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MESH FREE APPROACH FOR BENDING ANALYSIS OF BEAMS AND THIN PLATES WITH VARIOUS FIELD PARAMETERS

[1]K. Sai Anitha, [2]K. Mari Muthu

[1]Student, [2]Assistant professor

[1]Civil Engineering,

[1]Srinivasa Institute of Engineering and Technology, Amalapuram, India

ABSTRACT: The design addressed the study of beams and thin plates subjected to plane stress adopting the meshless approach, ELEMENT FREE GALERKIN. This involves a detailed study of the Element Free Galerkin Method consisting of its expression, mode of application, its advantages and disadvantages along with a brief study of the analysis of thin plates and beams. The presently developed EFG technique is a truly meshless technique, as it doesn't bear the mesh, either for the construction of the shape functions, or for the integration of the original weak form. To properly understand the reason and functioning of the technique, a MATLAB software was created to analyse a Timoshenko Beam Problem utilising EFGM. Several cases of plane stress were considered in the design similar as beams subordinated to point loads and slightly distributed loads, and plates with varying shapes and boundary conditions using MFree2D simulation package. The major points of this design were twofold. first, to test the delicacy of the Element Free Galerkin Method by comparing it to accurate theoretical values, and second, to solve some common issues related to plates by the Element Free Galerkin Method.

Keywords **- Numerical calculation, element-free Galerkin, meshless technique. moving least places, Ansys.**

1. INTRODUCTION

The finite element technique is a numerical approach for solving problems in engineering and mathematical physics. Analysis of FEM Is used to commonly in now a days Typical problems area of interest in engineering and mathematical physics that are solvable by use of the Structure analysis, fluid flow, mass transfer, and electromagnetic potential are all examples of element methods.

For problem involving complicated geometries, loading, and material properties, it is generally not possible to obtain analytical mathematical solutions. Analytical solutions are those that are provided by a mathematical equation that provides the values of the required unknown quantities at any place in the body (whole structure) and so are acceptable for an unlimited number of body locations Mesh free methods use distinct points known as nodes on the problem domain and the boundary to define the problem. These analytical solutions often need the solution of ordinary or partial differential equations, which are frequently achievable because to the complicated geometries, loads, and material characteristics. Hence, we need to rely on numerical methods, such as the FEM for acceptable solution these numerical methods yielding approximately values of unknown at discrete numbers of points in the continuum.

Hence this process of modelling a body by dividing it into an equivalent system of smaller bodies or units interconnected at points common to two or more elements (nodal points) and boundary lines and surfaces is called discretization. The solution for structural problems typically refers to determining the displacement at each node and the stresses within each element making up the structure that is subjected to applied loads. This technique has been used with great accuracy for determination of various parameters of important in the system of consideration. But the complexity of the problem statement increases, the accuracy of the FEM becomes an issue. Since this methos requires the presence of a predefine mesh for proper analysis to be carried, modelling of structure with complicated geometries requires a very fine mesh arrangement or some times more than one mesh thus increasing the time, load and cost of the computation. As a result, there is a need to look for different computational approaches that may produce higher accuracy in less time. A lot of effort has been done to develop mesh free methods of analysis as an alternative to FEM. In meshless methods, the problem domain and boundary are defined by separate points called nodes.

The Element Free Galerkin Method was developed by Belytschko in 1994, it is based on the Diffuse Element Method (Nayroles 1992).

The EFG method's main characteristics include:

- Moving least square method is used to create shape function
- Galerkin weak form cretes discretized equations.
- A background mesh is created to carry out integration to obtained the system matrices

The following numerical techniques are often thought to belong to the broad category of "meshfree" methods.

1. Element free galerkin method
2. Finite pointing method
3. Generalized finite difference method
4. Mesh free moving kriging interpolation method
5. Natural element method

The Element Free Galerkin Method's salient characteristics include

- Moving least squares (MLS) approximation is employed for the construction of the shape functions.
- Galerkin weak form with constrains is employed to develop the discrete system equations.
  Background cells are required to perform the numerical integration or computing system matrices.

2. REVIEW OF LITERATURE

Meshless approaches have clear advantages in adaptive processes as well. For the most of meshfree algorithms, there exist a priori error estimates. This allows the definition of adaptive refinement processes as in finite element computations: an a posteriori error estimate is computed and the solution is improved by adding nodes/particles where needed or increasing the order of the approximation until the error becomes acceptable. Meshless approaches have clear advantages in adaptive processes as well. For the most of meshfree algorithms, there exist a priori error estimates. This makes it possible to define adaptive refinement processes similar to those used in finite element calculations, where an a posteriori error estimate is computed and the solution is improved by including more nodes or particles where required or raising the order of the approximation until the error is acceptable. Meshfree techniques were developed more than 25 years ago, but only recently have they attracted a lot of attention. This method, known as a galerkin, may be thought of as a subset of moving least-squares (MLS) approximations.

