IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Variation of Cop and Robber Game and Dismantable Graphs

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Abstract

In graph theory, the game of cops and robbers is played on a finite, connected graph. The players take turns moving along edges as the cops try to capture the robber and the robber tries to evade capture forever. This game has received quite a bit of recent attention including several conjectures that have yet to be proven. In this paper, we restricted the movement of the robber in order to try to prove the conjectures that all graph is a cop win graph.

Keywords: cop number, graph, cop win, icosahedron, truncated tetrahedron

1. Introduction

In mathematics graphs are useful in geometry and certain parts of topology such as knot theory. ‘Algebraic graph theory’ has close links with ‘group theory’. Algebraic graph theory has been applied to many areas including dynamic systems and complexity. The paper written by ‘Leohard Euler’ on the ‘Seven Bridge of Konisberg’ and published in 1736 is regarded as the first paper in the history of graph. The of Konigsberg in Prussia (present-day Kaliningrad, Russia) was set on both sides of the Pregel River and included two large islands Kneiphof and Lomse- that were connected to each other via the two mainland portions of the city by seven bridges. The problem was to devise a walk through the city that would cross each of those bridges once and only once.

2. Preliminaries

Definition 2.1

A **pitfall** is a vertex, u, that has an adjacent vertex, v, such that all the neighbors of u are also neighbors of v.

In some literature, a pitfall could also be called a corner or a dominated vertex and its corresponding neighbor is called the dominating vertex. Going back to the examples, we can find pitfalls in each case. For the complete graphs, every vertex is simultaneously a pitfall and dominating vertex because all the vertices are connected to each other.

Definition 2.2

A **cycle** is a closed path in a graph that forms a loop. When the starting and ending point is the same in a graph that contains a set of vertices, the cycle of the graph is formed. The cycle of the graph is denoted by C_n

Definition 2.3

The icosahedron is one of the five platonic solids. A regular icosahedron has **30 edges, 20 equilateral triangles faces, with 5 faces meeting at each of its 12 vertices.** The icosahedron has the maximum number of faces and volume among all platonic solids for its surface area. The number of sides of Icosahedron is 20. Following are the characteristics of Icosahedron shape which have the number of faces, edges of the icosahedron are 20, 30 and 12 respectively. The face type of Icosahedron is a regular triangle. The number of sides at the verge of the Icosahedron is 3. The number of edges adjacent at the top of the Icosahedron is 5.

Definition 2.4

The truncated icosahedron contains no 3 or 4 cycles and has minimum degree $\delta(G) = 3$. it follows that $c(G) = 1$. In the small rhombicosidodehedron every vertex is part of a 3-cycles, two 4-cycles, and a 5-cycles. The only final configuration that leaves the robber no escape is if the cop occupy the corners of the square. At this point robber stays put until he gets a chance to escape. Therefore, robber does not have an escape strategy at every point of the graph, that cop will definitely capture him.

Theorem 2.5

If v is dominated by w then $c(G - v) = c(G)$.

Theorem 2.6

G is 1-cop win if and only if G is dismantle able.

Theorem 2.7

If H *is* a retraction of G then $c(H) \leq c(G)$

Theorem 2.8

If G is a graph with minimum degree δ(G) which has no 3- or 4-cycles, then c(G) ≥ δ(G)

Theorem 2.9

If G is planar then c(G) ≤ 3.

Theorem 2.10

Let u, v be vertices of a graph G. Let P be a shortest path between u and v. After a finite number of moves, a cop may prevent the robber from entering P .

3. Main Results

Theorem 3.1

Let u, v be vertices of a graph G. Let P be a shortest path between u and v. After a first move, if robber is restricted to enter into their previous vertices, then the graph G is a cop win graph.

(or)

A graph G is a cop win graph if and only if the robber is restricted to enter into anyone of their previous vertices.

Proof:

This is truly remarkable. How does a condition about not crossing edges relate to the cop number.

Consider a graph with **n** vertices. By using the above condition one cop is sufficient to catch the robber. Since, the graph is finite, the robber is caught in finite number of moves. Let **C** be the vertex occupied by the cop and **R** be the position occupied by the robber. The distance between or the number of edges between the cop and the robber must not be less than two. If it is less than two then the robber is caught in the first move.

If the distance between the cop and the robber is greater than or equal to two, the robber is safe from caught as he has an adjacent vertex to move, and that vertex will not be adjacent to the cop's vertex.

