



Sensex- a GARCH Volatility Analysis

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Abstract: Investors need volatility estimation for almost every kind of investment in order to maximize their returns. GARCH family models have been a major helper for the investor in this area. Present study investigates the capabilities of GARCH (1,1), under various error distributions, in effectively modelling volatility in the Indian market index Sensex.

Index Terms – Stock market, Volatility, GARCH, Model Estimation

I. INTRODUCTION

Volatility estimation has been extensively researched in past. The importance given to this area in the research field is because volatility inputs are required by all kinds of players in almost all kinds of markets with various tradable assets. Be it hedgers, speculators, arbitrageurs or the genuine investors, all require to know how much variations are expected in the return of the asset they are trading. Generalised Autoregressive Conditional Heteroskedastic (GARCH) model has been expansively utilized across assets and markets for volatility modelling and estimation. There have been so many versions of the basic GARCH model that have been developed till date, that now there is a whole family popularly known as the GARCH-family which can be used for volatility modeling. Out of all these, GARCH (1,1) still is the most idolized model in the area of volatility estimation. Moreover, Indian stock markets have grown prodigiously over the past few decades and has left its mark at the world investment platform. With this growth in the markets, not much emphasis has been given in testing the Indian markets, for volatility modelling. Present study tries to fill this gap by applying GARCH (1,1) model for modelling volatility of Sensex. There have been many studies conducted on application of GARCH model in volatility modelling of different assets, some of which includes; Akigray (1989), Hansen and Lunde (2001), Doidge and Wei (1998), Nelson (1991), Tully and Lucey (2007), Walsh and Tsou (1998), Madhusudan Karmarkar (2005), Awartani and Corradi (2005), Banumathy and Azhagaiah (2015), Pandey (2005), Seo and Kim (2020), Bergsli, Lind and Molnar (2022), etc

The present study gets segregated into following sections: first section introduces GARCH model briefly; second section explains the data and some preliminary tests performed; third section presents the results and discussions and last section concludes the study.

II. GARCH MODEL

The Generalized Autoregressive Conditional Heteroskedastic (GARCH) model was developed by Bollerslev in 1986 and is defined in its simplest form as follows:

$$X_t = \mu_t + \sigma_t \epsilon_t \quad \dots\dots\dots[1]$$

$$\sigma_t^2 = \omega + \alpha \epsilon_t^2 + \beta \sigma_{t-1}^2 \quad \dots\dots\dots[2]$$

which says that the conditional variance at time t depends not only on the squared error term in the previous time period [as in ARCH (1,1)] but also on its conditional variance in the previous time period.

III. DATA AND PRELIMINARY RESULTS

Present study uses the daily closing prices of Sensex from 2015 to 2020. The GARCH (1,1) is applied for volatility modeling of the index with two different error distributions, that is normal distribution and the student's t distribution. Sensex is an index reflecting average of 30 companies listed on Bombay Stock Exchange.

