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ANALYSIS OF HIGH TEMPERATURE NATURAL CONVECTION IN OPEN VERTICAL TUBES WITH VOLUMETRIC HEAT GENERATION

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ABSTRACT: Numerical analysis of natural convection in open ended ended two dimensional axisymmetric vertical tube has been carried out for a wide temperature range to find out the relation between non-dimensionalized number. Walls are maintained at constant wall temperature due to which convective flows were analyzed by varying the wall temperature and aspect ratio of the tube. The simulation is carried for turbulence model of natural convection. It was found that there is no single non dimensional number on which the flow can dependent. Numerical analysis of two-dimensional natural convection in open ended vertical tube has been carried out for a wide range of heat generation rate to find out the appropriate non-dimensional numbers governing the process. Buoyancy induced flows in a tube with adiabatic wall boundary condition and uniform volumetric heat generation rate in fluids were analyzed by varying heat generation rate and the geometrical aspect ratio of the tubes with and without considering surface radiation. Curve fits have been provided for non-dimensionalized mass flow rate versus other relevant non-dimensional numbers based on heat generated and geometry of the tube. Non-dimensional correlation has been found out for natural convection flow in an open adiabatic tube with internal heat generation and neglecting surface radiation. One more non-dimensional correlation has been found out for natural convection flow in an open adiabatic tube with internal heat generation and considering surface radiation.

Keywords: Natural convection, vertical tubes, heat generation rate, variable properties, non-dimensional numbers, buoyancy flow, Turbulent flow.

INTRODUCTION: Natural convection is an essential mode of heat and mass transfer from the point of view of cooling and mass transfer in absence of any other flow creating mechanism. It is the easiest and most inexpensive way to cool internal surfaces, create necessary flow in Bio-mass chulhas. Natural convection is particularly relevant in electronic cooling systems as one hand and biomass combustion systems on the other, the former deals with low temperatures (generally less than 100°C), the latter which includes rural household chulhas, and rural industrial devices in which Bio-mass is burned, deals with higher temperatures (upto 1200°C).

Analysis of high temperature natural convection systems is more complex than that of low temperature natural convection systems due to strong dependence of properties. Thus we cannot extend the results of low temperature work directly to the high temperature cases. In particular, the issue of identifying the relevant non-dimensional numbers which govern the high temperature natural combustion flow has not been addressed in the literature.

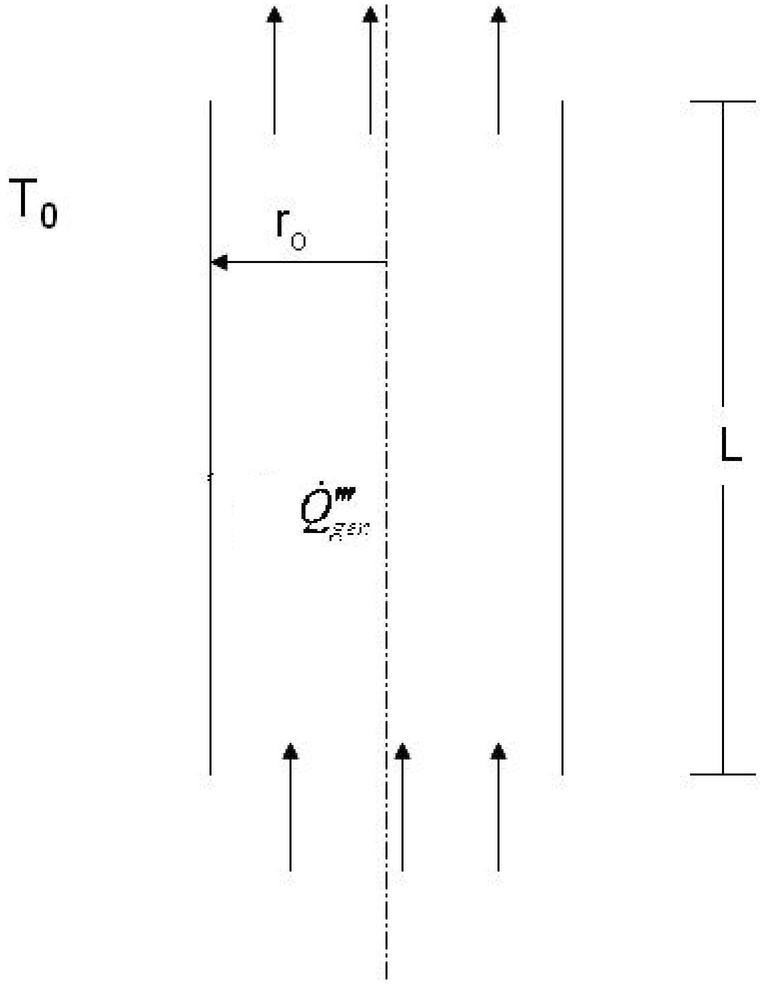
The focus of this project was to study high temperature natural convection in an open vertical tube through numerical simulation. For the results to be useful in combustion devices, the source of buoyancy is taken as the heat generated in the bulk of the gas.

PROBLEM DEFINATION: The problem that is studied is basically a chulha problem in which combustion is taking place inside the tube and air is being inducted because of this heat generation. Thus this problem is a coupled problem in which combustion, continuity, momentum and energy equations are solved simultaneously. Earlier, Ravi (2006) dealt with the similar problem and in his approach he analyzed the natural convection in open vertical tube assuming constant wall temperature. As a first step of approach, numerical analysis of the free convection heat transfer with volumetric heat generation in an open vertical adiabatic tube is considered here. Although simplified, this approach is more practical from the point of view of chulha problem. Thus, this project aims to conduct a numerical study of natural convection in an open vertical tube for a range of wall temperatures and geometrical aspect ratio. Simulation results are used to generate correlations for non-dimensional mass flow rate (NDMF) in terms of relevant non-dimensional parameters.

