



Technique of controlling the self-focusing length by using non-linear material and Electro-optic materials

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Abstract:

There are several uses of nonlinear optical materials such as optical switching, lens less optical focusing and defocusing etc. These non-linear materials can show its strong applications. The focal length of a material (if the material is used for self- focusing) depends on the applied power. Here in this chapter, a method of controlling the focal length of a nonlinear material based on the joint use of electro-optic material and a nonlinear crystal is proposed. The focal length of the nonlinear material depends upon the voltage applied to the electro-optic material. By changing this voltage in the electro-optic material, the focal length can be varied and this technique can be used as a focal length controller. A suitable electro-optic material and a nonlinear material can be used this purpose.

Key Words: Nonlinear materials, Self Focusing, Eletro-optic material, Focal length, Applied voltage.

Introduction:

When the refractive index of a material is depends on the applied electric field linearly is called Pockel effect and when it depends on the square of the applied field is called Kerr effect. The Kerr effect also called quadratic electro-optic effect [1]. The Kerr effect has a distinct from the Pockel's effect as has the induced index change is directly proportional to the square of the electric field instead of varying linearly with it. This refractive index variation is responsible for the nonlinear optical effects like self- focusing, self-phase modulation and modulation instability and is the basis for Kerr-lens mode locking. There are several applications of Kerr effects in optical switching, arithmetic and algebraic operations etc. [2, .3, .4, 5,.6, 7, and 8].

In the Kerr electro-optic effect, or DC Kerr effect, a slowly varying external electric field is applied across the sample material. Under the influence of the external signal the sample birefringent, with different indices of refraction for light polarized parallel or perpendicular the applied field. The difference in index of refraction, Δn , ($\Delta n = n - n_0$, where n and n_0 the refractive indices of the material with and without the applied of the external electric field respectively) is given by $\Delta n = \frac{1}{2} K E^2$, where λ is the wavelength of the applied light, K is a material constant, and E is the strength of the electric field. This difference in index of refraction helps the material to act as a wave plate when the polarization of light is perpendicular of the applied electric field. If the material is kept between two 'crossed' linear polarizer's, no light come out when electric field is turned off, and almost all the light is transmitted for the application of the optimum value of the electric field. A higher value of the Kerr constant allow as a good transmission with a smaller applied electric field. In this particular work, the author shows by the use of Kerr material how one can control the focal length here the focal length of a non-linear material actually controlled by applied voltage, and the system is behaves like an optical lens.

Self-focusing and De-focusing of a Gaussian beam by the use of non-linear material: Due to a Kerr type of lensing, an intense optical pulse propagating in a non-linear medium experiences a self-focusing, where the beam diameter is decreased compared to of a weaker pulse. The physical mechanism is based on a Kerr nonlinearity with positive χ^2 . In this situation, the higher optical intensities of near to the beam axis, as compared to the off axis intensity, causes an increased refractive index in the inner part of the beam. This modified refractive index distribution acts like a focusing lens. The effect, occurring in the case of a negative χ^2 nonlinearity, self-defocusing, where a reduced refractive index is seen on the beam axis.

A Kerr non-linear process which arises in a media exposed to intense electromagnetic radiation, and which produces a variation of the refractive index n as described by the formula $n = n_0 + n_2 I$, where n_0 and n_2 are the linear and non-linear components of the refractive index respectively, and I is the intensity of the light passing through it. The intensity distribution is taken spatially Gaussian, and the sign of the non-linear correction n_2 be either positive or negative, for self-focusing and defocusing [1.1, 1.9].

If the non-linear correction term n_2 is positive then in peripheral region the plane wave front takes a concave shape in the direction of the beam and is focused at the optical axis of the medium (Fig-1a). On the other hand if the n_2 is negative than central part of the beam goes faster than that of the peripheral region. Consequently, the plane wave front takes the shape of a convex shape direction of propagation and. Thus it defocused into the axis (Fig-1b).

