IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Effectiveness of Augmented Reality Content on Visual Thinking in Mathematics among Secondary School students with special reference to components of Visual Thinking

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Abstract:
Augmented Reality Content is the computer-generated input used to enhance parts of learner's physical world through mobile, tablet or smart glasses. This research aimed to identify the Effectiveness of Augmented Reality Content on Visual Thinking in Mathematics among Secondary School Students with special reference to components of Visual Thinking. To accomplish the research objective, the experimental method with a non- equivalent group design was adopted and it included an experimental group and a control group with the pre-test and post-test of both groups. The research was implemented on a random sample of 68 students and the research tools included the Visual Thinking Test, Lesson Transcripts based on Augmented Reality Content and Lesson Transcripts based on Activity-Oriented Method. The tools were pre- and post- tested to the ninth standard students of the academic year 2022. The results concluded that the experimental group surpassed the control group in visual thinking when the components of Visual Thinking were considered. Based on the results, the researcher recommended the involvement of augmented reality content in mathematics instruction at different levels of education.

Index Terms -
Effectiveness, Augmented Reality Content, Secondary School, Mathematics, Visual Thinking in Mathematics, Visual Discrimination Skill, Scientific Deduction Skill, Visual Reading Skill, Skill of analyzing and interpreting the visual shape

INTRODUCTION

AR Content is information that is relevant to a certain location, also called a point-of-interest (POI). (Visser, 2011). Augmented Reality is an area of research that aims to enhance the real world by overlaying digital information on top of it. AR applications enrich the perception of the context and it is a new way to interact with information. Augmented Reality can help to supply information in a context with limited information and enhance user experience. The creation of Augmented Reality Content can potentiate the integration of social media (Vera, 2016).

According to the National Council of Teachers of Mathematics [NCTM] (2015), technology use is essential for developing learner's mathematical skills, encouraging them to learn more profoundly and increasing their interest in Mathematics. The Augmented Reality Content helps students to understand complex subjects by providing 3D simulations of invisible situations that are hard to visualize (Cai et al., 2020). Visual Thinking in Mathematics is defined as the mental activity and skill that involves visual imagination or visual perception of external diagrams, is widespread in Mathematics, across levels, across subjects and across kinds of mathematical activity (Giaquinto, 2007). Augmented Reality Content has an effective role in developing thinking, especially visual thinking. Several studies have found that students who were taught with Augmented Reality Content performed significantly better than those who were not taught to them in achievement and visual thinking (Al-Ghamdi, 2020; Al Hilou, 2017; Al-Muqrin, 2020; Al-Salahat, 2019; Osamah et al., 2019; Salama, 2019).

Visual Thinking is a mental activity and skill that helps a person obtain information, represent it, interpret it, perceive it and memorize it, then express those ideas verbally. This is to achieve communication with others, and it is important for reasoning and problem solving (Mcloughlin & Krakwski, 2010). Surya et al. (2013) defined visual thinking skills as a group of processes, involving a set of abilities that inspire the student to visual thinking and meditation, and to interpret

The study titled "Effectiveness of Augmented Reality Content on Visual Thinking in Mathematics among Secondary School Students with special reference to components of Visual Thinking " is of great importance as it throws light into the extent to which AR Content enables learners to grasp abstract mathematical concepts and help them to create immersive educational experiences on their own, thereby enhancing the cognitive skills of the learners, especially Visual Thinking in Mathematics. The main goal of the paper was to prove the effectiveness of teaching Mathematics with Augmented Reality (AR) Content over Activity-Oriented Method in enhancing the Visual Thinking in Mathematics among Secondary School Students.

In the next sessions of the paper, the investigator will test the effectiveness of teaching Mathematics based on Augmented Reality Content over Activity-Oriented Method on Visual Thinking in Mathematics among Secondary School Students with reference to components of Visual Thinking as Visual Discrimination Skill, Scientific Deduction Skill, Visual Reading Skill and Skill of analyzing and interpreting the geometrical shape.