REVIEW OF LITERATURE: After a rigorous literature survey for the similar problems it was found that literature related to natural convection, induced because of volumetric heat generation in the fluid inside the tube is scarce. Thus the literature survey was carried in such a manner so as to include articles relating to the present geometry and similar geometry. Also, looking at the physics of this high temperature natural convection problem variable property formulation was studied. The articles reviewed include work with Boussinesq-constant property formulation, variable property formulation, phenomenon like backflow, transition from laminar to turbulent. Although Boussinesq approximation gives valid results at low temperature and are used widely, it cannot be used at higher temperature range. So the variable property analysis must be done for high temperature buoyancy flow cases.

Guo *et al.* (1993) analyzed the variable property effects on a natural convection flow in vertical ducts and studied both constant wall temperature and constant wall flux condition. They observed that mass flow rate doesn’t increase monotonically but has a maximum at a certain high temperature .They attributed this phenomenon to the fact that thermal drag increases more rapidly than the buoyancy and termed it as thermal clogging. For constant wall flux case, same trend was observed. They also found Boussinesq approximation giving erroneous results at higher temperatures for the same case.

MATHMATICAL FORMULATION



Vertical tube with volumetric heat generation and adiabatic wall

The focus of the present work is the buoyancy induced flow through a vertical tube open at both ends. Consider a vertical tube, with its inner radius r_o and height L. If the tube walls were adiabatic, with ambient at temperature T_o and heat is generated in the fluid inside the tube, this results in higher fluid temperature inside the tube as compared to ambient and hence lower fluid density as compared to ambient. If the two ends of the tube were open, this would set in buoyancy induced flow or natural convection through the tube entering at the bottom and exiting the top. Thus natural convection is governed by the laws of mass, momentum and energy. conservation. In a real situation, the heat could be generated by combustion of fuel (like Bio-mass). In that case, the species conservation will also govern the phenomena under consideration. However, it was assumed for simplification that the fluid is air in which volumetric heat generation is taking place inside the tube and flow is induced because of this heat generation. So, no combustion is considered inside the tube and hence species conservation equations are not considered. Thus, governing equations for the buoyancy driven flow under consideration are continuity, momentum and energy equations.

GOVERNING EQUATION: The geometry under consideration is axisymmetric. Hence, equations are considered in cylindrical co-ordinates and the problem is solved as 2-dimensional steady state problem. Since the pressure differences in a natural convection problem are very small, the flow is considered to be incompressible and laminar. The governing equations are thus given by:

Continuity Equation

$$\frac{\partial v_x}{\partial x} + \frac{1}{r}\frac{\partial (r v_r)}{\partial r} = 0$$

Axial Momentum Equation

$$\frac{\partial}{\partial x}\left(\rho v_x^2\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho v_r v_x\right) = -\frac{\partial p}{\partial x} + 2\frac{\partial}{\partial x}\left(\mu\frac{\partial v_x}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\mu\left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x}\right)\right] - \rho.g$$

Radial Momentum Equation

$$\frac{\partial}{\partial x}\left(\rho v_x v_r\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho v_r{}^2\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial x}\left[r\mu\left(\frac{\partial v_r}{\partial x} + \frac{\partial v_x}{\partial r}\right)\right] + \frac{2}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v_r}{\partial r}\right) - \frac{2}{3}\mu\frac{v_r}{r^2}$$

Energy Equation

$$\frac{\partial}{\partial x}\left(\rho c_p v_x T\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho C_p v_r T\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + Q'''_{gen}$$

Where, v_x and v_r are velocities in axial and radial directions respectively in m/s

ρ is density of the fluid inside the tube in Kg/m^3

μ is viscosity in N-s/ m^2

k is thermal conductivity in W/ m^2-K

C_p is specific heat of the fluid in J/Kg-K

Q'''_{gen} is volumetric heat generated in W/ m^3

Radiation Equation: Radiative heat transfer was included in this analysis as radiant heat flux is large compared to the heat transfer rate due to convection. Radiative heat transfer dominates in high temperature cases because radiative heat flux has a fourth order dependence on temperature.

The radiative transfer equation (RTE) for an absorbing, emitting, and scattering medium at position $\vec{r}$ in the direction $\vec{s}$ is given as:

$$\frac{dI(\vec{r},\vec{s})}{ds} + (a+\sigma_s)I(\vec{r},\vec{s}) = an^2\frac{\sigma T^4}{\pi} + \frac{\sigma_s}{4\pi}\int_0^{4\pi} I((\vec{r},\vec{s}\,')\phi(\vec{s},\vec{s}\,'))d\Omega'$$

where $\vec{r}$ = position vector

$\vec{s}$ = direction vector

$\vec{s}\,'$ = scattering direction vector

s = path length

a = absorption coefficient

n = refractive index

σ_s = scattering coefficient

σ = Stefan-Boltzmann constant (5.672×10^{-8} W/m^2 -K^4)

I = radiation intensity, which depends on position ($\vec{r}$) and direction ($\vec{s}$)

T = local temperature

ϕ = phase function

Ω' = solid angle

As already discussed, the fluid is assumed to be non-interfering medium where no scattering is taking place. So, scattering coefficient is taken as zero and refracting index is assumed to be unity. The equation for the change of radiation intensity, dI , along a path, ds, can be written as:

$$\frac{dI}{ds} + aI = \frac{a\sigma T^4}{\pi}$$

This above equation is integrated along a series of rays emanating from boundary faces. If a is constant along the ray:

$$I(s) = \frac{\sigma T^4}{\pi}\left(1-e^{-as}\right) + I_o e^{-as}$$

As, only surface radiation was considered gas absorption coefficient was assumed to be zero, thus

$$I(s) = \frac{\sigma T^4}{\pi} + I_o$$

where I_o is the radiant intensity at the start of the incremental path, which is determined by the appropriate boundary condition.

