DERIVATION OF CONTINUOUS MODEL OF RISK OF VICTIMIZATION

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From the discrete equation we can derive the continuous equations for the **Risk of victimization** $R(x,t)$, dynamic of **Risk of victimization** of burglary $Q(x,t)$ and the density of burglars $\rho(x,t)$.

We start the derivation of continuous equation of the dynamics of **Risk of victimization** $Q(x,t)$ by the notion of continuous limit.

We assume l is the grid spacing of square grid and time step is h. If we assume l is very small, then for $l \to 0$ as $h \to 0$ such that $\frac{l^2}{h} \to D$, where D is the coefficient of diffusion. For the sake of simplicity, we rename

$$\rho(x,t) \approx \rho(X,t)$$

where $X = lx \therefore x = \frac{X}{l}$

$$R(x,t) \approx R(\frac{X}{l},t) = \frac{1}{l}R(X,t) \approx R(X,t)$$

and $$Q(x,t) \approx Q(\frac{X}{l},t) = \frac{1}{l}Q(X,t) \approx Q(X,t)$$

as l is very small.

Similarly, we can write $Q(x,t+h) \approx Q(X,t+h)$, these can be written with following notion

Since, $aP(x,t) = P(ax,t)$, $bP(x,t) = P(x,bt)$ as the probability is independent of length or time for any constant *a*, *b*.

The second term of the equation is estimated as follows :

$$\frac{\eta}{2}\{Q(x-1,t)+Q(x+1,t)\} = \frac{\eta}{2}\left\{Q\left(\frac{X}{l}-1,t\right)+Q\left(\frac{X}{l}+1,t\right)\right\}$$

$$= \frac{\eta}{2}\left\{Q\left(\frac{X-l}{l},t\right)+Q\left(\frac{X+l}{l},t\right)\right\}$$

With the help of the above notion that the **Risk of victimization** is independent of length and time, further, as l is small,

$$Q(x-1,t) = \frac{1}{l}Q(X-l,t) \approx Q(X-l,t)$$

and $$Q(x+1,t) = \frac{1}{l}Q(X+l,t) \approx Q(X+l,t)$$

Thus the second term of equation can be written as :

$$\frac{\eta}{2}\{Q(x-1,t))+Q(x+1,t)\}$$

$$= \frac{\eta}{2}\left\{\frac{1}{l}Q(X-l,t)+\frac{1}{l}Q(X+l,t)\right\}$$

$$= \frac{\eta}{2}\{Q(X-l,t)+Q(X+l,t)\}$$

Allowing $Q(x,t)$ diffusion to the spatial location, we can re-write the right hand side of the above as

$$\frac{\eta}{2}\{Q(x-1,t)+Q(x+1,t)\} = \frac{\eta}{2}\{Q(X-l)+Q(X+l)\} \tag{1}$$

Now expanding the right hand side of the above term by Taylor's series
We have,

$$Q(X-l)=Q(X)-lQ_X(X)+\frac{l^2}{2}Q_{XX}(X)-O(l^3)$$

where $O(l^3)$ denotes the higher order terms consisting, l.

$$Q(X+l)=Q(X)+lQ_X(X)+\frac{l^2}{2}Q_{XX}(X)+O(l^3)$$

Now neglecting terms consisting higher power of l (as l is very small) and adding, we have,

$$Q(X-l)+Q(X+l)=2Q(X)+2\frac{l^2}{2}Q_{XX}(X)$$

Thus the relation (1) reduces to

$$\frac{\eta}{2}\{Q(x-1,t)+Q(x+1,t)\}=\eta\{Q(X)+\frac{l^2}{2}Q_{XX}(X)\} \tag{2}$$

Also the first term in right side of equation can be re-written as

$$(1-\eta)Q(x,t)=(1-\eta)Q(\frac{X}{l},t)\approx(1-\eta)Q(X,t) \tag{3}$$

as

$$Q(x,t)\approx Q(\frac{X}{l},t)=\frac{1}{l}Q(X,t)\approx Q(X,t)$$

Again left hand side of equation can be re-written as :

$$Q(x,t+h)=Q(\frac{X}{l},t+h)=\frac{1}{l}Q(X,t+h)\approx Q(X,t+h) \tag{4}$$

Using relation (2), (3) and (4), the equation can be re-written as

$$Q(X,t+h)=\left[(1-\eta)Q(X,t)+\eta\left\{Q(X)+\frac{l^2}{2}Q_{XX}(X)\right\}\right](1-\beta_1 h)$$
$$+\alpha\rho(X,\mathrm{t})R(X,t)h$$

$$\Rightarrow \quad Q(X,t+h)-Q(X,t)=-\beta_1 hQ(X,t)+\frac{l^2}{2}Q_{XX}(X)$$
$$-\frac{\eta l^2}{2}Q_{XX}(X)\beta_1 h+\alpha\rho(X,\mathrm{t})R(X,t)h$$

In spatial diffusion, $Q(X,t)\approx Q(X)$, $\rho(X,t)\approx\rho(X)$, $R(X,t)\approx R(X)$ and dividing both sides by h and taking limit as $h\to 0$, we have

$$\lim_{h\to 0}\frac{Q(X,t+h)-Q(X,t)}{h}$$

$$= \lim_{h\to 0}\left\{-\beta_1 Q(X)+\frac{\eta l^2}{2h}Q_{XX}(X)+\frac{\eta l^2}{2}Q_{XX}(X)\beta_1+\alpha\rho(X)R(X)\right\}$$

or $$Q_t(X)=\lim_{h\to 0}\left\{-\beta_1 Q(X)+\frac{\eta l^2}{2h}Q_{XX}(X)+\frac{\eta l^2}{2}Q_{XX}(X)\beta_1+\alpha\rho(X)R(X)\right\}$$

as $h\to 0$, when $l\to 0$ and then assuming $\frac{l^2}{h}\to D$ (constant, say), $l^2\to 0$, then we have the resulting equation as

$$Q_t(X)=-Q(X)\beta_1+\frac{\eta}{2}DQ_{XX}(X)+\alpha\rho(X)R(X)$$

For convenience this equation can be written as

$$\frac{\partial Q}{\partial t}=\frac{\eta D}{2}Q_{XX}-\beta_1 Q+\alpha\rho R$$

$$\Rightarrow \frac{\partial Q}{\partial t}=\frac{\eta D}{2}\nabla^2 Q-\beta_1 Q+\alpha\rho R \qquad (5)$$

where $\Delta Q=div(\nabla Q)=\nabla^2 Q=\sum_{i=1}^{n}\frac{\partial^2 Q}{\partial X_i^{\,2}}=Q_{XX}$ and Δ is a spatial laplacian and D is the diffusive coefficient.

The equation (5) is the continuous equation for the dynamic of **Risk of victimization** of burglary events, called the reaction – diffusion equation.

Since $R(x,t)=R_0+Q(x,t)$

Renaming, $R(x,t)\approx R(X,t)$, $Q(x,t)\approx Q(X,t)$ and considering spatial diffusion of **Risk of victimization**,

$$R=R_0+Q\Rightarrow Q=R-R_0$$

$$\therefore \frac{\partial Q}{\partial t}=\frac{\partial R}{\partial t},$$

So the continuous equation, in terms of R is obtained by putting $Q=R-R_0$ in equation (5) as

$$\frac{\partial R}{\partial t}=\frac{\eta D}{2}\nabla^2(R-R_0)-\beta_1(R-R_0)+\alpha\rho R \qquad (6)$$

Conclusion

This continuous equation represents a nonlinear partial differential equation of **Risk of victimization** of burglary, also called a reaction- diffusion equation.

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