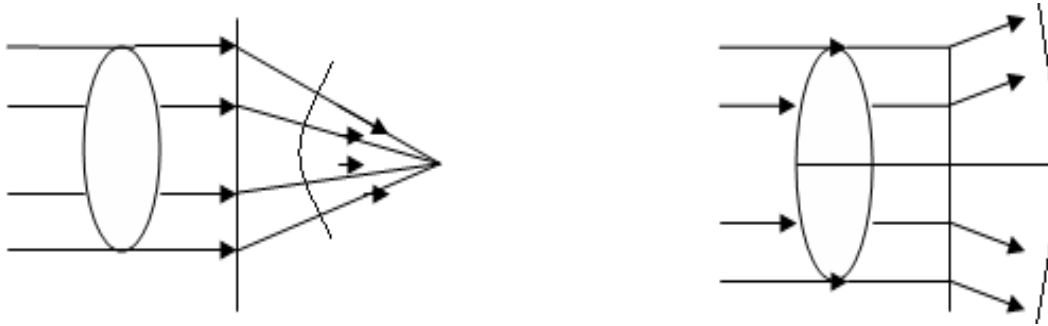


Fig-1. Focusing and defocusing in a non-linear medium. a) $n_2 < 0$, b) $n_2 > 0$

Electro-optic material as an Amplitude modulator:

Electro-optic modulator is an optical device in which an electrical signals exploiting the electro-optic effect and is used to modulate a proper beam of light. The modulation may be used to change the phase, frequency, amplitude, or polarization of the modulated beam. Modulation bandwidth at the gigahertz range is possible with the use of a laser based coherent controlled modulators. [10,11,12,13,14].

Certain materials change their optical properties when they are exposed to an electric field. This is caused by the forces that distort the positions and orientations of the molecules the material. The electro-optic effect gives the change in the refractive index from low frequency electric field to high one up to new range [15,16,17,18,19,20].

Some electric-optic materials are massively used as amplitude modulator such as Potassium di-deuterium phosphate (KD*P), Beta-barium borate (BBO), also Lithium niobate (LiNbO_3), Lithium Tantalite (LiTaO_3) and Ammonium dihydrogen phosphate ($\text{NH}_4\text{H}_2\text{PO}_4$, ADP) etc. In addition to these there are also some organic types of special polymer modulators. A schematic diagram of LiNbO_3 based electro-optic modulator is shown in the fig 2.