BOUNDARY CONDITIONS

The following two cases are analyzed here:

Laminar flow with heat generation and neglecting wall radiation.

Laminar flow with heat generation and considering wall radiation.

In both cases same boundary conditions were used for energy and momentum equations, except radiation boundary conditions in the second case.

Energy equation

For energy equation it was assumed that fluid at ambient temperature T_o enters the vertical tube. At the axis, the radial gradient of temperature considered to be zero. Wall was assumed to be adiabatic so no heat is conducted to the tube. Volumetric heat generation rate was assumed to be constant.

Momentum equations

The pressure boundary conditions were taken according to Aihara (1973), the stagnation pressure at inlet was taken to be equal to the ambient pressure. At outlet, the static pressure was taken to be equal to the ambient pressure. At axis, the radial velocity and the radial gradient of axial velocity was considered to be zero. In addition to this, no slip condition was also employed at tube wall.

Boundary	Position	Continuity and momentum	Energy equation
Inlet	x=0	$p_{stag} = p_a$	$T = T_o$
Exit	x=L	$p_s = p_a$	$\frac{\partial T}{\partial r} = 0$
Axis	r=0	$v_r = 0, \frac{\partial \rho v_x}{\partial r} = 0$	$\frac{\partial T}{\partial r} = 0$
Wall	$r=r_o$	$v_r = 0, v_x = 0$	$\frac{\partial T}{\partial r} = 0$

Radiation equation : For solving radiation equation, wall was considered to be gray and diffuse. Wall emissivity was considered to be 1 and gas absorption coefficient was taken as zero, for taking only surface radiation in account. It was assumed that black body absorption is taking place at both inlet and outlet so emissivity of the flow inlet and flow outlet was taken as unity.

NON-DIMENTIONAL GOVERNING EQUATION : The above equations can be non-dimensionalized in different ways. In literature it was found that either Grashof number or Rayleigh number are the relevant non-dimensional parameters that are governing the buoyancy flows. Considering this fact, non-dimensionalization is done in such a manner that Grashof number term appears in the equation. Here two sets of non-dimensionalized equations are obtained.

First method of non-dimensionalization

Above equations were non-dimensionalized below using following non-dimensional variables corresponding to reference temperature.

$$R = \frac{r}{r_o}, \hat{\mu} = \frac{\mu}{\mu_o}, \hat{k} = \frac{k}{k_o}, \hat{C}_p = \frac{C_p}{C_{po}},$$

$$X = \frac{x}{r_o}, \hat{\rho} = \frac{\rho}{\rho_o}, V = \frac{v_r r_o}{v_o}, U = \frac{v_x r_o}{v_o}$$

$$\theta = \frac{T - T_o}{T_{ref} - T_o}, \hat{p}_d = \frac{(p - p_a) r_o^2}{\rho_o v_o^2}$$

Where,

$$\Delta T_{ref} = T_{ref} - T_o = \frac{Q'''_{gen} r_o^2}{k_o} \quad \text{and,}$$

$p = p_d + p_a$, p_a being ambient pressure follows the relation $\frac{\partial p_a}{\partial x} = -\rho_o g$.

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{1}{R}\frac{\partial (RV)}{\partial R} = 0$$

Axial Momentum equation

$$\frac{\partial}{\partial X}(\hat{\rho} U^2) + \frac{1}{R}\frac{\partial}{\partial R}(R\hat{\rho} UV) = -\frac{\partial \hat{p}_d}{\partial X} + 2\frac{\partial}{\partial X}\left(\hat{\mu}\frac{\partial U}{\partial X}\right) + \frac{1}{R}\frac{\partial}{\partial R}\left[R\hat{\mu}\left(\frac{\partial U}{\partial R} + \frac{\partial V}{\partial X}\right)\right] + \theta.Gr$$

Radial Momentum equation

$$\frac{\partial(\hat{\rho} UV)}{\partial X} + \frac{1}{R}\frac{\partial(\hat{\rho} RV^2)}{\partial R} = -\frac{\partial \hat{p}_d}{\partial R} + \frac{\partial}{\partial R}\left[\hat{\mu}\left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial R}\right)\right] - \frac{2}{3}\frac{\hat{\mu} V}{R^2} + \frac{2}{R}\frac{\partial}{\partial R}\left(R\hat{\mu}\frac{\partial V}{\partial R}\right)$$

Energy Equation

$$\frac{\partial}{\partial X}(\hat{\rho}\hat{C}_p U\theta) + \frac{1}{R}\frac{\partial}{\partial R}(R\hat{\rho}\hat{C}_p V\theta) = \frac{1}{\Pr_o}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\hat{k} R\frac{\partial \theta}{\partial R}\right) + \frac{\partial}{\partial X}\left(\hat{k}\frac{\partial \theta}{\partial X}\right) + 1\right]$$

Where, Prandtl number and Grashof number are defined as

$$Gr = \frac{g\beta Q'''_{gen} r_o^5}{k_o \nu_o^2}, \Pr = \frac{\nu_o}{\alpha_o}$$

where β is defined as $\beta = \frac{1}{T_o}$

T_o = reference temperature (293.16 K)

Second method of non-dimensionalization

Later, the need of analyzing the effect of geometry and temperature on the flow was felt. So it was decided to separate out terms representing geometrical aspect and temperature aspect from the Gr, and thus retaining Gr as a relevant non-dimensional parameter. So, the governing equations were non-dimensionalized such that effect of heat generation and geometry could be studied separately through two different non-dimensional parameters.

