ECONOMIC ORDER QUANTITY WITH BACKORDERS ALLOWED MODEL AND ITS IMPLICATION ON TOTAL COST OF INVENTORY IN COMPARISON WITH BASIC EOQ MODEL

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ABSTRACT:

Basic EOQ model is formulated by considering various restrictions which are in terms of assumptions. One of among this is the backorders not allowed. This paper will describe and formulate the EOQ model with backorders are allowed with simple differential calculations. This paper also gives the explanation for the implication of this model on trade-offs between the components of total cost of inventory. A numerical example is also included for better comparison of the two models with numbers. At last the critical consideration on backordering cost is also provided for making review decision among the two models.

KEYWORDS:

EOQ, Economic Order Quantity, Backordering, Shortages, Costs of Inventory, SCM.

INTRODUCTION

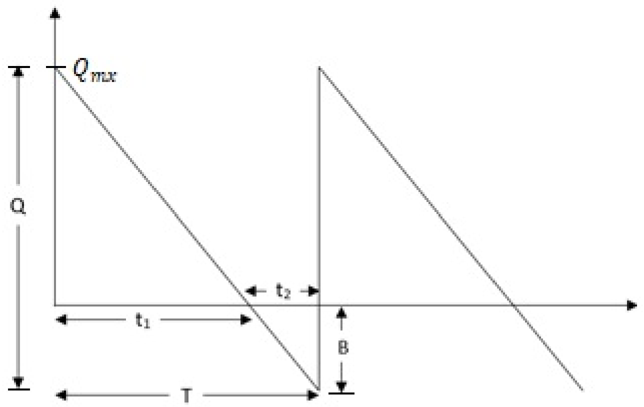
Any inventory management system should answer the basic two questions of

1. How much to order &
2. When to order

The answer to the first question is the order quantity to be decided (which is an optimized quantity) by considering the objective of to minimize the total cost.

Economic Order Quantity, better known as EOQ, is used to decide the optimum quantity which will minimize the total cost. As per the basic EOQ model [8,10], several restrictions are made in terms of assumptions. One of the assumption, backorders allowed, is relaxed for this model.

Assumptions of EOQ model with backorders allowed:



1. There is constant rate of demand per unit of time
2. There is an instantaneous replenishment of the desired quantity, when it is ordered.
3. Cost price of the item will remain constant for the period.
4. Per order ordering cost is fixed.
5. Inventory carrying or Holding cost is fixed as per unit per year
6. Shortages are relaxed and backorders are allowed
7. Backorder cost is fixed as per unit per order

Figure 1 EOQ model with backorders allowed

Explanation of Components of Total Cost:

$$Total\ cost = Item\ Cost + Ordering\ cost + Carrying\ Cost + Backorder\ Cost$$

Item cost: it is the total cost associated with buying the item. Which is decided simply by multiplying the quantity with the unit cost price of that item.

Ordering Cost: It is the cost incurred every time when an order is placed for an item. Which mainly include the cost of office personnel and office expenses, transportation, inspection, rejection, taking follow up, delay etc.

$$Ordering\ Cost = (no.of\ orders\ per\ year) \times (order\ cost\ per\ order)$$

$$= \frac{D}{Q} \times C_O$$

Carrying or Holding Cost: It is the cost incurred for holding the inventory for a certain period of time. It is based on the average inventory. The main part of this cost is the interest cost which is to be considered for holding the amount of money for certain period as inventory. Other part includes the cost of storing, electricity, safety, security, personnel staff, obsolesce, theft, etc.

$$Carrying\ Cost = (Average\ Inventory\ held\ for\ the\ year) \times (carrying\ cost\ per\ unit\ per\ year).$$

$$= (average\ invetory\ for\ t_1 + avg.Inv.for\ t_2) \times C_C$$

$$= \left(\frac{\frac{(Max\ Inv. + Min.Inv.) \times t_1}{2} + \frac{(Max\ Inv. + Min.Inv.) \times t_2}{2}}{T}\right) \times C_C$$

$$= \frac{(Q_{mx} + 0) \times t_1 + 0}{2T} \times C_C$$

$$= \frac{(Q_{mx}) \times t_1}{2T} \times C_C$$

Backorder or Shortage cost: It is the cost incurred for not meeting the due date or not satisfying the customer order because of the shortage of the material.

This situation can be handled in two ways:

1. One way is to cancel all the orders for that particular period when the material is in shortage. Which is called loss of sales situation.
2. The other way is to carry forward the order to the next period of schedule by delaying and complete the order. This situation is called backordering which is carrying the unmet demand to the next period and met the demand.

Usually when it is not specified the backorder situation is considered and the production continues with the backorders.