Figure 1: Descriptive statistics and histogram of Sensex return series

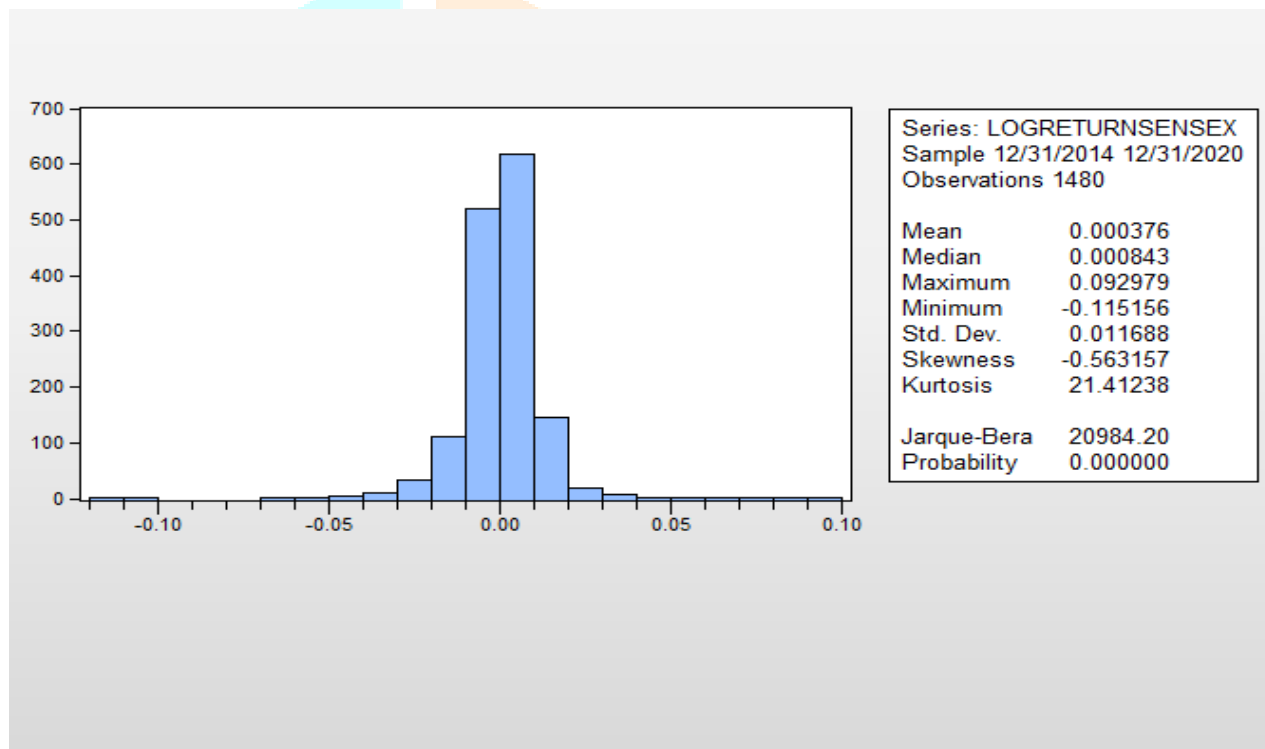
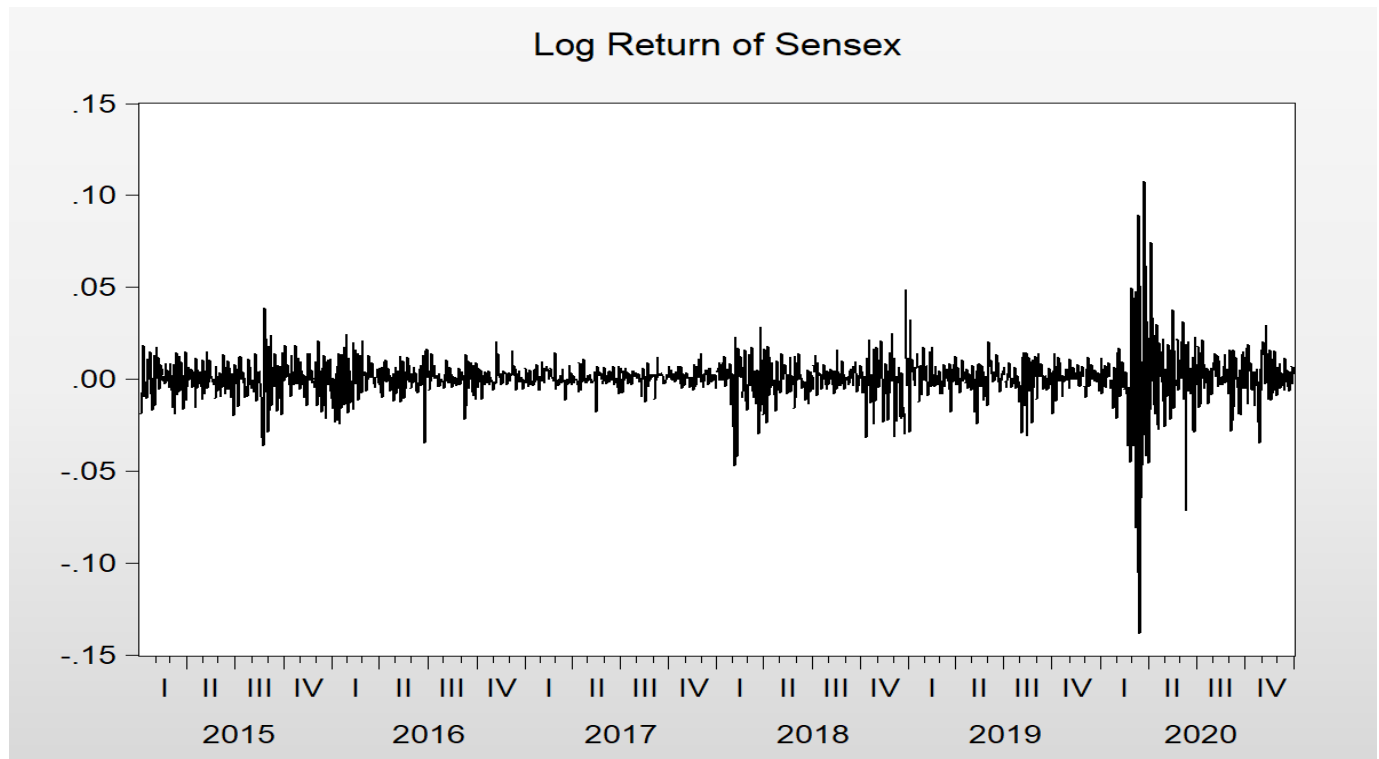