In the second method of non-dimensionalization the axial momentum equation and energy equation were modified as:

Axial Momentum equation

$$\frac{\partial}{\partial X}\left(\hat{\rho} U^2\right) + \frac{1}{R}\frac{\partial}{\partial R}\left(R\hat{\rho}UV\right) = -\frac{\partial \hat{p}_d}{\partial X} + 2\frac{\partial}{\partial X}\left(\hat{\mu}\frac{\partial U}{\partial X}\right) + \frac{1}{R}\frac{\partial}{\partial R}\left[R\hat{\mu}\left(\frac{\partial U}{\partial R} + \frac{\partial V}{\partial X}\right)\right] + \frac{\theta - 1}{\theta}\frac{g.r_o^3}{\nu_o^2}$$

Energy Equation

$$\frac{\partial}{\partial X}\left(\hat{\rho}\hat{C}_p U\theta\right) + \frac{1}{R}\frac{\partial}{\partial R}\left(R\hat{\rho}\hat{C}_p V\theta\right) = \frac{1}{\Pr_o}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\hat{k}R\frac{\partial \theta}{\partial R}\right) + \frac{\partial}{\partial X}\left(\hat{k}\frac{\partial \theta}{\partial X}\right)\right] + \frac{Q'''_{gen} r_o^2 \alpha_o}{\nu_o k_o T_o}$$

Here, $\theta = \frac{T_{ref}}{T_o}$ and rest of the variables represents the same quantities as defined earlier in above section.

After, non-dimensionalizing the governing equations possibility of $\frac{Q'''_{gen} r_o^2 \alpha_o}{\nu_o k_o T_o}$ and $\frac{g.r_o^3}{\nu_o^2}$ as relevant non-dimensional numbers was found. Here, the term $\frac{Q'''_{gen} r_o^2 \alpha_o}{\nu_o k_o T_o}$ represents the temperature aspect. Examination of non-dimensional number $\frac{g.r_o^3}{\nu_o^2}$ reveals that when it is divided by aspect ratio i.e. $\frac{L}{r_o}$ it becomes a function of geometry of the tube.

Thus Nd1 $= \frac{g.r_o^3}{\nu_o^2}\frac{r_o}{L}$, Nd2 $= \frac{Q'''_{gen} r_o^2 \alpha_o}{\nu_o k_o T_o}$

SOLUTION METHODOLOGY

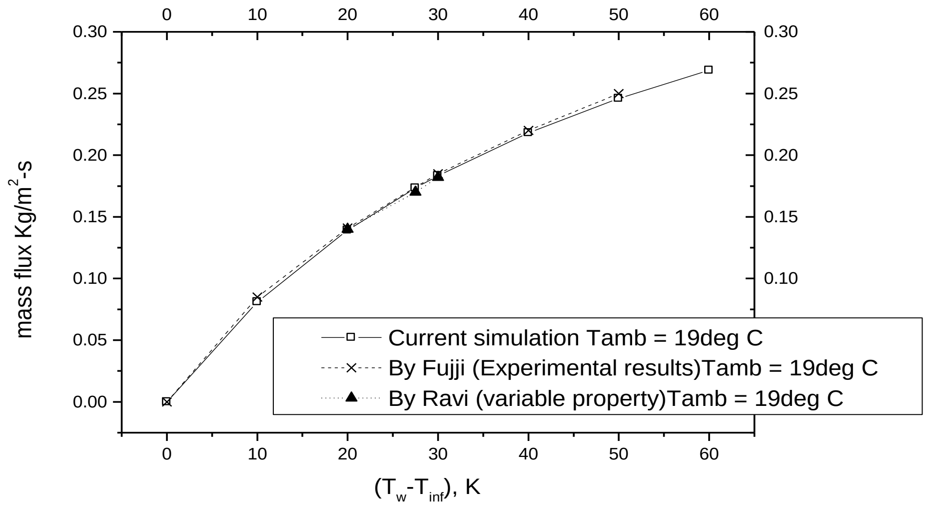
USING CFD SOFTWARE FLUENT® : Most of the fluid flow and heat transfer processes can be represented in the form of differential equations when laws governing these phenomena are applied to them. Every CFD software discretizes these equations using some scheme and divides domain into small finite numbers of volume or area element, and assuming a profile between these grid points it solves the problem. FLUENT® divides the domain into a finite number of control volumes thus replacing the continuous information contained in the exact solution of the differential equations with discrete values. Then it integrates the governing equations over these control volumes. Then finite difference approximation forms of derivatives present in the corresponding advection, diffusion and source terms in the integral form of the governing equations are substituted. Thus a set of algebraic equations are obtained. These equations are solved in an iterative manner with the help of some guessed values for the solution until the convergence criteria are met.