Lancaster has a thorough overview of MLS approximants (1981). Evidently, Nayroles et al. (1992) used moving least square approximations for the first time in the diffuse element approach was created using Galerkin weak form (DEM). The approach was improved and expanded to include discontinuous approximations by Belytschko et al. (1994), who dubbed it element-free Galerkin (EFG). Book of “Mesh Free Methods: Going Beyond the Finite Element Method”, G.R. Liu provides a detailed explanation of the EFG approach. There are several numerical examples used to demonstrate convergence studies and the impact of variables like the number of nodes, quadrature points, support size domains, etc. Moreover, G.R. Liu and Y.T. Gu produced a source code for FORTRAN that was used to analyse the EFG Method, and their book "An Introduction to Mesh Free Techniques and their Programming"[4] published the code. Source codes for were also produced by J. Dolbow and T. Belytschko[5]. their article "An introduction to programming the mesh-less Element Free Galerkin Method," which describes the EFG technique. The technique and flowchart for programming the EFGM for both 1D and 2D problem domains were thoroughly documented in the study. The paper really clearly explains the method and the theory that behind it. The MATLAB source codes for the same are also provided in addition to this. A uniform node distribution and a Gaussian quadrature order of 4 are assumed in order to simplify the problem.

3. MESHFREE METHODS BASED ON THE MOVING LEAST SQUARES APPROXIMATION

Using moving least squares approximation, meshless approaches the original creator of the moving least squares (MLS) approximation was

working on surface construction and data fitting (Lancaster and Salkauskas, 1981). As the MLS may offer a continuous approximation for a field function across the entire issue domain, it was essential for the creation of numerous MFree weak-form approaches. It is currently commonly used for creating MFree form functions in many different sorts of MFree techniques.

3.1 Formulation of MLS shape functions

The following approximate function in the form of series as:

$$(u)^h(x) = \sum_{j=1}^{m} p_j\,(x)\ a_j\,(x) = p^T\ a\ (x)$$

a(x) is the vector of coefficients given by where m is the number of monomials (polynomial basis)

$$a^T\,(x) = \{a_0\,(x)\ a_1\,(x) \ldots\ldots\ldots\ldots\ldots\ldots\ a_m\,(x)\}$$

It is important to notice that a(x) is an arbitrary function of x. The estimated values of the field function and the nodal parameters $u_1=u(x_1)$ are then used to generate a functional of weighted residual

$$J=\sum_{i=1}^{n} w(x-x_1)\ [u^h\,(x-x_1)\ (x_1)]^2$$

$$J=\sum_{i=1}^{n} w(x-x_1)\ [p^T\,(x_1)\ a(x)-u_1]^2$$

Were $w(x-x_1)$ is weight function the sufficient nodes are used

$$\frac{\partial J}{\partial a} = 0$$

This results in the coefficient vector being expressed as follows:

$$a(x) = A^{-1}\,(x)B(x)u_s$$

where A is the MLS moment matrix denoted by

$$A(x) = \sum_{i=1}^{m} w(x)p^T\,(x_1)\ p(x_1)$$

where $w(x) = w(x-x1)$ and B(x) has

$$u^h\,(x) = \sum_{i=1}^{n} \Phi(x)u_1$$

Φ(x) is MLS shape function

$$\Phi(x) = \sum_{i=1}^{m} p_I\,(x)A^{-1}(x)B(x)j_i u_i = p^T\ A^{-1}\ B_1$$

This brings us to another key moment about applying the essential boundary conditions to the weak form, because we cannot apply boundary conditions directly to u_1. As a result, we use Lagrange multipliers to enforce the necessary boundary condition.

3.2 Formulation of EFG

The system's partial differential equations and boundary conditions (2D issue) may be expressed as

$$L^T\sigma + b = 0$$

Boundary conditions are given by

$$U = \bar{u} \quad \text{on } \Gamma_t$$

L= divergent operator

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \end{bmatrix}$$

$$\int_{\Omega} \delta(Lu)^T (cLu)\, d\Omega - \int_{\Omega} \delta u^T b\, d\Omega - \int_{\Gamma_t} \delta u^T \bar{t}\, d\Gamma -$$

$$\int_{\Gamma_u} \delta\lambda^T (u - \bar{u})\, d\Gamma - \int_{\Gamma_u} \delta u^T \lambda\, d\Gamma = 0$$

$$\delta U^T [KU - F] + \delta \Lambda^T (G^T U - Q) + \delta U^T G \Lambda = 0$$

$$\delta U^T [KU + G\Lambda - F] + \delta \Lambda^T (G^T U - Q) = 0$$

Above equation can be written as

$$\begin{cases} KU + G\Lambda - F = 0 \\ G^T U - Q = 0 \end{cases}$$

4. STATISTICAL ANALYIS AND DISCUSSIONS

A MATLAB programme for linear static was created as a first test to understand the EFG approach. Timoshenko and Goodier developed this solution.

Consider a beam of length L=100mm and depth D=36mm subjected to a traction force F=1000KN at free end the beam subjected to completely in elastic nature, as illustrated in Figure 1

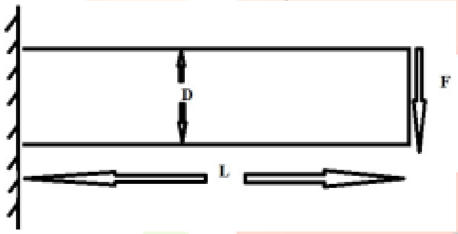

Figure 1 Timoshenko beam problem

The exact solution of the Timoshenko beam is given by the following equations, given by Timoshenko and Goodier

$$\sigma_{xx} = -Fy(L-x)/I$$

$$\sigma_{yy} = o$$

$$\sigma_{xy} = -Fy(D^2/4-y^2)/I$$

The solution of the stresses obtained by the EFG program created in MATLAB is shown in the following pages. The error of the EFGM technique is presented as a percentage error as well as a global energy error.