Figure 2: Sensex log return series for the period 2015 to 2020



The descriptive statistics is given in figure 1 for the two indices along with the histograms. As can be seen from the figure mean of the return series is close to zero, thereby indicating that the squared returns can be used as a measure of variance for the series. Sensex return series have negative skewness and very high kurtosis. The Jarque-Bera test of normality is 20984.2 for Sensex with probabilities zero, thereby indicating non-normality in the series. Figure 2 shows the Sensex log return series graph, which depicts the volatility clustering in Sensex returns indicating the appropriateness of using a GARCH model.

4. RESULTS AND DISCUSSIONS

The GARCH (1,1) model is applied to the Sensex return series with two different error distributions, namely the normal distribution and the Student's t distribution. The daily close prices are taken for the period 2015 to 2020. The estimation results for GARCH (1,1) with the two different error distributions are shown in table 1 and 2 respectively. Table 1 shows the estimations results of GARCH (1,1) with normal error distribution. As can be seen from table 1, GARCH (1,1) estimation results show that the estimated variance coefficients are significant at 5% as well as 1 % level of significance. The ARCH and the GARCH terms are 0.110456 and 0.856228 respectively which add up to less than one indicating a stationary variance process. The log likelihood value is 4776.235 and the AIC and BIC criterion are -6.451975 and -6.434060 respectively.

Table 1: GARCH (1,1) estimation results with normal error distribution

Dependent Variable: LOGRETURNSensex
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 02/21/23 Time: 09:33
Sample (adjusted): 1/01/2015 12/31/2020
Included observations: 1479 after adjustments
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000832	0.000209	3.979537	0.0001
LOGRETURNSensex(-1)	-0.033700	0.028247	-1.193035	0.2329
Variance Equation				
C	4.04E-06	8.87E-07	4.554176	0.0000
RESID(-1)^2	0.110456	0.009412	11.73604	0.0000
GARCH(-1)	0.856228	0.015713	54.49030	0.0000
R-squared	0.001568	Mean dependent var		0.000376
Adjusted R-squared	0.000892	S.D. dependent var		0.011692
S.E. of regression	0.011687	Akaike info criterion		-6.451975
Sum squared resid	0.201740	Schwarz criterion		-6.434060
Log likelihood	4776.235	Hannan-Quinn criter.		-6.445296
Durbin-Watson stat	2.051685			

Table 2 shows the estimation results for GARCH (1,1) model with student's t error distribution. Again the table shows the estimated variance coefficients are significant at both 5% and 1% levels of significance. The ARCH and GARCH terms are 0.083758 and 0.880175 respectively adding up to less than one displaying stationarity in variance process. The log likelihood value is 4845.506 and the AIC and BIC criterion are -6.544294 and -6.522787 respectively.

