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Volume Between Regular Polyhedron And Inscribed Sphere

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Abstract:

In this paper, the volume between a regular polyhedron and inscribed sphere has been obtained for all the five platonic solids.

Key words: Volume, regular polyhedral, platonic solids and sphere.

Introduction:

A polyhedron with faces of identical regular polygon is called a regular polyhedron. A regular polyhedron has all side lengths equal as well as all angles equal. There are five regular polyhedra, namely; tetrahedron, cube, octahedron, dodecahedron and icosahedrons. The regular polyhedra are also called platonic solids.

A tetrahedron has four faces (equilateral triangles), four vertices and six edges. The four vertices of a tetrahedron are at equal distance from each other. A cube has six faces (identical squares), eight vertices and 12 edges. An octahedron has eight faces (identical triangles), six vertices and 12 edges. A dodecahedron has twelve faces (identical pentagons), twenty vertices and thirty edges. An icosahedron has twenty faces (equilateral triangles), twelve vertices and thirty edges.

In this paper, the volume between a regular polyhedron and inscribed sphere has been obtained for all the five platonic solids.

Analysis:

The volume between inscribed regular polyhedron and inscribed sphere has been discussed for all the five platonic solids as follows:

Case 1: Volume between a regular tetrahedron and inscribed sphere: This is shown in figure 1 given below:

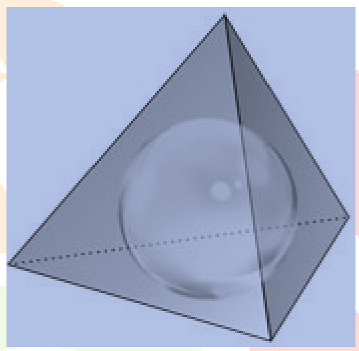

Figure 1

Let the radius of the sphere be *r* units and length of circumscribed regular tetrahedron be *x* units. Then

$$x = \frac{12}{\sqrt{6}} r. \qquad \ldots(1)$$

Now volume of sphere is given by

$$A_1 = \frac{4}{3}\pi r^3 \text{units},$$

and volume of circumscribed regular tetrahedron is given by

$$A_2 = \frac{x^3}{6\sqrt{2}} = \frac{24}{\sqrt{3}} r^3 \text{units}.$$

Therefore required volume is given by

$$A = A_2 - A_1 = \frac{24}{\sqrt{3}} r^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\left(6\sqrt{3} - \pi\right) r^3 \text{ units.}$$

Case 2: Volume between a cube and inscribed sphere: This is shown in figure 2 given below:

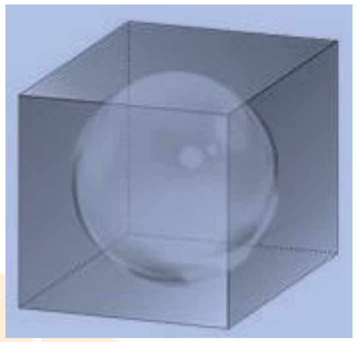

Figure 2

Let the radius of the sphere be *r* units and length of circumscribed cube be *x* units. Then

$$x = 2r. \qquad \ldots(2)$$

Now volume of sphere is given by

$$A_1 = \frac{4}{3}\pi r^3 \text{units,}$$

and volume of circumscribed cube is given by

$$A_2 = x^3 = 8r^3 \text{units.}$$

Therefore required volume is given by

$$A = A_2 - A_1 = 8r^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}(24 - \pi) r^3 \text{ units.}$$

Case 3: Volume between a regular octahedron and inscribed sphere: This is shown in figure 3 given below:

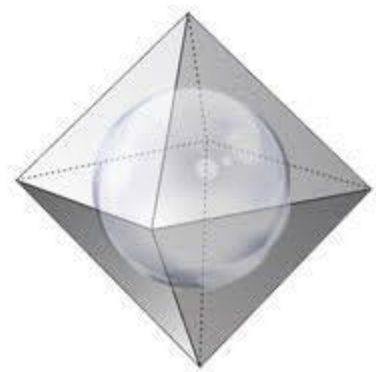

Figure 3

Let the radius of the sphere be *r* units and length of circumscribed regular octahedron be *x* units. Then

$$x = \sqrt{6}r. \quad \ldots(3)$$

Now volume of sphere is given by

$$A_1 = \frac{4}{3}\pi r^3 \text{units.}$$

and volume of circumscribed octahedron is given by

$$A_2 = \frac{\sqrt{2}}{3}x^3 = 4\sqrt{3}r^3 \text{ units.}$$

Therefore required volume is given by

$$A = A_2 - A_1 = 4\sqrt{3}r^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\left(3\sqrt{3} - \pi\right)r^3 \text{ units.}$$

Case 4: Volume between a regular dodecahedron and inscribed sphere: This is shown in figure 4 given below:

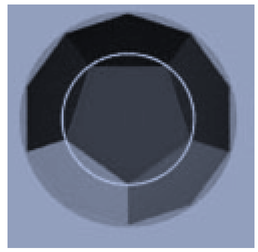

Figure 4

Let the radius of the sphere be r units and length of circumscribed regular dodecahedron be x units. Then obviously

$$x = \frac{4r}{\sqrt{10+\frac{22}{\sqrt{5}}}}. \quad \ldots(4)$$

Now volume of sphere is given by

$$A_1 = \frac{4}{3}\pi r^3 \text{units},$$

and volume of circumscribed dodecahedron is given by

$$A_2 = \frac{\left(15+7\sqrt{5}\right)}{4}x^3 = \frac{16\left(15+7\sqrt{5}\right)}{\left(10+\frac{22}{\sqrt{5}}\right)\sqrt{10+\frac{22}{\sqrt{5}}}}r^3 \text{ units}.$$

Therefore the required volume is given by

$$A = A_2 - A_1 = \frac{16\left(15+7\sqrt{5}\right)}{\left(10+\frac{22}{\sqrt{5}}\right)\sqrt{10+\frac{22}{\sqrt{5}}}}r^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\left(\frac{12\left(15+7\sqrt{5}\right)}{\left(10+\frac{22}{\sqrt{5}}\right)\sqrt{10+\frac{22}{\sqrt{5}}}} - \pi\right)r^3 \text{ units}.$$

Case 5: Volume between a regular icosahedron and inscribed sphere: This is shown in figure 5 given below:

Figure 5

Let the radius of the sphere be *r* units and length of circumscribed regular icosahedron be *x* units. Then obviously

$$x = \sqrt{3}(3 - \sqrt{5})r. \quad \ldots(5)$$

Now volume of sphere is given by

$$A_1 = \frac{4}{3}\pi r^3 \text{units},$$

and volume of circumscribed icosahedron is given by

$$A_2 = \frac{5(3+\sqrt{5})}{12}x^3 = 10\sqrt{3}(7 - 3\sqrt{5})r^3 \text{ units}.$$

Therefore required volume is given by

$$A = A_2 - A_1 = 10\sqrt{3}(7 - 3\sqrt{5})r^3 - \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\left(15\sqrt{3}(7 - 3\sqrt{5}) - 2\pi\right)r^3 \text{units}.$$

Concluding remarks:

Here the volume between sphere and circumscribed all the five platonic solids have been obtained.

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