator of any elementary line contrast among GCA effect for CDC plan is obtain as,

$V(\hat{g}_i - \hat{g}_j) = \frac{2k}{\lambda v(v-2)}$ (4.2)

Now, efficiency factor of existing CDC plan compare to RBD in *r* replication is defined as,

$$\text{Efficiency (E)} = \frac{V(\hat{g}_i - \hat{g}_m)_{RBD}}{V(\hat{g}_i - \hat{g}_j)_{CDC}} = \frac{\frac{2}{r(v-2)}\sigma_r^{\,2}}{\frac{2k}{\lambda v(v-2)}\sigma^2}$$

$E = \frac{\lambda v(v-2)}{rk(v-2)} = \frac{\lambda v}{rk}$ if $\sigma_r^2 = \sigma^2$ (4.3)

Where σ^2 is error mean square of CDC plan, σ_r^2 is error mean square of RBD and r denotes the number of crosses of a CDC plan.

Hence, efficiency factor of the proposed design relative to RBD under the assumption of equai intra block variance is increasing as *v* increase. Finally for large value of *v*(line), efficiency factor tends to 1.

Table 4.1: Variance and Efficiency factor

v	b	r	k	Λ	$v(g_i - g_j)_{RBD}$	E
7	21	3	3	1	0.1333333	0.7777778
11	55	5	5	2	0.0444444	0.88
19	171	9	9	4	0.0130719	0.9382716

Remark: The efficiency factor of exiting CDC plan compare to randomized block design is same as the efficiency factor of a balanced incomplete block design.

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REFERENCE

[1] Agarwal, S. C. And Das, M. N. (1990). Incomplete block designs for partial diallel crosses. Sankhya B 52, 75-81.
[2] Bose, R.C. (1939). On the construction of balanced incomplete block designs. Ann. Eug. 9,353-399
[3] Dey A, Midha CK (1996). Optimal designs for diallel crosses. Biometrika 83(2),484–489
[5] Divecha, J. And Ghosh, D.K. (1994). Incomplete block designs for complete diallel crosses and their analysis. Journal of Applied Statistics, Vol.21,No. 5.
[6] Divecha, J. And Ghosh, D.K. (1994). Incomplete block designs for complete diallel crosses and their analysis. Journal of Applied Statistics, Vol.21,No. 5.
[7] Fisher, R.A. and Yates, F. (1963). Statistical Tables for Biological, Agricultural and Medical Research. Oliver and Boyd,Edinburg.
[8] Gilbert, N. E. G.(1958): Diallel cross in plant breeding. Heredity 12, 477–492.
[9] Ghosh, D. K. And Divecha, J. (1997). Two associate class partially balanced incomplete block designs and partial diallelcrosses.Biometrika, 84,1, 245-248.
[10] Ghosh, D.K. and Biswas, P.C. (2000). Robust designs for diallel crosses against the missing of one block. Journal of Applied Statistics, 27(6), 715 – 724.
[11] Gupta, S. And S. Kageyama (1994). Optimal complete diallel crosses, Biometrika, 81,420-424.
[12] Hinkelmann, K., and K. Stern(1960): Kreuzungspläne zur Selektionszüchtung bei Waldbäumen. Silvae Genetica 9, 121–133.
[13] Kempthorne, O.1956. The theory of the diallel cross. Genetics, 41, 451–459.
[14] Kempthorne, O., and R. N. Curnow (1961): The partial diallel cross. Biometrics 17, 229–250.
[15] Yates, F . (1936). Incomplete Randomized blocks. Ann. Eugen., 7, 121-140.