The assumptions that user make in FLUENT® while solving a problem numerically are termed as schemes. The solution greatly depends upon the discretization scheme that one opts, as the accuracy and computing power required both depends upon the discretization scheme. Schemes used while carrying simulation here, are:

- Momentum, Energy : Second order upwinding
- Pressure interpolation scheme: Body force weighted
- Pressure-velocity coupling: SIMPLE

RESULTS AND DISCUSSION : In course of the literature review, no paper was found reported on the buoyancy induced flow due to the heat generation in the fluid inside a vertical tube. Thus, as starting point buoyancy induced flow through a tube due to wall heating was simulated using FLUENT®, for which data was available in the literature for comparison. The experimental results of Fujii et al. (1988) were reproduced as discussed below. Subsequently, simulations were carried out for open tube with buoyancy-induced flow due to heat generation. Firstly, the simulations were carried out without considering radiation and later surface radiation was included. The results for all these cases are presented in this chapter.

REPRODUCTION OF FUJJIS RESULTS : Fujii *et al*. (1988) performed experiments on an isothermal tube in a non-stratified ambient and demonstrated the dependence of fluid properties on temperature, which in turn affects the average mass velocity. Ravi (2006) also reproduced results of Fujii *et al*. (1988) and validated his simulations.



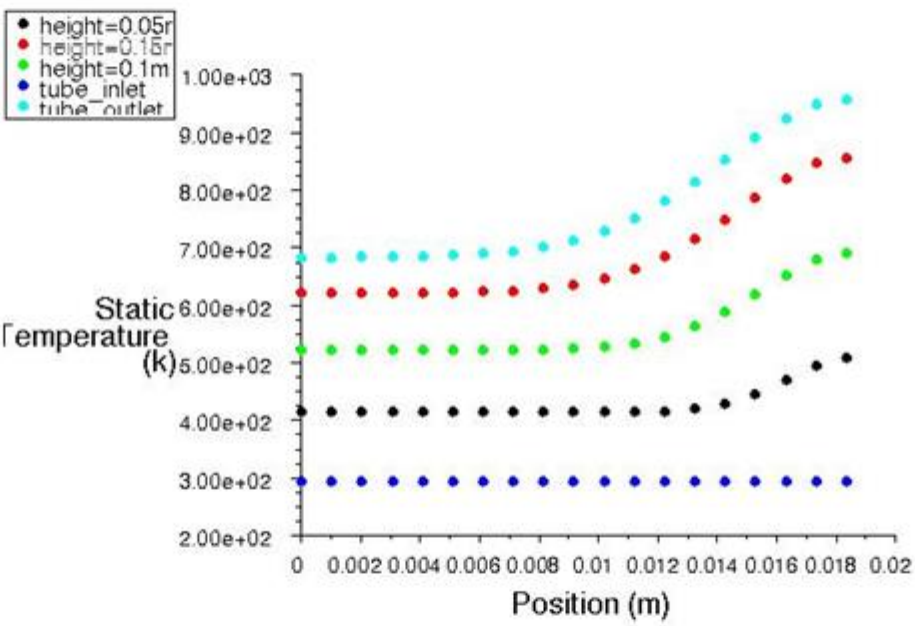
Comparison of mass flux rate with Fujii (1988) experiment and Ravi (2006) simulation for variable propertyHere the results of the Fujii *et al*. (1988)

experiments on isothermally heated vertical tubes were reproduced successfully. A comparison of the simulation results with experimental result of Fujii *et al*. (1988) and simulation results of Ravi (2006) . The simulation for T_{amb} as 19°C matches well with that of Ravi (2006) and Fujii *et al*. (1988) for the same ambient temperature.Variation of mass flow rate and heat transfer rate with grid size.

S.No.	N_x	N_y	Mass Flow rate(10^{-4}Kg/m^2 s)
1	100	22	9.0105
2	150	22	9.0108
3	100	30	9.0098
4	150	30	9.0097

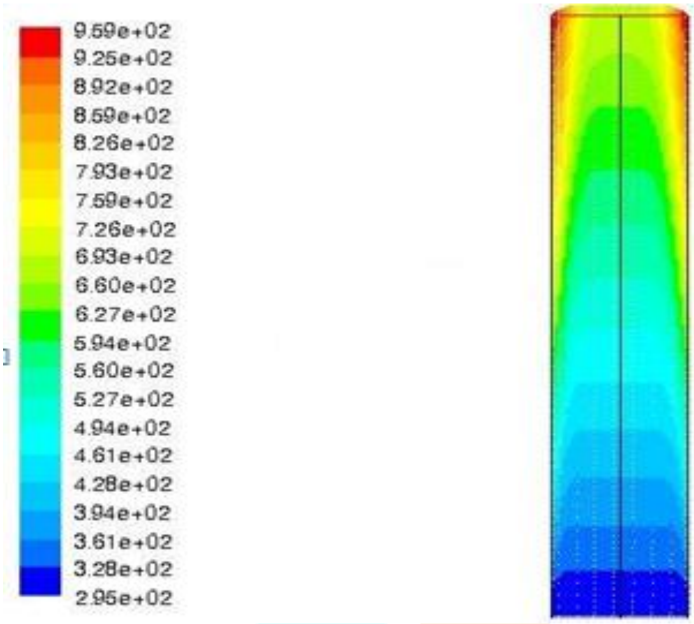
Simulations with heat generation without considering radiation

As a starting point, the buoyancy induced flow inside the tube with adiabatic wall was simulated where volumetric heat generation is taking place in the fluid. The velocity profiles and temperature profiles of the flow are shown below for a case when radiation was not considered. Radius of the simulated tube was 18.33 mm and height of the simulated tube was 183.3 mm. Heat generated inside the tube was 15x10^5 W/m^3.