Components of Backorder or Shortage Cost:

1. Loss of profit
2. Loss of opportunity
3. Cost of additional capacity
4. Cost of rescheduling
5. Cost of under-utilized capacity
6. Increased transportation cost
7. Loss of customer goodwill

$$Backorder\ Cost = (Avg.Backorder\ quantity\ generated\ for\ the\ year) \times (backorder\ cost\ per\ unit\ per\ year)$$

$$= (Avg.Backorder\ generated\ during\ period\ t_2) \times C_B$$

$$= \left(\frac{\frac{(Max\ Qnty. + Min.Qnty.) \times t_2}{2}}{T}\right) \times C_B$$

$$= \left(\frac{(B + 0) \times t_2}{2T}\right) \times C_B$$

$$= \frac{B \times t_2}{2T} \times C_B$$

Formulation of Model:

Table-1 Notations used for the model:

TC	Total cost
Q	Optimal Order Quantity
Q_{mx}	Maximum inventory level
B	Backorders (Quantity) Generated
C_O	Ordering cost per order
C_C	Carrying cost/holding cost per unit per year
C_B	Backorder cost

$$Total\ cost = Item\ Cost + Ordering\ cost + Carrying\ Cost + Backorder\ Cost \tag{1}$$

Now here the Item cost component will remain constant throughout the calculations because it depends on the cost price and annual demand and both are constant. Thus, this component will be ignored from further calculations.

Substituting all the values from previous calculations into Eq. (1)

$$TC = \frac{D}{Q} \times C_O + \frac{(Q_{mx}) \times t_1}{2T} \times C_C + \frac{B \times t_2}{2T} \times C_B \tag{2}$$

Now this equation has several variables like, Q, B, Q_{mx}, t1, t2, T, which are unknown. But we can establish relationships from the graph between these variables.

From the graph it is seen that,

$$Q = Q_{mx} + B \Rightarrow Q_{mx} = Q - B \tag{3}$$

Now inventory will decrease from Q_{mx} to zero at a rate of d within time t_1, we can write this as,

$$d \cdot t_1 = Q_{mx}$$
$$t_1 = \frac{Q_{mx}}{d} \tag{4}$$

Similarly, backorders will generate from zero to negative of B level at the rate of d within the time period t_2, we can write this as,

$$d \cdot t_2 = B$$
$$t_2 = \frac{B}{d} \tag{5}$$

Now, substituting Eq.(4) and Eq(5) into Eq (3),

$$Q = Q_{mx} + B$$

$$= d \cdot t_1 + d \cdot t_2$$

$$= d \cdot (t_1 + t_2)$$

$$= d \cdot T$$

$$T = \frac{d}{Q} \tag{6}$$

Now from Eq.(4), Eq.(5), and Eq.(6), we can write,

$$\frac{t_1}{T} = \frac{Q_{mx}}{Q} = \frac{Q - B}{Q} \quad \&\ also\ \frac{t_2}{T} = \frac{B}{Q}$$

Substituting these values in Eq. (2),

$$TC = \frac{D}{Q} \times C_O + \frac{(Q - B) \times (Q - B)}{2Q} \times C_C + \frac{B \times B}{2Q} \times C_B$$

$$TC = \frac{D}{Q} \times C_O + \frac{(Q - B)^2}{2Q} \times C_C + \frac{B^2}{2Q} \times C_B$$

Now this equation is having only two variables, Q and B, and other dependencies are substituted with common variables.

To minimize the TC we need to differentiate it with respect to B and Q separately and equate it with zero to find the values of Q and B.

$$\frac{dTC}{dB} = 0 + \frac{C_C}{2Q}\left(2(Q-B)(-1)\right) + \frac{C_B \times 2B}{2Q}$$

$$= \frac{C_c(Q-B)}{Q} + \frac{BC_B}{Q}$$

$$0 = QC_c - BC_c + BC_B$$

$$B(C_c + C_B) = QC_c$$

$$B = \frac{QC_c}{(C_c + C_B)}$$

This will give us the value of Backorder quantity.

$$\frac{dTC}{dQ} = DC_O\left(-\frac{1}{Q^2}\right) + \frac{C_c}{2}\left(\frac{[Q\cdot 2\cdot(Q-B)\cdot(1)] - [(Q-B)^2\cdot(1)]}{Q^2}\right) - \frac{B^2C_B}{2Q^2}$$

$$\frac{dTC}{dQ} = -\frac{DC_O}{Q^2} + \frac{C_c}{2}\left(\frac{2Q^2 - 2QB - Q^2 + 2QB - B^2}{Q^2}\right) - \frac{B^2C_B}{2Q^2}$$

$$\frac{dTC}{dQ} = -\frac{DC_O}{Q^2} + \frac{C_c(Q^2 - B^2)}{2Q^2} - \frac{B^2C_B}{2Q^2}$$

$$0 = -\frac{DC_O}{Q^2} + \frac{C_c(Q^2 - B^2)}{2Q^2} - \frac{B^2C_B}{2Q^2}$$

$$0 = -2DC_O + C_c(Q^2 - B^2) - B^2C_B$$

Substitute the value of B,

$$2DC_O = C_c\left(Q^2 - \frac{Q^2C_c^2}{(C_c + C_B)^2}\right) - \frac{Q^2C_c^2C_B}{(C_c + C_B)^2}$$