Need and Significance of the study

The advancement and popularity of handheld devices and sensing technologies has enabled researchers to implement more effective learning methods (Ogata, Li, Hou, Uosaki, El-Bishouty, & Yano, 2011). Most of mobile learning studies emphasize the adoption of digital learning aids in real-life scenarios (Sharples, Milrad, Arnedillo- Sanchez, & Vavoula, 2009; Ogata & Yano, 2004; Wong & Looi, 2011). However, regarding supplementary mobile learning aids, the interaction between digital learning aids and the actual environment needs to be emphasized to enable students to effectively manage and incorporate personal knowledge (Wu, Lee, Chang, & Liang, 2013). Augmented Reality Content combines human senses (e.g., sight, sound and touch) with virtual objects to facilitate real-world environment interactions for users to achieve an authentic perception of the environment (Azuma, 1997).

The use of Augmented Reality Content in Mathematics allows the learner to experience virtual objects in another way without losing contact with the environment in which they are. According to a study by Orozo, Esteban and Trefftz (2006), the learners who were taught Mathematics using Augmented Reality Content were able to infer generalizations, considering what they observed, and predict what would happen with the graphical representation of other functions. The learners also showed greater ease in understanding the mathematical concepts taught. Augmented Reality Content in Mathematics contributes to increased understanding of mathematics content, in particular learning geometry and 3D shape.

Augmented Reality Content reinforces the teaching and learning process, and encourage learners to pursue tasks. Also, AR Content develops positive trends towards learning mathematics. By incorporating Augmented Reality Content in teaching Mathematics, students may improve visual thinking by focusing on the shapes, sketches and images provided in the situation and the actual relationships involved in it, and trying to find meaning for the material they contain (Campbell et al., 1995).

Ahmad (2020) also mentioned that the benefits of Augmented Reality Content in Mathematics learning are that these content increases learner's confidence and understanding. It contributes to increased understanding of mathematics content, in particular learning geometry and 3D shape. The Augmented Reality Content improves the geometry visualization of learners by providing interesting and entertaining visual content to see engineering objects from different angles and this made learning Mathematics more effective.

Sun and Chen (2019) in the article "Utilizing free augmented reality app for learning geometry at elementary school in Taiwan: Take volumetric measurement of compound body for example" studied on Augmented Reality Content in Mathematics education and observed that AR Content positively affected interaction besides encouraging students to participate in learning activities with less cognitive effort and enhance their learning performance.

Estapa and Nadolny (2015) conducted a study examining the effect of Augmented Reality Content on the success and motivation of students in a Mathematics lesson, they found that AR Content had a positive impact on the success and motivation of students.

Studies were conducted on the benefits of Augmented Reality Content in facilitating the mathematics learning (Ahmad, 2020., Sun and Chen, 2019., Estapa and Nadolny, 2015).

The effect of Augmented Reality Content on Mathematics learning especially learning geometry and 3D shape (Orozo, Esteban and Trefftz, 2006) were also conducted. These studies prove that the effective intervention of Augmented Reality Content in Mathematics can improve learner's confidence and understanding in Mathematics. But, from the review conducted in the related field, it is clear that no studies were conducted to find the effectiveness of Augmented Reality Content on Visual Thinking in Mathematics of Secondary School Students. So, the investigator made efforts to integrate Augmented Reality Content in the teaching of Mathematics and developing visual thinking skills in students by focusing on the shapes, sketches and images provided in the situation and the actual relationships involved in it, and trying to find meaning for the material they contain.

Incorporating Augmented Reality Content in teaching Mathematics will enhance the visualization skills of students and contributes to increased understanding of Mathematics content, in particular, learning geometry and 3D shapes. Learners are able to visualize the mathematical concepts in a broad spectrum by the use of AR Content in teaching Mathematics.

Visual Thinking plays an imperative role in mathematical problem-solving. The learning experiences become more meaningful when more senses are involved. The use of Augmented Reality Content in classroom fosters the understanding of abstract mathematical concepts. Visual images or diagrams may illustrate causes of a definition, thereby giving learners a more vivid grasp of its applications. The visual images play an extremely important role in the thinking and learning of Mathematics.

The use of Augmented Reality Content in Mathematics will foster the visual thinking which is based on the connection between the sense of vision and mental activity aimed at obtaining and interpreting visual stimuli apparent from the external environment. Visual Thinking plays a vital role in instruction learning processes. Ammar and Kabani (2011) pointed to the importance of visual thinking in the field of developing students' visual language skill, developing the ability to understand visual messages surrounding the educational process members from every side as a result of scientific and technical progress, and the ability to solve problems, by selecting and defining visual concepts, understanding abstract concepts and associated processes, developing students' abilities to make visual comparisons and reaching conclusions easily, and facilitating the development of students' abilities to discover similarities and differences in the visual scene (Elsayed, 2021).