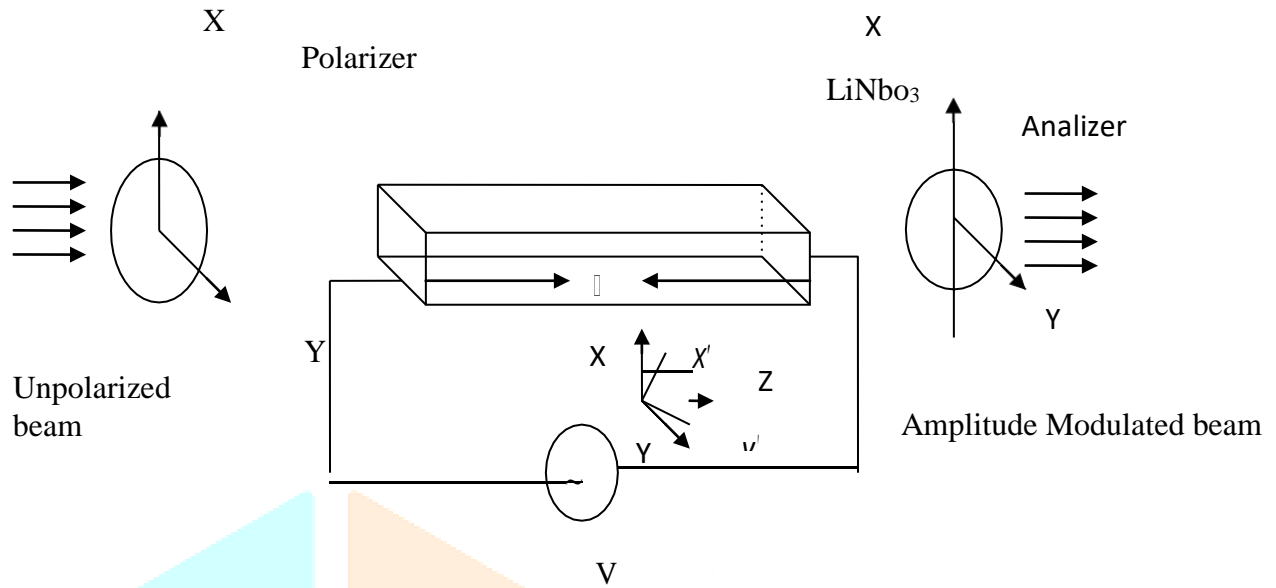


Fig-2 An electro-optic amplitude modulator using LiNbO_3



Gaussian beam:

Gaussian beam has its transverse electric field and intensity distribution which is well approximated by Gaussian functions. Many lasers emits beams that has a Gaussian profile, for that reason the laser is said to be operating on the fundamental transverse mode, or "TEM₀₀ mode" in the laser's optical resonator. When this beam is refracted by a diffraction-limited lens, a Gaussian beam is transformed into another Gaussian beam [13,14].

The beam profile of a Gaussian beam is shown in fig3.