The below table 1 shows result of EFGM for first 30 quadrature points

Table 1 Result of EFGM

NODE NUMBER	STRESS(XX) N/mm^2	STRESS(YY) N/mm^2	STRESS(XY) N/mm^2
1	391.0747	-4.1317	0.7889
2	378.1780	-2.9634	-11.3414
3	357.6244	-2.1966	-25.7798
4	340.2911	-1.8312	-36.0242
5	387.9402	-2.1914	-0.2942
6	374.0728	-1.8346	-11.6402
7	353.3518	-1.7335	-25.5455
8	336.2587	-1.6839	-35.6780
9	382.4394	-0.8510	-1.3763
10	367.6004	-1.1212	-11.9967
11	346.8371	-1.4734	-25.3917
12	330.185	-1.6013	-35.32010
13	378.2045	-0.2871	-2.0884
14	362.6185	-0.8413	-12.3280
15	341.6608	-1.3989	-25.4682
16	325.1980	-1.6028	-35.2725
17	330.6604	-1.4330	-41.5631
18	311.9606	-0.7696	-51.5186
19	286.9381	-0.9345	-62.2734
20	267.9847	-0.8021	-69.8860
21	326.8750	-1.4009	-51.2087
22	308.6971	-0.8512	-62.1940
23	284.1575	-1.0187	-69.9980
24	265.3647	-0.9134	-40.7892
25	321.1214	-1.3983	-50.8137
26	303.6470	-0.9016	-61.9282
27	279.8461	-1.0703	-69.8748
28	261.4137	-1.0113	-40.7034
29	316.3157	-1.4279	-50.6846
30	299.2659	-0.9566	-61.8054

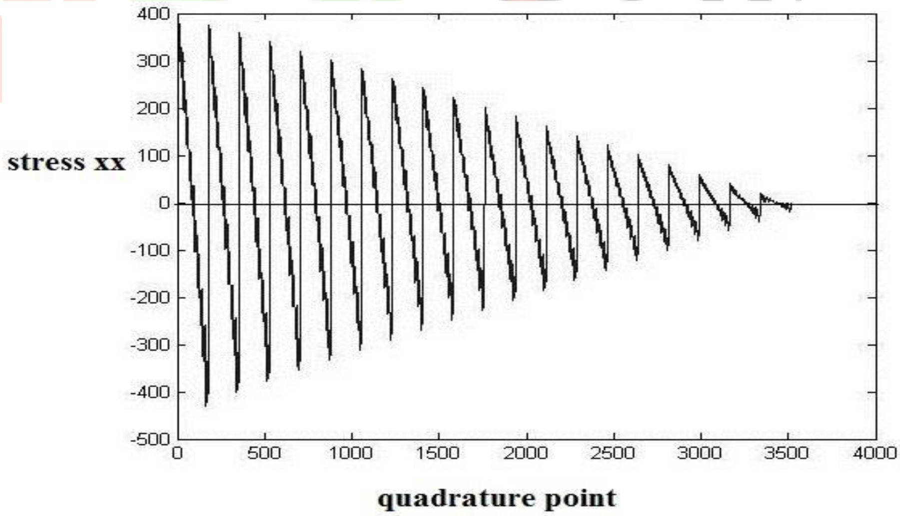

Figure 2 Variation of σ_{xx} with quadrature points

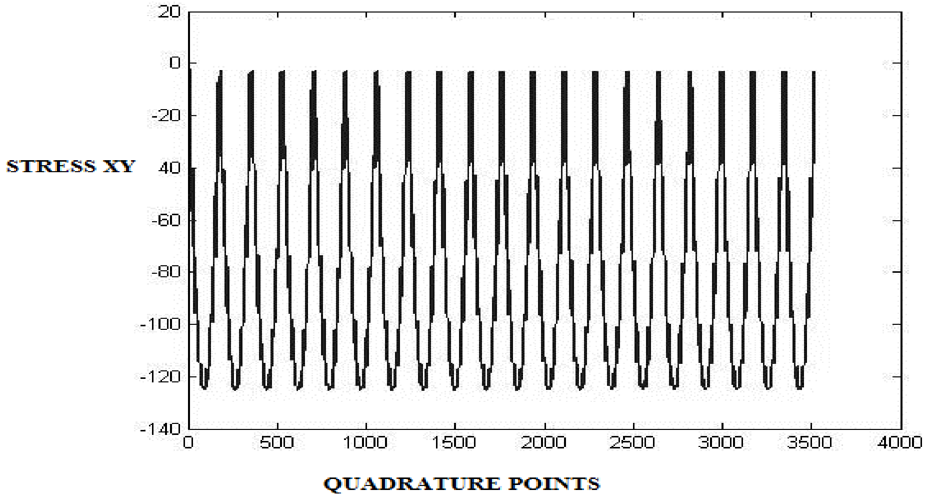

Figure 3 Variation of σ_{xy} with quadrature points

The following figures display the changes in direct stress (σ_{xx}) and shear stress (σ_{xy}) with quadrature points. For the purpose of computing the field parameters, arbitrary points called quadrature points are selected from the plate domain. In this example, 3200 quadrature points were selected. It must be remembered that the obtained values of the stresses depend on where the point is on the plate domain. Even if both points are equally distance from the fixed point, it is obvious that the stress will be compressive (negative) if the point is placed below the neutral axis and tensile (positive) if it is positioned above the neutral axis.