The four components of Visual Thinking were identified by the investigator as Visual Discrimination Skill, Scientific Deduction Skill, Visual Reading Skill and Skill of analyzing and interpreting the visual shape. Visual Discrimination Skill means the students' ability to identify the different or similar shape among a group of presented shapes (Al-Shalwi, 2010). According to Steussy (1984), scientific deduction skills are the skills that enhance the use of scientific processing skills in order to justify a particular conclusion in scientific inquiry. According to Abd Al-Reda and Fadel (2019) the skill of visual reading is the ability to determine the dimensions and nature of the shape or image presented. Skill of analyzing and interpreting the geometrical shape is defined by Duran and Bekdemir (2013) as the skill to perceive, express, interpret, assess and utilize spatial or visual information mathematically.

The study is expected to induce conscious efforts and techniques by authorities in the field to help student's meet mathematics-related goals. On the basis of above consideration, it was found necessary that a study incorporating Augmented Reality Content in Mathematics with special reference to components of visual thinking be undertaken.

I. RESEARCH METHODOLOGY

The methodology section outlines the plan and method that how the study is conducted. This includes Universe of the study, sample of the study, Data and Sources of Data, study's variables and analytical framework. The details are as follows:

3.1Population and Sample

The population for the study consists of secondary school students. The sample for the study included students from two class divisions of standard IX, of which 35 students are considered as the experimental group and other 31 students as the control group.

3.2 Data and Sources of Data

For this study primary data has been collected. The data was collected from the students after conducting a Visual Thinking Test in Mathematics to two divisions of Standard IX. The tool was pre-tested and post-tested to Experimental and Control groups and the observations were note down.

3.3 Theoretical framework

Variables of the study contains dependent and independent variable. The study used pre-specified method for the selection of variables. The study used the Visual Thinking in Mathematics as dependent variable.
Teaching using Augmented Reality Content and Activity method were the independent variables for the study.
According to Chang et al. (2010), several researchers have suggested that students and trainees can strengthen their motivation for learning and enhance their educational realism-based practices with virtual and augmented reality. Inspite of a great amount of research during the last two decades, adopting AR in learning and training is still quite challenging because of issues with its integration with traditional learning methods, costs for the development and maintenance of the AR system, and general resistance to new technologies. Now, however, AR promises to attract and inspire learners with the exploration and control of materials from diverse perspectives that have not been taken into consideration in real life; AR in education and training is thus believed to have a more streamlined approach with wider user adoption than ever before, due to the improvements in computer and information technology. Kerawalla, Luckin, Seljeflot, and Woolard (2006) stated that even though many AR applications have been developed for educational and training purposes since the advent of AR in the late 1960s, AR's potential and pragmatic employment has just begun to be explored and utilized in real life. He emphasized that AR has the potential to further engage and motivate learners in discovering resources and applying them to the real world from a variety of diverse perspectives that have never been implemented in the real world. With AR technology, teachers and students can collaborate by interacting with each other for some issues on shapes or arrangements. According to Chang et al. (2010), an AR application called Construct3D specifically designed for mathematics and geometry education with 3D geometric construction models (as cited in Kaufmann, 2006; Kaufmann & Schmalstieg, 2002; Kaufmann, Schmalstieg, & Wagner, 2000). This application allows multiple users such as teachers and students to share a virtual space collaboratively to construct geometric shapes by wearing head-mounted displays that enable users to overlay computer-generated images onto the real world.

3.4Statistical tools and econometric models

This section elaborates the proper statistical/econometric/financial models which are being used to forward the study from data towards inferences. The detail of methodology is given as follows.

3.4.1 Descriptive Statistics

Descriptive Statics has been used to find the maximum, minimum, standard deviation, mean and normally distribution of the data of all the variables of the study. Normal distribution of data shows the sensitivity of the variables towards the periodic changes and speculation.