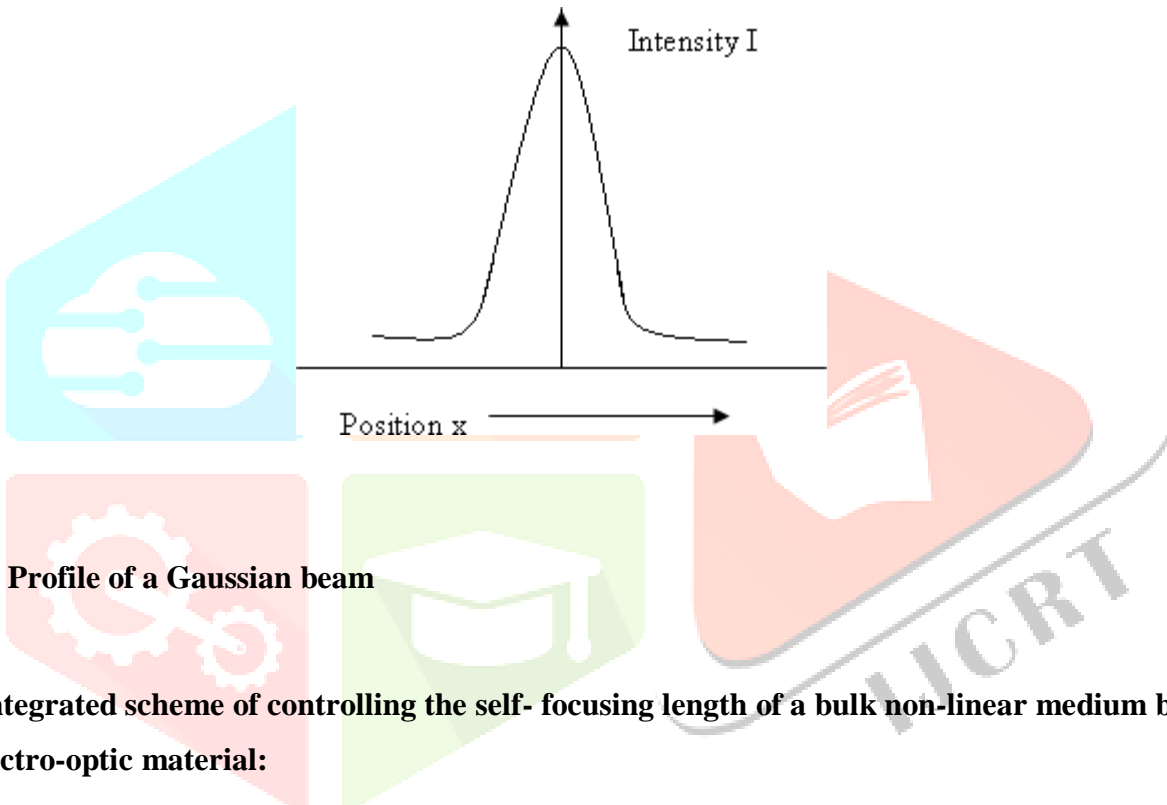


Fig-3 Profile of a Gaussian beam

An integrated scheme of controlling the self- focusing length of a bulk non-linear medium by the use of electro-optic material:

The refractive indices of the electro-optic modulator is [9]

$$n_x = n_0 - \frac{1}{2} n_0^3 r E_z \dots\dots\dots(1)$$

$$n_y = n_0 + \frac{1}{2} n_0^3 r E_z \dots\dots\dots(2)$$

Where ‘r’ is the material constant. E_z , is the applied field along z direction.

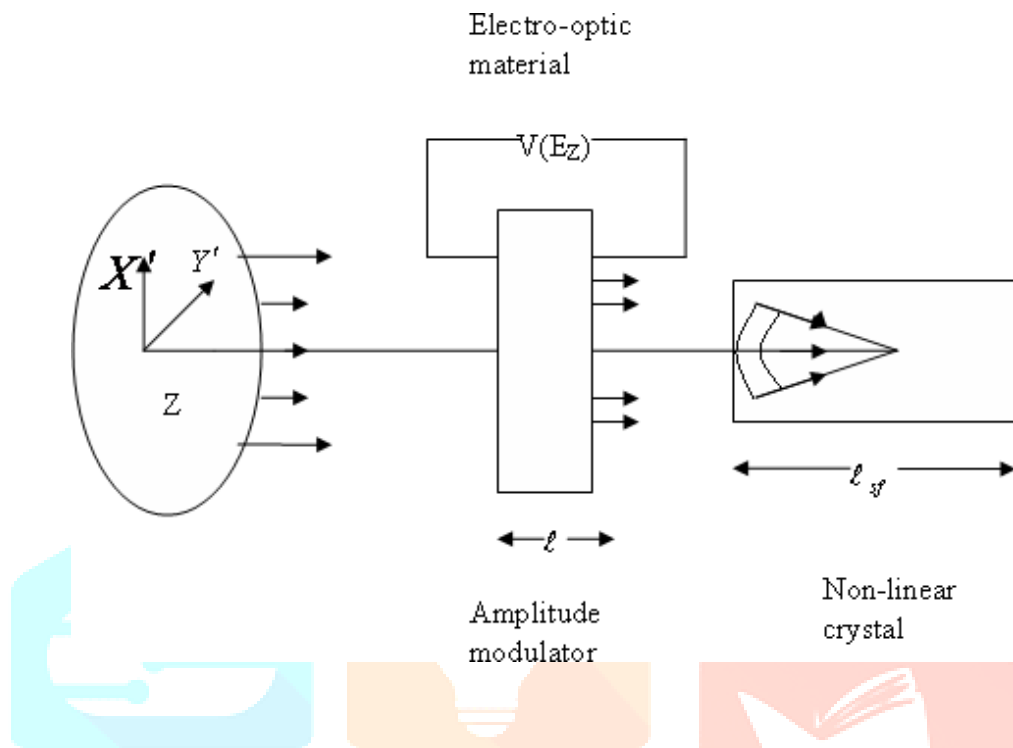


Fig-4 Scheme of controlling the focal length of Gaussian beam by use of E-o modulator and a Kerr type of non-linear material