By determining the percentage error or by evaluating the global strain energy error e_{norm}, the mistake in the EFG Method may be determined in two different ways. We can determine the e_{norm} value to determine how quickly the technique will converge. The statistics below demonstrate how the total number of nodes affects the value of error calculated using both techniques.

Table 2 Convergence table for EFG method

Numberof nodes	σxx (N/ mm^2) (EFG)	σxx (N/ mm^2) (Exact)	σyy (N/ mm^2) (EFG)	σyy (N/ mm^2) (EFG)	σxy (N/ mm^2) (EFG)	σxy (N/ mm^2) (EFG)
231	-1.6611	-1.5849	0.0339	0	-1.2234	-1.1492
1071	-0.6322	-0.6384	0.0094	0	-0.5782	-0.5766

Table 3 Error values for different number of nodes

Number of Nodes	Error e_{norm}	Percentage error in σ_{xx}	Percentage error in σ_{yy}	Percentage error in σ_{xy}
231	0.0074	4.8%	------------	6.4%
1071	0.0034	0.97%	------------	0.27%

It is obvious that the EFG approach is not particularly accurate for low numbers of nodes, but accuracy increases significantly as the number of nodes increases. Also, it should be noted that when the number of nodes increases, the calculation time does not change significantly.

4.1 Cantilever Beam with Uniformly Distributed Load

Length of cantilever beam is 2m and depth is 0.5m, subjected to a traction force of 1KN. The elastic behaviour of beam is assumed and young's modulus is 3x10^{7}N/m^2.

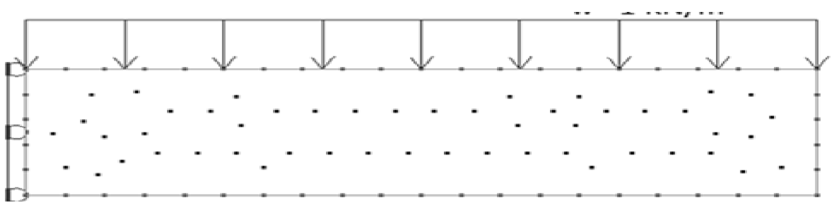

Figure 4 Cantilever beam with udl

The beam was manually analysed, and the stresses from the EFG approach were calculated using MFree2D. There is a comparison of the precise and obtained stresses. Two portions were evaluated in the domain analysis.

Section 1-1 at x=0m and Section 2-2 at x=1m

Table 4 Comparison of stresses obtained by EFG method and exact analysis

Sect. No.	σ_{xx} (N/mm^2) EFG Method	σ_{xx} (N/mm^2) Exact Analysis	Percentage Error
1-1	49850	48000	3.85%
2-2	11600	12000	3.33%