Let us consider, the first move of cop and robber is over. Now they both are in the vertex adjacent to their initial vertex. Now for the robber, the initial vertex of the robber is blocked. (ie.,) He cannot move back to that vertex.

Now consider, the second move of the cop and robber are over. If still the robber is not caught the previous vertex of the robber is blocked.

Similarly, by blocking the robber to enter into one of this previous vertex at finite number of moves the robber will be finally caught.

The number of moves may be equal to or less than the number of vertices.

Theorem 3.2

The min cop (G) for any graph is one.

Proof

The goal of this work is to investigate the cop number of familiar graphs. We are particularly interested in finding with cop number at most 1. One of the most useful tools in this endeavor is the following Aigner and Fromme.

In other words, a cop eventually "guard" any shortest path he chooses. If the robber ever steps on that path, he will be caught. Of course, it is not enough to simply guard shortest paths, the cops must actively pursue the robber.

Since by using this variant, after a finite a number of moves all the vertices in the graph except the cop's vertices is blocked. So, the robber must step into that vertex and he is caught.

Dismantlable graphs

Suppose i and j are nodes of a graph H such that $N(i) \leq N(j)$. This operation is called a fold of graph H and we say vertex i folded onto vertex j. A finite graph H is said to be dismantlable if there exists a sequence of folds reducing H to a graph with one vertex.

The closed neighbourhood N[v] of a vertex v in a given is the set of vertices consisting of v itself and all other vertices adjacent to v. The vertex v is said to be dominated by another vertex w when $N[v] \subset N[w]$. That is, v and w are adjacent, and every other neighbor of v is also a neighbor of w. Nowakowski and Winkler call a vertex that is dominated by another vertex an irreducible vertex.

Dismantling order

A dismantling order or domination elimination ordering of a given is an ordering of the vertices such that, if the vertices are removed one-by-one in this order, each vertex (except the last) is dominated at the time it is removed. A is dismantlable if and only if it has a dismantling order.

Theorem 3.3

A graph of Icosahedron is a cop win graph.

Proof

We show that the icosahedron is a cop win graph by providing a strategy for a cop to capture a robber irrespective of their or his starting position. This strategy consists of two different phase.

The first is for the cop to choose a vertex as their initial position and then the robber chooses his. The second phase, the cop move to capture the robber, who will have chooses one of the remaining vertices.

Placement Phase

The cop will place themselves on one of the vertices. While the cops are placing themselves and then waiting for their turn, the robber will either be incidentally caught, or end up on one of the remaining vertices.

Capture Phase

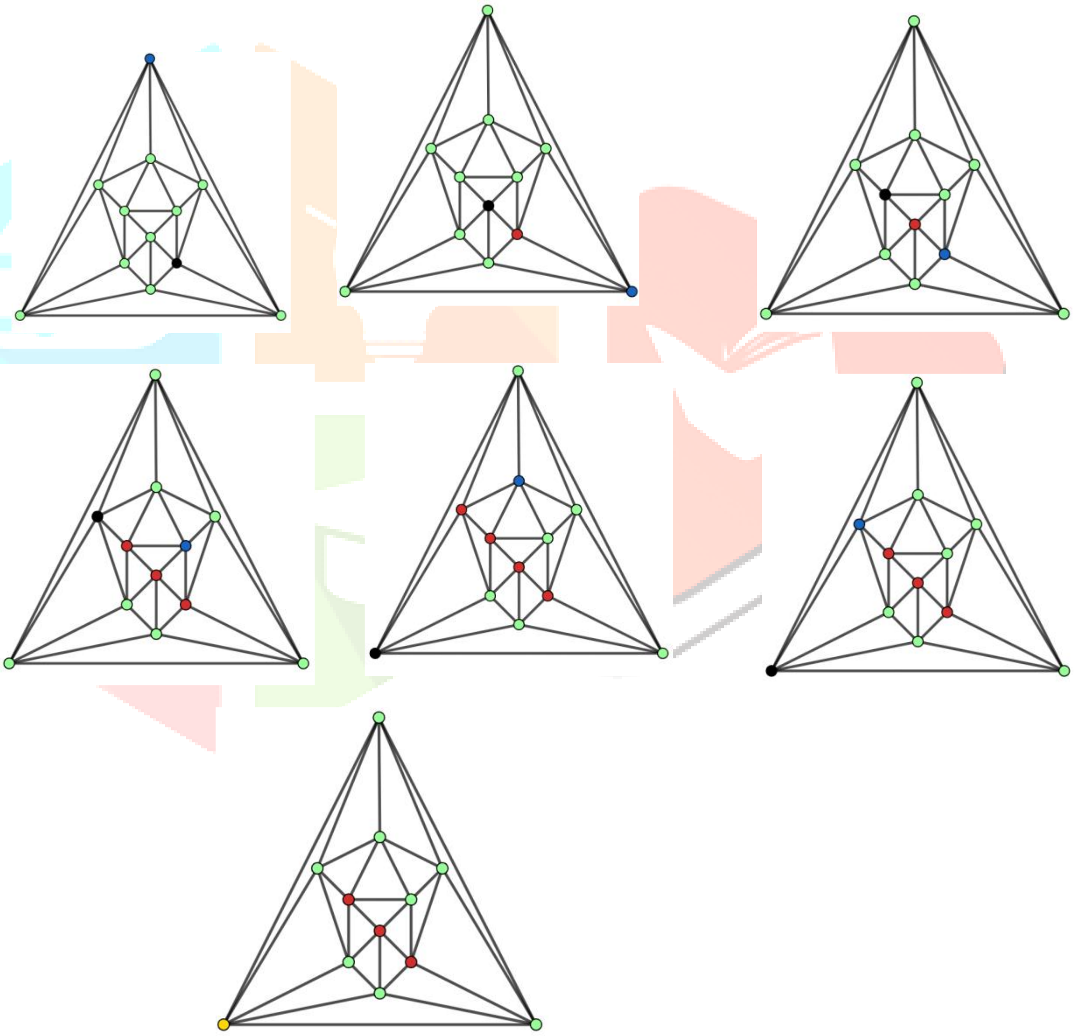
Accordingly, with the cop's move, the second phase begins. The graph appears as such, with the cop on vertices marked in blue and robbers on one of remaining vertices marked in black, and the vertices marked in red are restricted vertices for the robbers to move on.