First a linearly polarized wave polarized along the x-direction and (x' is one of the bi- axial direction of the Electro-optic material) traveling along the z-direction through electro-optic material is considered (fig 4), We have applied an external electric field E_z along the Z-direction (C-axis), then the output wave at $Z= \ell$ (where ℓ is the length of the electro-optic material along z) would be given by

$$\epsilon_x(\ell) = \epsilon_x(0) \left\{ i\omega t - n_0 \left(\frac{\omega}{c} \right) \ell + \frac{\omega}{2c} n_0^3 r E_z \ell \dots \dots \dots \right\} \quad (3)$$

Here $\epsilon_x(z)$ and $\epsilon_y(z)$ are the X' and Y' components of the electric field of the used

light. In a similar manner, a beam polarized along the Y-direction (where Y' is the other bi-axial direction of Electro-optic modulator) the output wave at Z= ℓ will be given by

$$\epsilon_{y'}(\ell) = \epsilon_{y'}(0) \exp\{[\omega t - n_0(\omega/c)\ell + (\omega/2c)n_0^3 r E_z \ell]\} \quad (4)$$

Now consider an incident wave polarized along the y direction is taken then it can be decomposed into two linearly polarized waves along two orthogonal direction as X and Y as these two components will have equal amplitudes and will be in phase Z=0, i.e; at the input of the crystal. Thus the two components which were in phase at Z=0 now develop a phase difference which is a function of the applied electric field (Ez). Thus the retardation at Z= ℓ between the two components will be

$$\gamma = (\omega/c)n_0^3 r_{63} E_z \ell = \omega n_0^3 r_{63} V / C \quad \dots\dots\dots (5)$$

V=Ez ℓ is the voltage applied across the crystal. One can define the ‘half wave’ Voltage Vπ as the voltage required to develop a phase shift of π between the two orthogonal polarization components

$$\text{So, } \gamma = \pi = (\omega/c)n_0^3 r_{63} V_{\pi} \quad \dots\dots\dots (6)$$

$$V_{\pi} = \frac{\lambda_0}{2n_0^3 r_{63}}$$

Substituting the values of εx and εy given by the equations (1) and (2), the expression of the total field (ε) becomes

$$\epsilon = A \exp\{i[\omega t - (n_0/c)\omega\ell + (\omega/2c)n_0^3 r_{63} E_z \ell]\} [-\exp(-i\gamma)] \quad \dots\dots\dots (7)$$

$$\gamma = (\omega/c)n_0^3 r_{63} E_z \ell = \pi(V/V_{\pi}) \quad \dots\dots\dots (8)$$

Thus the intensity of the output beam is given by

$$I_0 = \frac{1}{2} \text{Re}[\epsilon\epsilon^*] = \frac{1}{2} A^2 \sin^2 \frac{1}{2} \gamma \quad \dots\dots\dots (9)$$

Where V=EzL is the applied voltage. The intensity of the input beam (Ii) is given by

$$I_i = \frac{1}{2} A^2 \quad \dots\dots\dots (10)$$

Thus

$$I_0/I_i = \sin^2\left(\frac{1}{2} \pi V/V_\pi\right) \dots \dots \dots (11)$$

I_0/I_i is the transmission coefficient of the electro-optic modulator.

$$\frac{I_0}{I_i} \approx \left[\frac{\pi^2 V_0(t)}{4 V_\pi^2} \right] \dots \dots \dots (12)$$

Again the nonlinear refractive index of the Kerr type crystal is

$$n = n_0 + n_2 I_0 \dots \dots \dots (13)$$

Putting the values of I_0 from equation (12) in equations (13)

$$n = n_0 + n_2 I_i \left[\frac{\pi^2 V_0^2(t)}{4 V_\pi^2} \right]$$

$$n - n_0 = n_2 I_i \frac{\pi^2 V_0^2(t)}{4 V_\pi^2}$$

$$\Delta n = n_2 I_i \frac{\pi^2 V_0^2(t)}{4 V_\pi^2} \dots \dots \dots (14)$$

We know the focal length (L_{sf}) of the Gaussian beam in a non-linear medium can be

expressed as [1]

$$L_{sf} = \frac{1}{k} \sqrt{\frac{n_0}{2\Delta n}} \dots \dots \dots (15)$$

where a is the radius of the beam [1] (fig.5)

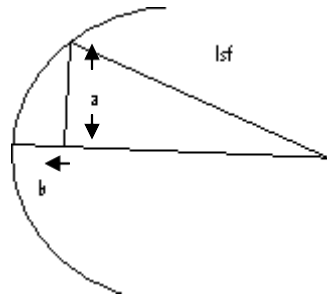


Fig-5 Calculation of self-focusing length.