Table 2: GARCH estimation results with student t error distribution

Dependent Variable: LOGRETURNSensex
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 02/21/23 Time: 09:33
Sample (adjusted): 1/01/2015 12/31/2020
Included observations: 1479 after adjustments
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000776	0.000206	3.765585	0.0002
LOGRETURNSensex(-1)	-0.007684	0.025988	-0.295679	0.7675
Variance Equation				
C	3.59E-06	1.12E-06	3.215873	0.0013
RESID(-1)^2	0.083758	0.016439	5.095130	0.0000
GARCH(-1)	0.880175	0.021753	40.46147	0.0000
T-DIST. DOF	5.575279	0.689914	8.081127	0.0000
R-squared	-0.000267	Mean dependent var		0.000376
Adjusted R-squared	-0.000944	S.D. dependent var		0.011692
S.E. of regression	0.011698	Akaike info criterion		-6.544294
Sum squared resid	0.202111	Schwarz criterion		-6.522797
Log likelihood	4845.506	Hannan-Quinn criter.		-6.536280
Durbin-Watson stat	2.104861			

By comparing the estimation results of the two GARCH models, one can see a few points. Firstly, according to the Log likelihood criteria, which should be maximized in any given case, the student's t distribution performs better as it has the Log likelihood value as 4845.506 which is higher than the value 4776.235 for normal distribution. So, if an investor takes the decision according to Log likelihood criteria he will prefer student's t error distribution while implementing a GARCH model on Sensex. But, the results would be totally opposite if the decision criterion is AIC or BIC or Hannen-Quinn values. As per these values which should be minimized for a given case, the normal error distribution should be preferred if these are to be minimized for Sensex return series. The Durbin-Watson test for both the case is more than two thereby indicating the absence of serial correlation in the residuals. This shows both the models are able to remove the ARCH effects quite satisfactorily.

V. CONCLUSIONS

The reason why so much importance is given to volatility modeling in the research arena is because volatility inputs are required by all kinds of players in almost all kinds of markets with various tradable assets. Generalized Autoregressive Conditional Heteroskedastic (GARCH) model has been expansively utilized across assets and markets for volatility modelling and estimation. Present study tries to investigate how GARCH model performs in Indian set up by comparing the model's performance with two different error distributions, namely the normal error distribution and the student's t error distribution. The results show that GARCH (1,1) with normal error distribution fits the data better if AIC, BIC or Hannen-Quinn criterions are followed. Oppositely, if Log likelihood criteria is followed then the GARCH (1,1) with student's t error distribution should be preferred since its value is much higher than that of GARCH (1,1) with normal distribution. The investor should be cautious of the sensitivity of the choice of criteria utilized in model estimation.

REFERENCES

- [1]. Akgiray, V. (1989). "Conditional Heteroscedasticity in Time Series of Stock Returns: Evidence and Forecasts", *Journal of Business*, 62, 55-80.
- [2]. Awartani, B. M., & Corradi, V. (2005). Predicting the volatility of the S&P-500 stock index via GARCH models: the role of asymmetries. *International Journal of Forecasting*, 21, 167-183.
- [3]. Banumathy, K., & Azhagaiah, R. (2015). Modelling Stock Market Volatility: Evidence from India. *Managing Global Transitions: International Research Journal*, 13(1).
- [4]. Bergsli, L.Ø., Lind, A.F., Molnár, P. and Polasik, M., 2022. Forecasting volatility of Bitcoin. *Research in International Business and Finance*, 59, p.101540.
- [5]. Doidge, C. and Wei, J.Z. (1998). "Volatility Forecasting and the Efficiency of the Toronto 35 Index Options Market", *Canadian Journal of Administrative Sciences/ Revue Canadienne des Sciences de l'Administration*, 15(1), 28-38.
- [6] Hansen, P. and Lunde, A. (2001). "A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)?" Working Paper Series, Brown University.
- [7]. Madhusudan, Karmarkar. (2005), "On the Predictive Ability of Several Common Models of Volatility: A Study on S&P CNX Nifty", IIM (L) Working Paper Series.

- [8]. Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the econometric society*, pp.347-370.
- [9]. Pandey, A. (2005). "Volatility Models and their Performance in Indian Capital Markets", *Vikalpa*, 30(2), 27.
- [10]. Seo, M. and Kim, G., 2020. Hybrid forecasting models based on the neural networks for the volatility of bitcoin. *Applied Sciences*, 10(14), p.4768.
- [11]. Tully, E. and Lucey, B.M., 2007. A power GARCH examination of the gold market. *Research in International Business and Finance*, 21(2), pp.316-325.
- [12]. Walsh, D.M. and Tsou, G.Y.G. (1998). "Forecasting Index Volatility: Sampling Interval and Non-trading Effects", *Applied Financial Economics*, 8(5), 477-485.