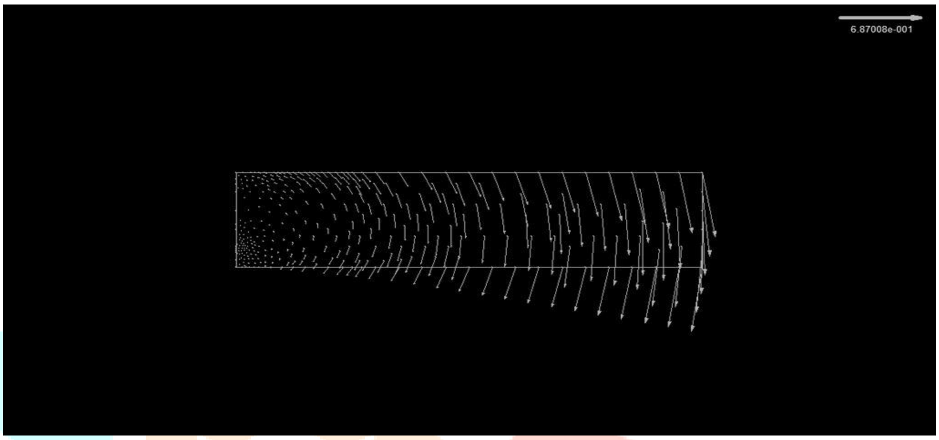

Figure 5 Displacement field vector

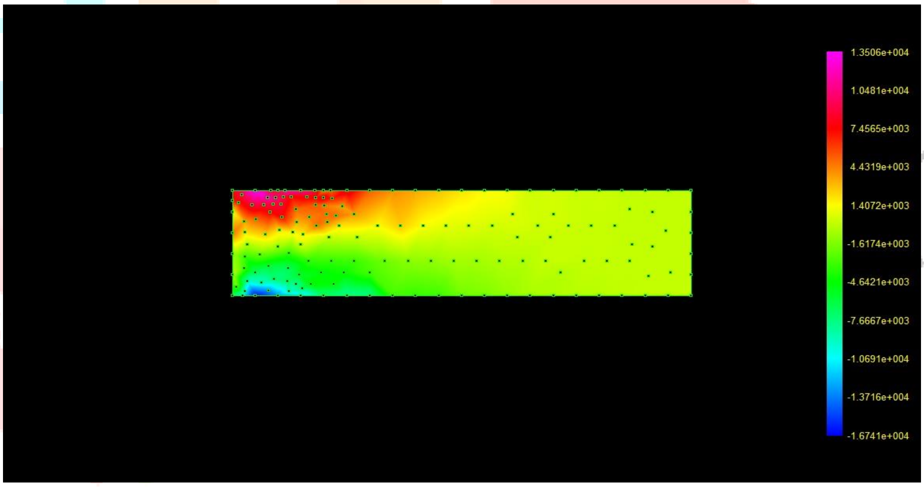

Figure 6 $\boldsymbol{\sigma}_{XX}$ Field vector

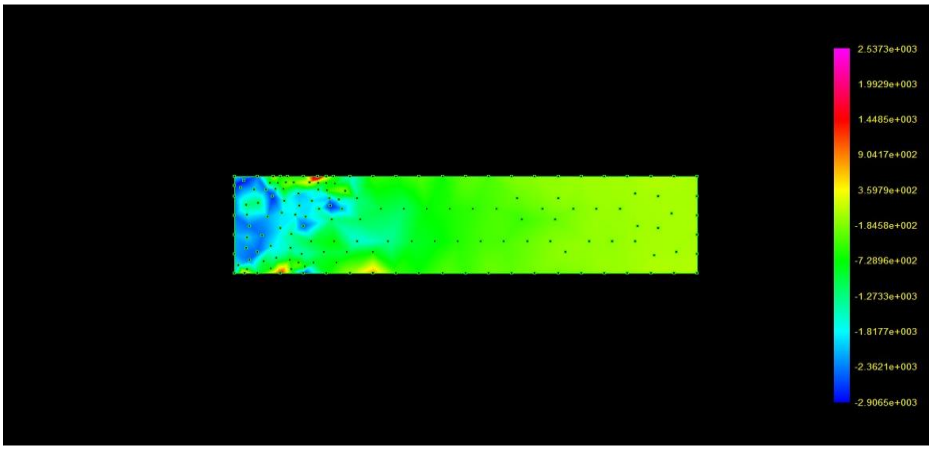

Figure 7 Shear stress field vector

It is discovered that the fluctuation of field parameters is comparable to the prior example. This is to be expected given that the test parameters are nearly identical, with the main difference being the existence of a distributed load rather than a single load. The shear stress field parameter increases as one moves away from the fixed end. This is due to the fact that the effective load acting is proportional to the distance from the fixed end. The variations of σxx and σxy for both of the preceding sections are displayed on the next pages.