$$\frac{2DC_O}{C_cQ^2} = \frac{C_c^2 + C_B^2 + 2C_cC_B - C_c^2 - C_cC_B}{(C_c + C_B)^2}$$

$$\frac{2DC_O}{C_cQ^2} = \frac{C_B^2 + C_cC_B}{(C_c + C_B)^2}$$

$$\frac{2DC_O}{C_cQ^2} = \frac{C_B(C_B + C_c)}{(C_c + C_B)^2}$$

$$\frac{2DC_O}{C_cQ^2} = \frac{C_B}{(C_B + C_c)}$$

$$\frac{2DC_O(C_B + C_c)}{C_cC_B} = Q^2$$

$$Q = \sqrt{\frac{2DC_O(C_B + C_c)}{C_cC_B}}$$

$$Q = \sqrt{\frac{2DC_O}{C_c}\cdot\frac{C_B + C_c}{C_B}}$$

$$Q = \sqrt{\frac{2DC_O}{C_c}\cdot\left(1 + \frac{C_c}{C_B}\right)}$$

This value will give us the total optimal order quantity.

Now substituting the value of Q in B we get,

$$B = \sqrt{\frac{2DC_O}{C_C \cdot \left(1 + \frac{C_C}{C_B}\right)}}$$

Formula comparison:

Table 2

Entity	Basic Model	Backorders Allowed
Q	$Q = \sqrt{\frac{2DC_O}{C_C}}$	$Q = \sqrt{\frac{2DC_O}{C_C} \cdot \left(1 + \frac{C_C}{C_B}\right)}$
Q_{mx}	--	$Q_{mx} = Q - B$
B	--	$B = \sqrt{\frac{2DC_O}{C_C \cdot \left(1 + \frac{C_C}{C_B}\right)}}$
TC	$\frac{DC_O}{Q} + \frac{QC_C}{2}$	$\frac{DC_O}{Q} + \frac{(Q-B)^2 C_C}{2Q} + \frac{B^2 C_B}{2Q}$

Numerical Example:

Consider D=50000 units per year, Co=200 rs. Per order, Cc=5 rs. Per unit per year, C_B=25 rs. Per unit per year.

Table 3

	Q	Q_{mx}	B	Ordering Cost	Carrying Cost	Backorder Cost	TC
EOQ	2000	--	--	5000	5000	--	10000
EOQ With Backorder	2191	1826	365	4564	3804	761	9129

Implications of The Backorder Model with Total Cost Trade-offs:

1. When we allow backordering, the total cost comes down (*Table* 3). This can be because of certain things.
    a. Order cycle becomes smaller as the EOQ value is higher while allowing backordering. Which in turn reduces the total ordering cost.
    b. The EOQ value is higher because the backorder quantity is also included in it.
    c. By allowing the backorders actually lowers the maximum quantity level. And that in turn lowers the average quantity level of inventory.
    d. This actually lowers the carrying cost because with lower level of average inventory to be held for lesser period of time.
2. The total cost is a trade of between the total ordering cost and the total of both carrying as well as the shortage cost put together.
    a. At optimum situation, the total ordering cost equal to the total cost of both carrying and shortage together.
    b. Here for every quantity level, Inventory cost comes down and backorder cost goes up. But eventually the order cost balances the other two costs and the total cost comes down.
3. Total cost variation very much depends on the value of C_B. *(Table 4)*
    a. If we keep increasing the value of C_B the EOQ keeps decreasing and TC keeps increasing
    b. And at a very large or infinity value of C_B, both EOQ and TC will eventually become equal to the basic EOQ model values where the backordering is not allowed.

Table 4 (Comparison of TC with different values of C_B)

C_B	Q	Q_{mx}	B	Ordering Cost	Carrying Cost	Backorder Cost	TC
50	2097.62	1906.93	190.69	4767.31	4333.92	433.39	9534.63

100	2049.39	1951.80	97.59	4879.50	4647.14	232.36	9759.00
200	2024.85	1975.46	49.39	4938.65	4818.19	120.45	9877.30
500	2009.98	1990.07	19.90	4975.19	4925.93	49.26	9950.37
1000	2004.99	1995.02	9.98	4987.55	4962.73	24.81	9975.09
2000	2002.50	1997.50	4.99	4993.76	4981.31	12.45	9987.52
5000	2001.00	1999.00	2.00	4997.50	4992.51	4.99	9995.00

Conclusion:

The result is very much dependent on the value of C_B. If we know the C_B accurately and enough protection related to inventory additional inventory are included in the system then this model becomes applicable. If we are not sure about the C_B then it is better to assume C_B to be infinity and use the basic model.

But, it is seen from the calculations *(Table 4)* that any backorder cost smaller to infinity will give us minimum total cost as compared to basic model which is without allowing the backorder quantity.

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