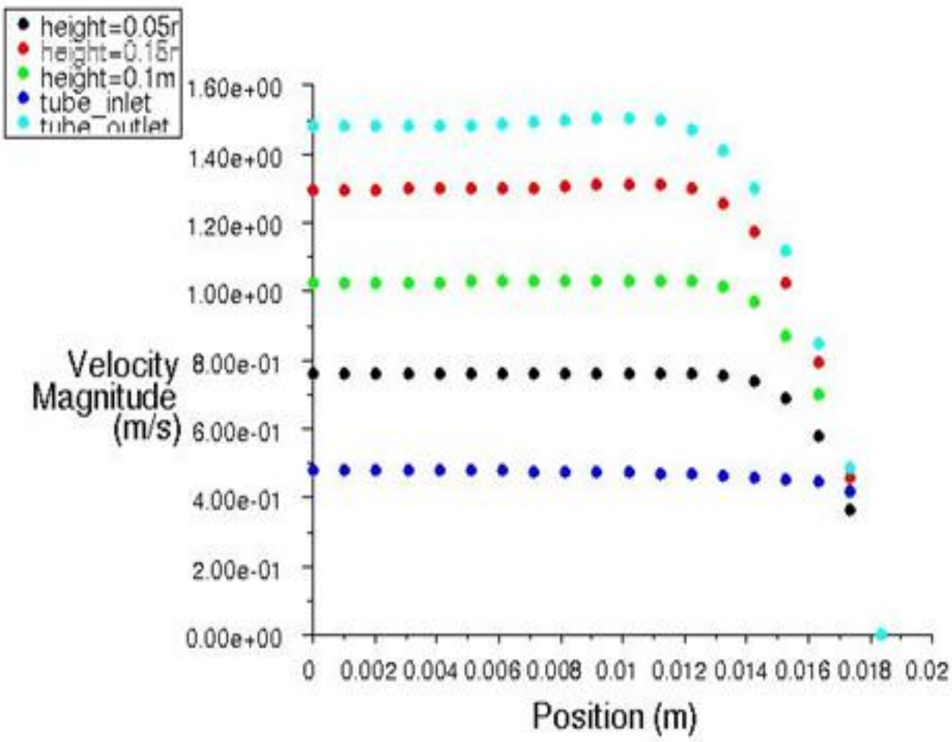


Temperature profile of the buoyancy induced flow inside the tube

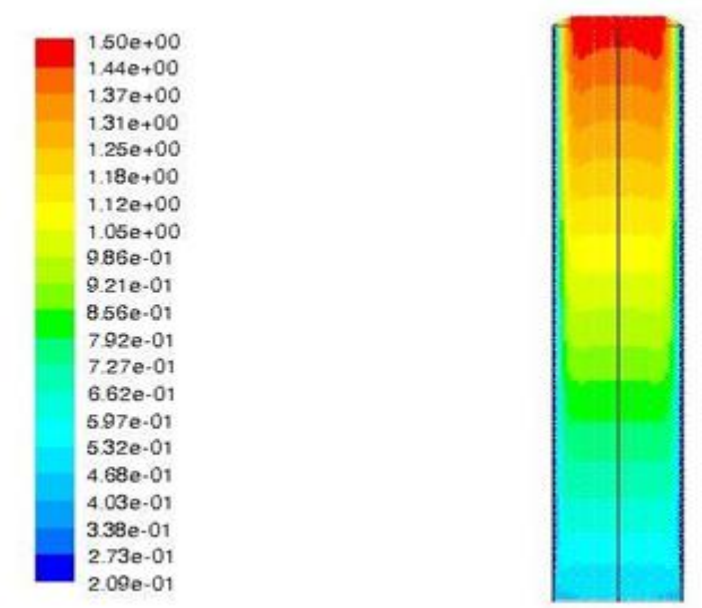
it was found that temperature increases along the height of the tube and temperature was highest at the wall at a given height. This profile reveals that maximum temperature exists near the outlet of the tube in the vicinity of wall. This result can be explained on the basis of the fact that velocities are zero at the wall (No slip condition) and thus no heat transfer takes place through adiabatic wall. Therefore heat accumulates at the adiabatic wall, causing high temperatures at walls.



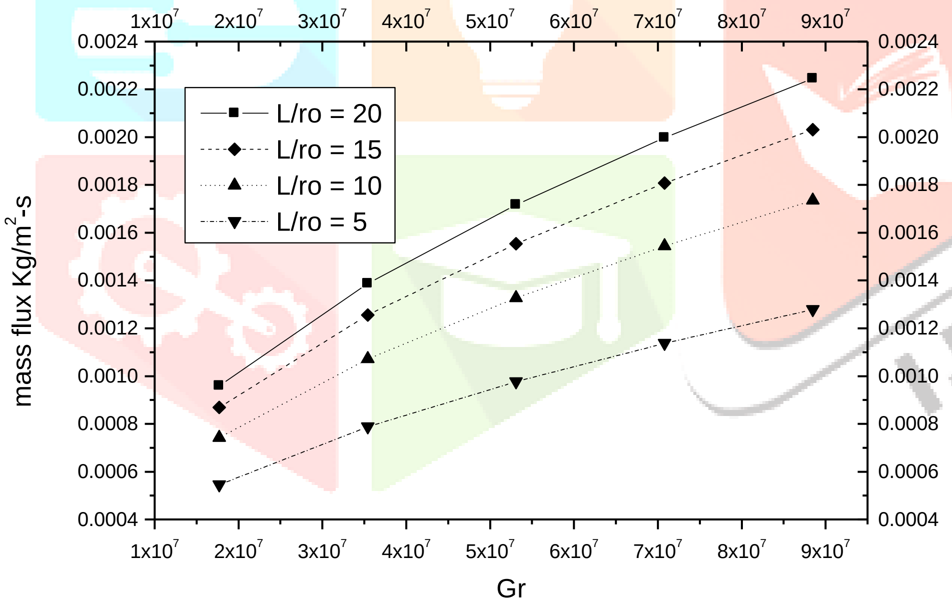
Temperature distributions inside the tube neglecting radiation