Graphs for section 1-1

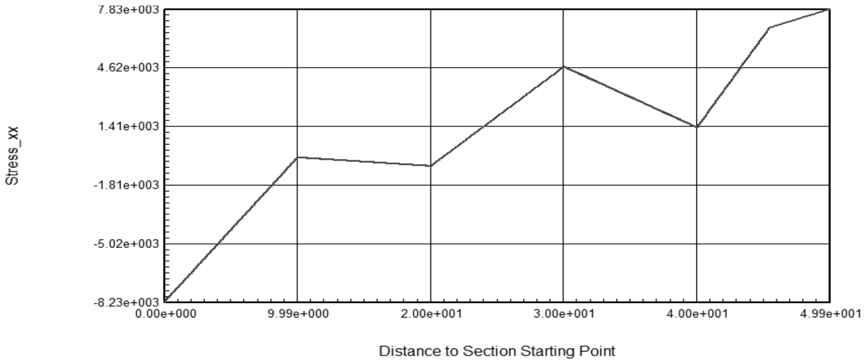

Figure 8 Plot for stress at section 1-1

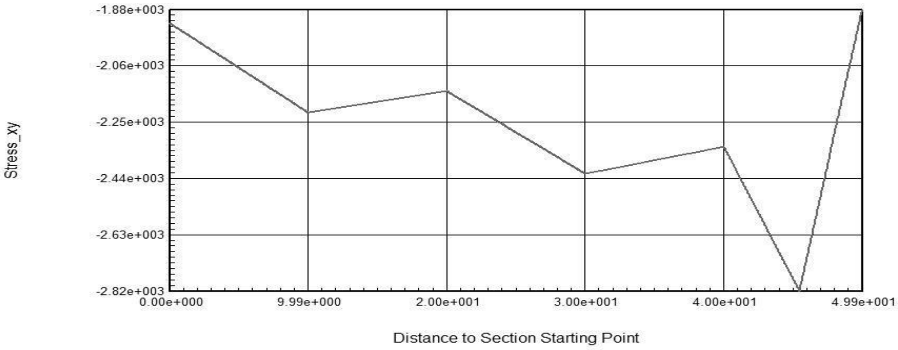

Figure 9 Plot for shear stress at 1-1

Graphs for section 2-2

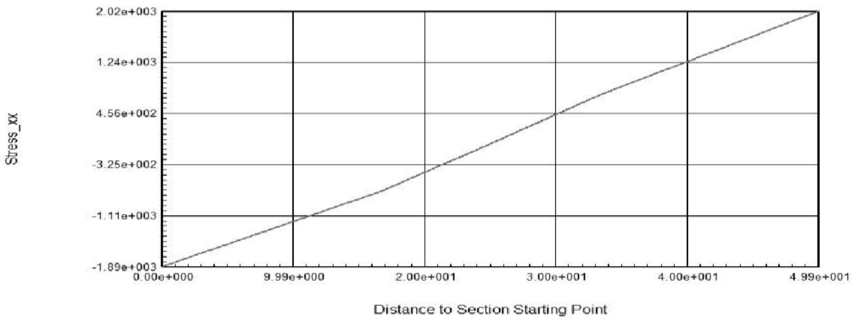

Figure 10 $\boldsymbol{\sigma}_{XX}$ at section 2-2

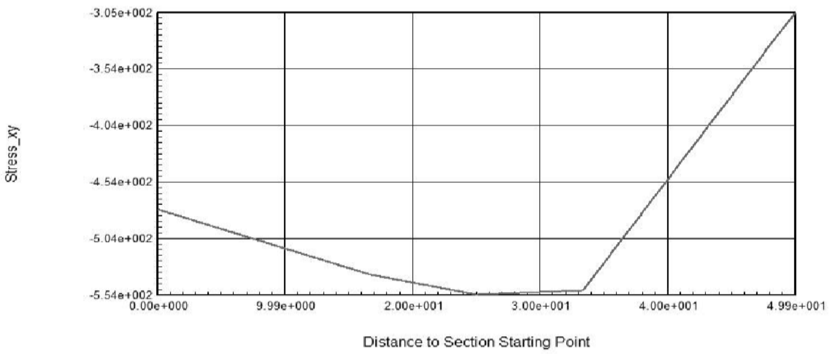

Figure 11 σ_{XY} at section 2-2

4.2 Square plate with one edge fixed and subject to udl

A thin 2m square plate is subjected to a distributed load of 1000N/m in the x direction. It is fixed on the opposite side of the force's action. All other sides are unrestricted. The material is considered to be entirely elastic, with a Young's Modulus of 3×10^7 N/m^2.

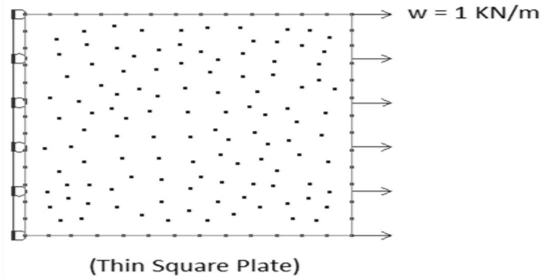

Figure 12 Square plate with one edge fixed and subject to udl

Timoshenko and Goodier [16] describe an analytical model of stresses on an infinite plate. Normal traction (Nx) is applied in the direction ox. The perforated plate's stress distributions (and) are described by

$$\sigma_{xx} = N_x\left[1 - \frac{R^2}{r^2}\left(\frac{3}{2}\cos(2\theta) + \cos(4\theta)\right) + \frac{3}{2}\frac{R^2}{r^2}\cos(4\theta)\right],$$

$$\sigma_{yy} = N_x\left[-\frac{R^2}{r^2}\left(\frac{3}{2}\cos(2\theta) - \cos(4\theta)\right) - \frac{3}{2}\frac{R^4}{r^4}\cos(4\theta)\right],$$

$$\sigma_{xy} = N_x\left[1 - \frac{R^2}{r^2}\left(\frac{3}{2}\sin(2\theta) + \sin(4\theta)\right) + \frac{3}{2}\frac{R^4}{r^4}\sin(4\theta)\right],$$

PLANE42 elastic elements with nodes and u.d.l are used in the FEM simulation with ANSYS software [17]. Modelling approach using EFG employs the generation ANSYS points in its methods for a fair distribution of points (Figure 12)

This problem has not been thoroughly investigated. The problem's main purpose is to demonstrate the use of the EFG technique in the computation of several field parameters, particularly stress (xx and xy) and displacement. The plate problem is a relatively common one, and it was chosen to make understanding and interpretation of the results as simple as possible. Plots of the field parameters are displayed, as well as variations of these values along two section lines is shown.

Sections taken:

Section 1-1 x=0 (along fixed end) and Section 2-2 (along the diagonal)