If the robber is on a vertex marked in black, then he is in one of the edges, that cop cannot immediately capture him

The greencoloured vertices are the possible vertices that a robber can move. Instead, the cops can make the moves marked in blue to the circled vertices for the respective green vertices. In each case, the vertices the cops move a dominating set of the neighbourhood of the robber,

Therefore, after a finite of moves all of the vertices are blocked for the robbers to move. So, on the final turn, he definitely must move to the cop's vertex and he is caught.

Diagrammatic Representation



vertex occupied by the cop

vertex occupied by the robber

vertices that a robber can move.

vertices that are restricted for the robber to enter

vertices occupied by cop and the vertex restricted for the robber

vertex in which cop cature the robber.

Conclusion

By this strategy, one cop can capture a robber on a Icosahedron graph irrespective of starting position, so the graph is one cop-win graph.

Theorem 3.4

A graph of the truncated tetrahedron is a cop win graph.

Proof

We show that the truncated tetrahedron is a cop win graph by providing a strategy for a cop to capture a robber irrespective of their or his starting position. This strategy consists of two different phase.

The first is for the cop to choose a vertex as their initial position and then the robber chooses his.

In second phase, the cops move to capture the robber, who will have choses one of the remaining vertices

Placement Phase

The cops will place themselves on one of the vertices. While the cops are placing themselves and then waiting for their turn, the robber will either be incidentally caught, or end up on one of the remaining vertices.