Now, putting the value of Δn from equation (14) we can get,

$$L_{sf} = a \sqrt{\frac{n_0}{2n_2 I_1 \frac{\pi^2 V^2(t)}{4 V_\pi^2}}} \dots\dots\dots (16)$$

It is known that input intensity is I_i , where

$I_i = E_0^2$ (E_0 is the amplitude of the electric field strength of the light at the time of introduction in the modulator of the axis)

Thus the expression of the focal length can be written as,

$$L_{sf} = \frac{aV_\pi}{\pi V(t)} \sqrt{\frac{2n_0}{n_2 E_0^2}} \dots\dots\dots (17)$$

$$L_{sf} = \frac{2V_\pi}{\pi V(t)} \sqrt{\frac{2n_0}{n_2 E_0^2}} \dots\dots\dots (18)$$

Now $E(r)$ can be written in a radial function (where r is the radial position in the circular beam)

As
$$E(r) = E_0 \sqrt{1 - \frac{r^2}{a^2}} \dots\dots\dots (19)$$

For the mean value of the energy flux density, we obtain the expression

$$\langle S \rangle = v \epsilon E^2 / 2 = [\epsilon C E_0^2 / (2n_0)] (1 - r^2/a^2)$$

$$= (\epsilon_0 n_0 C E_0^2 / 2) (1 - r^2/a^2) \quad (20)$$

The energy flux of the beam is given by

$$P = \int_0^a \langle S \rangle d\sigma = \pi \epsilon_0 n_0 C E_0^2 \int_0^a (1 - r^2/a^2) r dr \quad (21)$$

Where σ is the cross-sectional area of the beam. After integration the total energy flux (P) can be written as

$$P = \pi \epsilon_0 n_0 C E_0^2 a^2 / 4 \quad (22)$$

$$E_0 = \sqrt{\frac{P^2}{\pi \epsilon_0 n_0 C a}} \quad (23)$$

Result:

Thus the self focal length

$$L_{sf} = \frac{V_\pi}{\pi V(t)} \frac{a^2 \sqrt{\pi \epsilon_0 n_0 C}}{\sqrt{P}} \sqrt{\frac{n_0}{2n_2}}$$

The result obtained in equation (24) can be used to obtain the focal length in Carbon bi sulphide, or in any other non-linear medium.

For Carbon bisulphide, $n_0 = 1.62$, $n_2 = 0.22 \times 10^{-19} \text{ m}^2/\text{w}^2$, thus for a power $P = 10 \text{ MW}$ and beam radius $a = 1 \text{ cm}$,

((the LiNbO_3 is used as Electro-optic modulator before the non-linear material) ℓ_{sf}

become 6.92 m (considering $V_\pi = 64 \text{ V}$ and $V = 64 \text{ v}$ also in equation (.24))).

Again if the applied voltage $V=640V$ in $LiNbO_3$ the $I_0 \ell_{sf}$ becomes 0.692 m.

Conclusion:

From the above analytical treatment it is seen that if an electro-optic Pockel cell is used before a Kerr-cell which extends the self-focusing then one can easily control the focal length of the self-focusing system by applying the desired amount of voltage at the electro-optic material. Similarly the defocusing length can also be controlled by the same mechanism. The whole scheme may extend a tremendous application in optical communication through optical fiber. These mechanisms can help the coupling of desired amount of light intensity in an optical fiber from a source in case of data communication. To use it in the application domain one can use a suitable electro-optic material and a suitable simple Kerr non-linear medium.

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