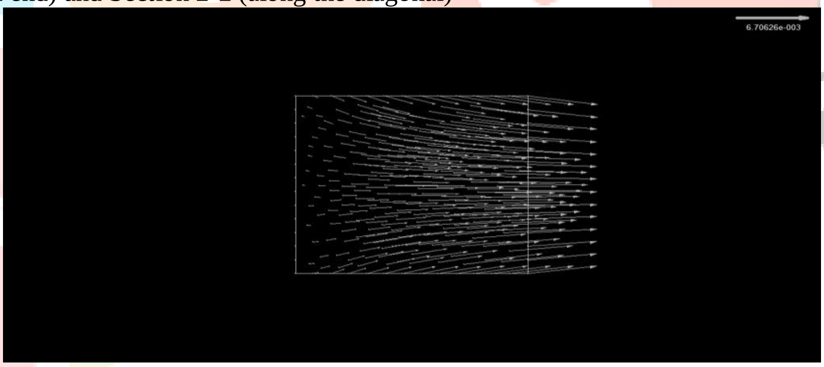

Figure 13 Displacement vector

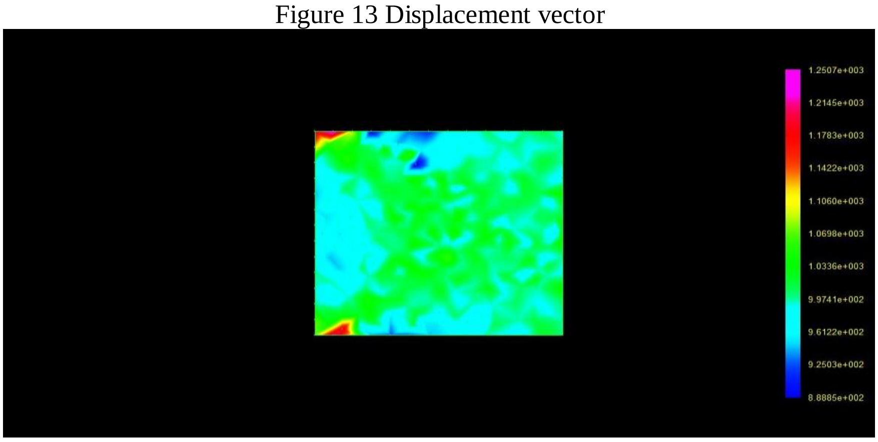

Figure 14 Field vector σ_{xx}

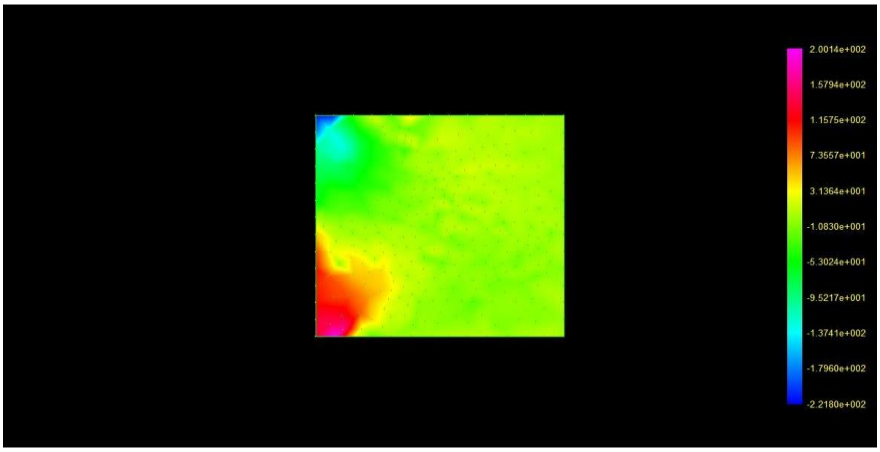

Figure 14 Field vector σ_{xy}

The field vector displacement illustrates that nodes closest to the fixed end are displaced less than nodes further away. The shifted placement of the nodes demonstrates the influence of the imparted force on the plate domain once again. The field vector σxx indicates that uniform stress is applied to the whole plate domain (997.4 N/m^2 to 1033.6N/m^2). The stress is highest in the plate's corners at the fixed end (1178.3 N/m^2). The field vector σ_{xy} is comparable to that of σ_{xx}, except that the shear is highest towards the fixed end and uniform throughout the plate. The graphs located on the following page illustrate the fluctuation of these parameters at the pre-defined sections.