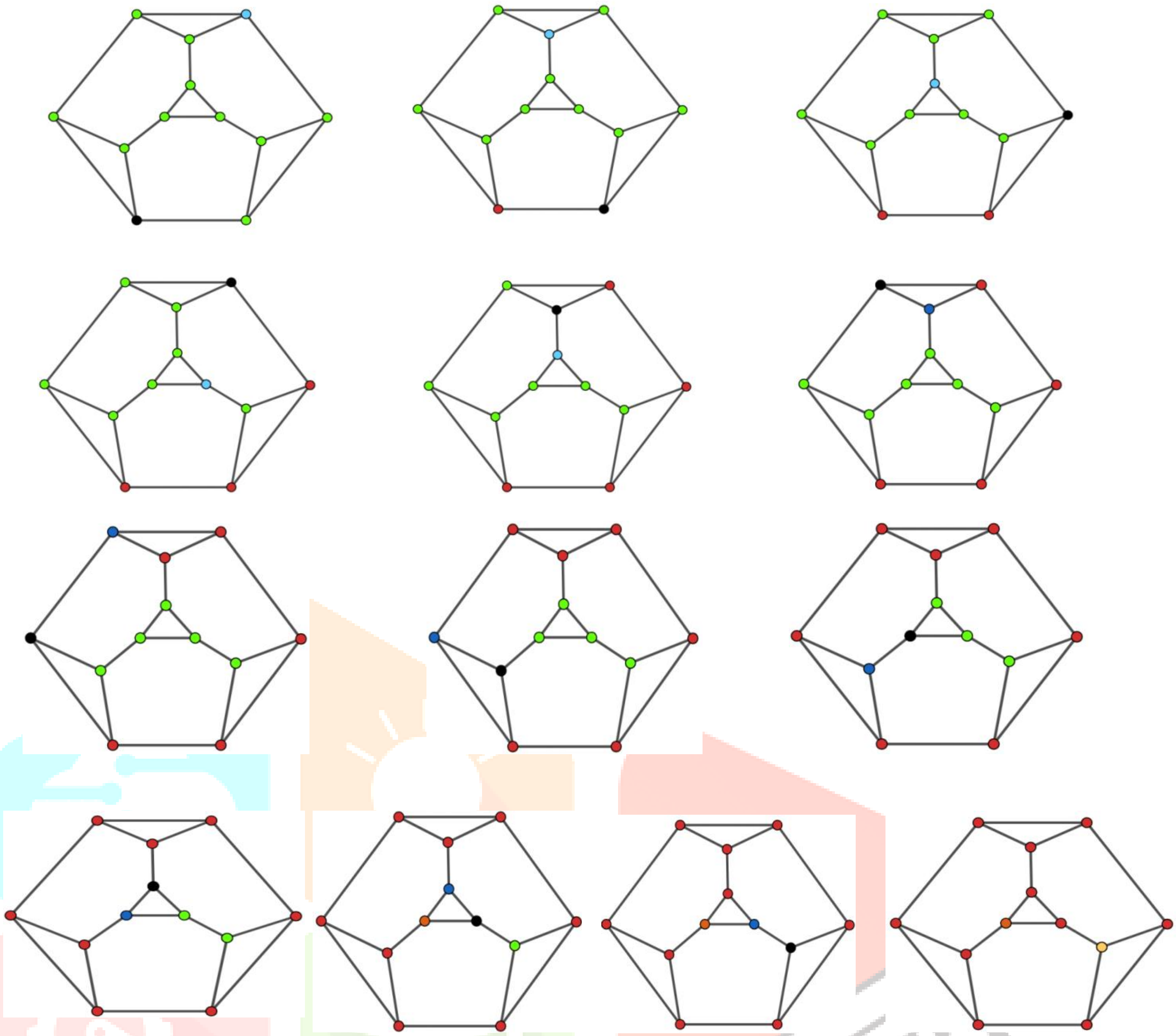
Capture Phase

Accordingly, with the cop's move, the second phase begins. The graph appears as such, with the cop on vertices marked in blue and robbers on one of remaining vertices marked in black, and the vertices marked in red are restricted vertices for the robbers to move on. If the robber is on a vertex marked in black, then he is in one of the edge, that cop cannot immediately capture him.

The green coloured vertices are the possible vertices that a robber can move. Instead, the cops can make the moves marked in blue to the circled vertices for the respective green vertices. In each case, the vertices the cops move to a dominating set of the neighbourhood of the robber.

In every phase, the robber and cops move to their adjacent vertices. In each phase, once the robber moves to the adjacent vertices his previous vertices are restricted to move for him.Therefore, after a finite number of moves all of the vertices are blocked for the robbers to move. So, on the final turn, he definitely must move to the cop's vertices and he is caught.

Diagrammatic Representation



vertex occupied by the cop

vertex occupied by the robber

vertices that a robber can move.

vertices that are restricted for the robber to enter

vertices occupied by cop and the vertex restricted for the robber

vertex in which cop cature the robber.

Conclusion

By this strategy, one cop can capture a robber on a truncated tetrahedron irrespective of their starting positions, so the graph is one cop-win graph.

4. Conclusion

One of the most intriguing parts of math is that simple puzzles and childish games can lead to profound and often complex questions. The game of cops and robbers is a good example of this. Based on the set of rules that can be explained to an elementary school student, cops and robbers has led to many papers, conjectures and theorems. Beginning with a basic notion of how the game is played and some definition, we managed to build a characterization for all the graphs which are cop-win. Chapter 4 tells us that any graph that can be completely decomposed to a single vertex by successively removing pitfalls must be cop-win. Even restricting in the rules of the Game of Cop and Robber answers have been found, but there are still many questions left to answer.

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