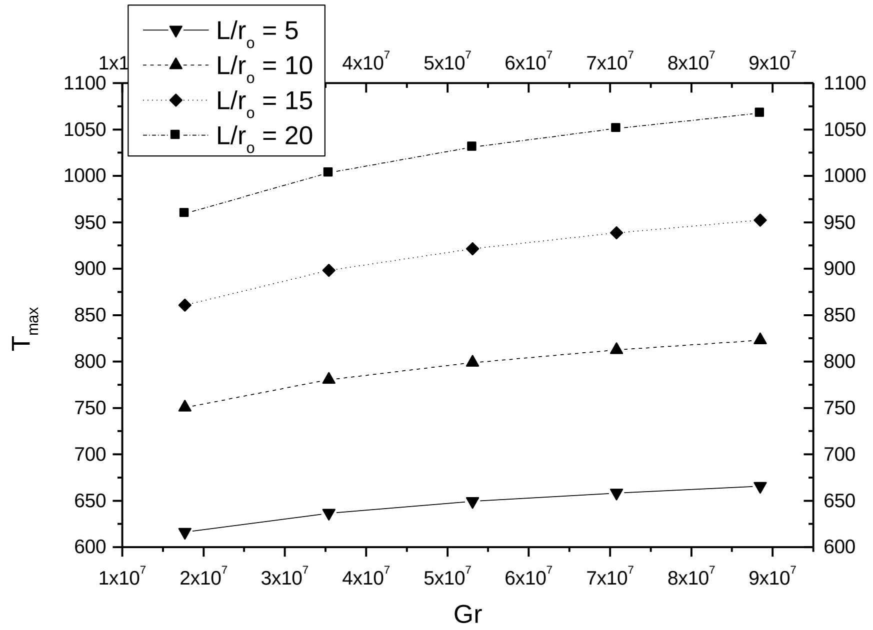
Velocity profile of the buoyancy induced flow inside the tube



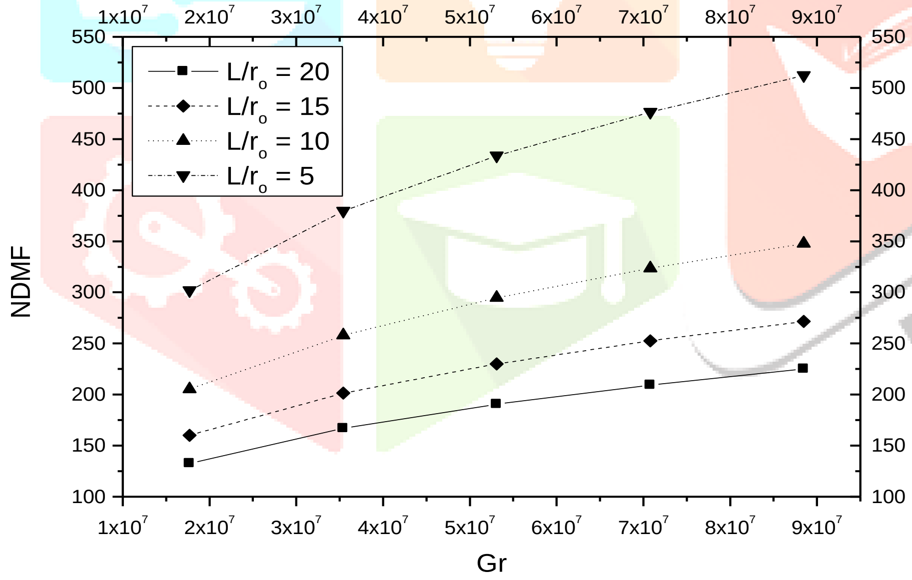
Velocity distributions inside the tube neglecting radiation



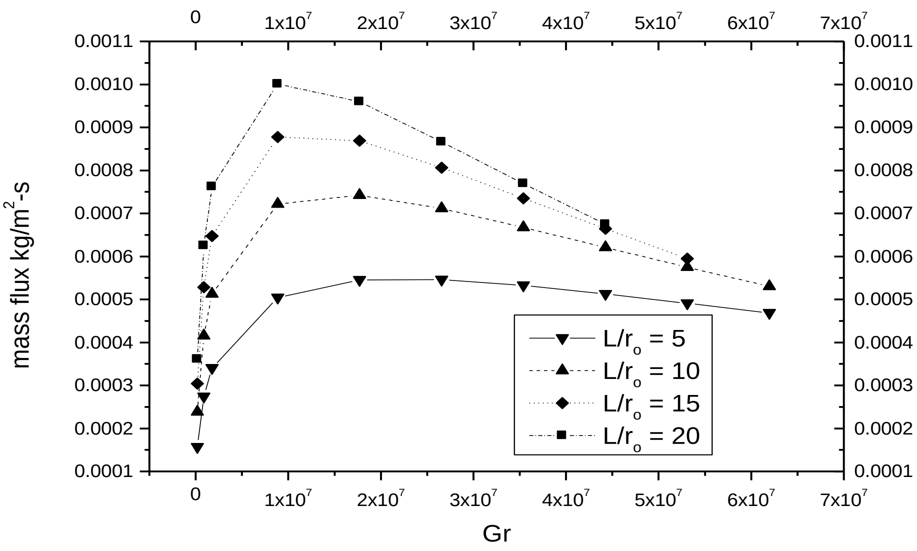
Variation of mass flux with Gr ($Q'''_{gen} = 10^6$ W/m^3)