Graph For Section 1-1

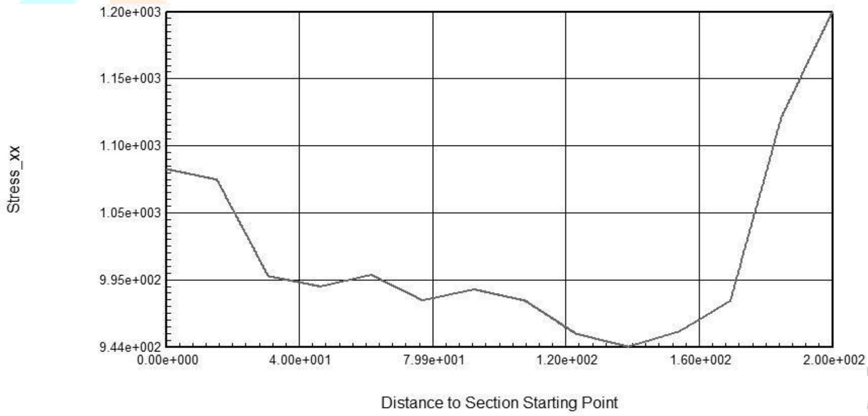

Figure 15 σ_{XX} at section 1-1

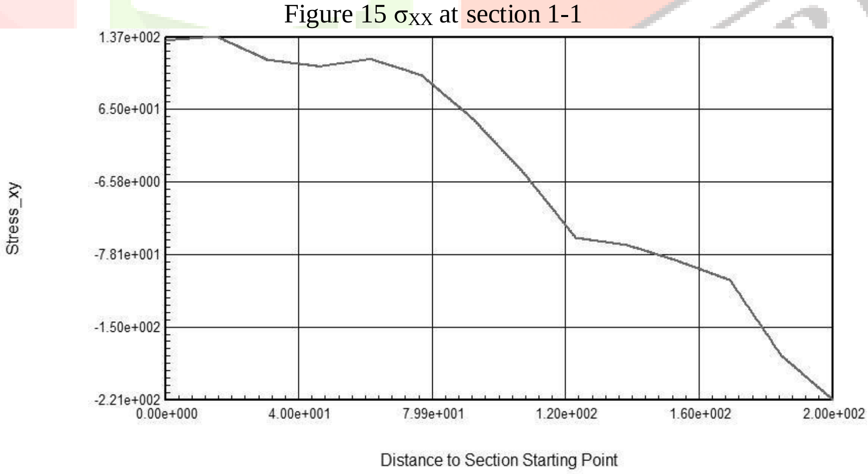

Figure 15 σ_{XY} at section 1-1

Graphs for section 2-2

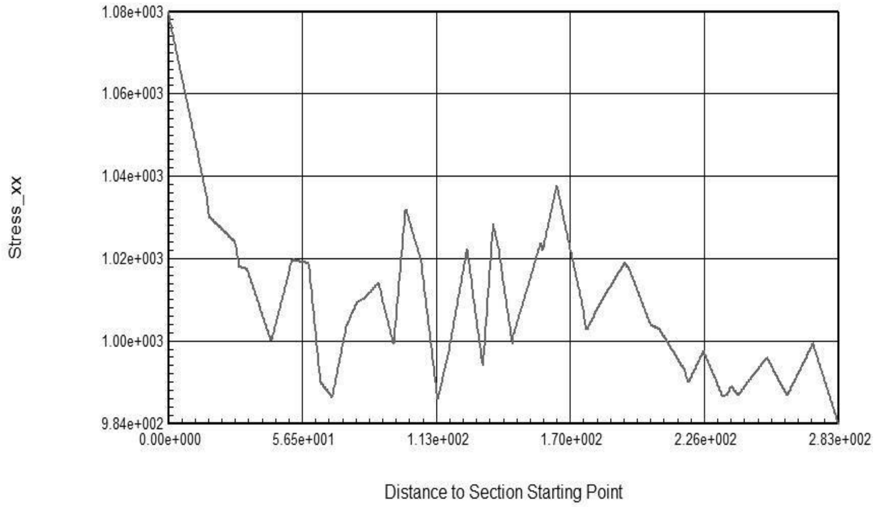

Figure 16 σ_{XX} for section 2-2

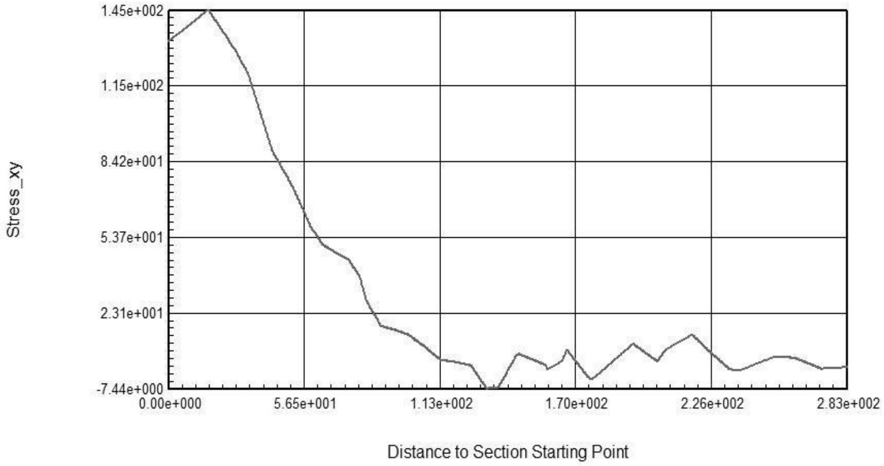

Figure 17 σ_{XY} at section 2-2

The displacement and stress in the plate have been accurately simulated by the Meshfree algorithm. The numerical results achieved with the EFG approach may be considered good and are nearly identical to the Analytics solution.

5. CONCLUSION

The mesh-free Element Free Galerkin technique is used to analyse a variety of structures or problem domains. A complete study of the approach showed that it is not entirely mesh-free since it uses a background mesh for integration**.**

The purpose of the first three numerical discussed was to show the EFGM's precision. The MFree2D software tool was used to solve the remaining numerical problems (including the final four) and to write the EFG analysis for the first technique. The findings reached were as follows.

- The number of nodes directly affects the EFGM's accuracy. The accuracy of the EFGM automatically improves with an increase in node count.
- Similar to this, we may increase the quadrature points while using the same number of nodes to reduce the error value. The backdrop mesh may be modified to achieve this. As the mesh is not predetermined, it may be improved without changing the domain. As a result, the overall computation time is unaffected.

Hence, the EFGM proves to be a very accurate analytical approach for elasto-statics with the right selection of the number of nodes and the quadrature points.

It is clear from the information above that the Element Free Galerkin approach may be used to precisely and effectively analyse 2D objects with a variety of geometries and loads.

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