Variation of T_{max} with Gr (Q'''_{gen} = 10^6 W/m^3 varying r_o)



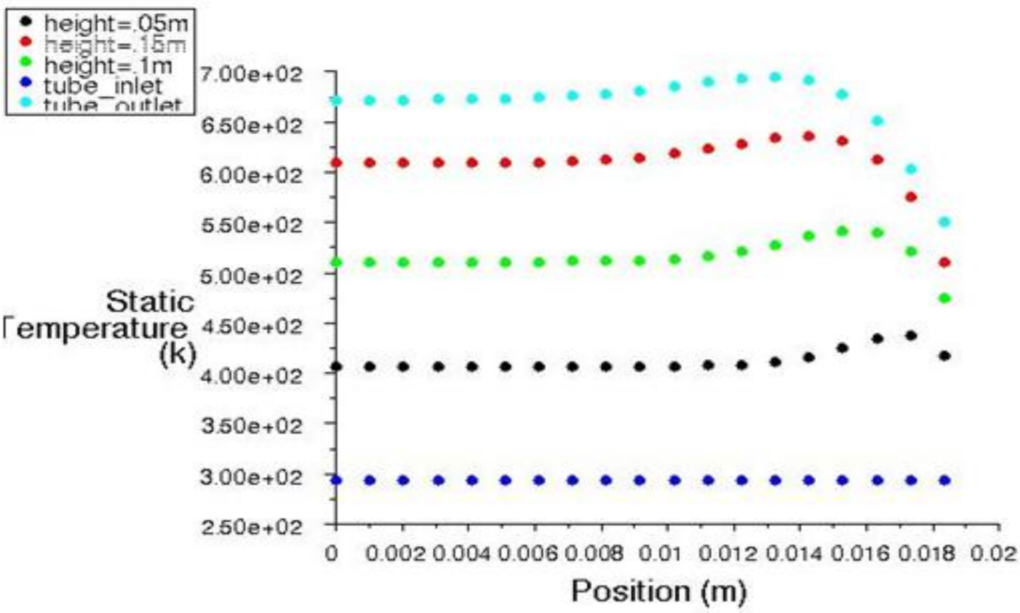
Variation of NDMF ($\tilde{m} = \frac{\dot{m}}{\mu_o L}$) with Gr (Q'''_{gen} = 10^6 W/m^3 varying r_o)



Variation of mass flux with Gr (r_o = 19.8 mm varying Q'''_{gen})

Simulations with heat generation and considering surface radiation

At high temperatures radiation cannot be neglected and thus effect of radiation was included. Thus the same problem is numerically analyzed considering surface radiation and variable property effect. Before starting the simulations for determining correlations, it was decided to study the velocity and temperature profiles of the buoyancy induced flow inside the adiabatic tube, where volumetric heat generation is causing the flow. Length of the simulated tube was 183.3 mm and radius was 18.33 mm, volumetric heat generation inside the tube was $15x10^5$ W/m^3.



Temperature profile of the buoyancy induced flow inside the tube

Above Figure shows temperature profiles of the buoyancy induced flow due to the volume heat generation inside an adiabatic tube considering surface radiation in account. It was found that temperature increases along with the height of tube. When, this profile was analyzed for a particular height it was found that maximum temperature doesn't exist at the wall unlike the earlier case when radiation effect was neglected. It was also observed that maximum temperature and average temperature were lower when we considered radiation effect account

CONCLUSION BASED PRESENT STUDY

A numerical investigation for a two-dimensional natural convection in open ended vertical tube has been performed with volumetric heat generation. A wide range of heat generation rate, radius and aspect ratio have been considered to find out the appropriate non-dimensional numbers governing the process and correlations between them. Based on computational results the following conclusions can be drawn:

- Non-dimensionalizing the governing equations and boundary conditions resulted in a set of non-dimensional numbers like Gr, aspect ratio, temperature and Pr. It was found that Gr is not a relevant non-dimensional parameter for the given problem as non-dimensional mass flow rate varied when different dimensional entities were varied individually keeping Gr as constant.
- It was found that heat generation in the fluid and geometry of tube were affecting non-dimensional mass flow rate in a different manner when varied individually keeping other constant. Need of separating Gr into two non-dimensional number was felt such that both represents the different aspect of flow. Nd1 and Nd2 were discovered as the relevant non-dimensional numbers for the given problem and were separated out from Gr itself. Nd1 represents the geometrical aspect of the flow and Nd2 represents heat generated aspect.
- Many sets of simulations were carried out varying dimensional quantities within range, and it was found that T_{max} has a strong dependence on Q_{gen}, for both the cases, considering and neglecting radiation.
- NDMF doesn't increase monotonically with Nd2, after reaching the peak value it decreases because of the drastic decrease in density. But for the second case studied i.e. considering radiation, NDMF becomes constant after reaching the peak.

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