3.4.2 Inferential Statistics (Test of significance of difference between means, Analysis of Variance - ANOVA and Analysis of Covariance-ANCOVA)

The term *inferential statistics* refers to applying statistical analysis with observed data for the purpose of making inferences to that which cannot be observed (Seaman,2018) Inferential Statistics allow the researcher to make inferences or "infer" the value of a population parameter and thus make larger statements about a population (generalizability) from a relatively small sample (Westfall,2004)

- **3.4.2.1 Significance of difference between means**

The t distribution (or the z distribution if the sample is large) is used to determine the level of statistical significance of an observed difference between sample means. Once a t value is obtained, we can determine how often a difference score of a given magnitude between samples of a given size will occur when there is no difference in the population. Generally, educational researchers choose to reject the null hypothesis if the t value reaches a significance level of p<0.05[6] (Borg, 2003)

$$t = \frac{M1-M2}{\text{SED}}$$

in which SE_D = Standard error of the difference of means

$$SE_D = \sqrt{(SE_{M_1})^2 + (SE_{M_2})^2}$$

SEm_1 = Standard error of the mean of the first sample

SEm_2 = Standard error of the mean of the second sample

As $SE_{M_1} = \frac{\sigma_1}{\sqrt{n_1}}$ and $SE_{M_2} = \frac{\sigma_2}{\sqrt{n_2}}$ we can substitute

$$SE_D = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

σ_1 = Standard deviation of pre-test
σ_2 = Standard deviation of post-test
M_1 = Mean of pre-test
M_2 = Mean of post-test
n_1 = Number of samples in the pre-test group

n_2 = Number of samples in the post-test group

- **3.4.2.2 Analysis of Variance (ANOVA)**

Analysis of Variance (ANOVA) is used to determine whether there is a significant difference between two or more means at a selected probability level. The concept underlying ANOVA is that the total variation, or variance of scores can be attributed to two sources- variance between groups (variance caused by the treatment) and variance within groups (error variance) (Gay,1996)

The simplest case of ANOVA is the one-way ANOVA where the groups are varied across only one factor and each group has the same sample size. Suppose there are 'k' groups and within each group there are 'n' samples taken for each group for a total sample size of nk (Thayer, 2001)

F* is a fraction (ratio) with the observed "variance" associated with the treatment in the numerator and "variance" associated with the error in the denominator

$$F^* = \frac{SS_T/df_T}{SS_E/df_E}$$

- **3.4.2.3 Analysis of Co-variance (ANCOVA)**

Analysis of Covariance (ANCOVA) is a statistical procedure that forms part of the general linear model. It can be thought of as combination of two other methods within this family of statistical models: analysis of variance (ANOVA) and linear regression. ANCOVA can be used in experimental designs to remove the effect of one or more confounding variables (Julius, 2018). ANCOVA is commonly used when there is a pre-test post-test design, where a test is given before participants are allocated to the experimental condition and then the same test is given after the experimental condition. Here, the pre-test scores are used as covariate.
The experiment was done using intact classroom groups which are non -equivalent. Analysis of covariance is the method that enables the researcher to equate the pre-experimental status of the groups in terms of relevant known variables. Difference in the initial status of the groups can be removed statistically so that they can be compared as though their initial status has been equated (Best & Kahn, 1996).

Thus, in the present study, the technique ANCOVA was adopted for sharper experimental comparison of performance between experimental and control groups. The steps adopted for the computation of Analysis of Covariance is discussed below.
Let X_1, X_2, X_3... be the initial scores and Y_1, Y_2, Y_3…… be the final scores of the experiment. Before going through the actual steps of Analysis of Covariance, various sums and means are computed. Th following formulae are used for the computation of various sums and means.

$\Sigma X = \Sigma X_1 + \Sigma X_2 + \Sigma X_3 + \ldots\ldots$
$\Sigma Y = \Sigma Y_1 + \Sigma Y_2 + \Sigma Y_3 + \ldots\ldots$
$\Sigma X^2 = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \ldots\ldots$
$\Sigma Y^2 = \Sigma Y_1^2 + \Sigma Y_2^2 + \Sigma Y_3^2 + \ldots\ldots$
$\Sigma XY = \Sigma X_1 Y_1 + \Sigma X_2 Y_2 + \Sigma X_3 Y_3 + \ldots\ldots$

After computing these sums and means, the following steps were adopted.

Step 1: Computation of Correction Terms (C's)

Different corrections are applied to different sum of squares and these can be computed using the following formulae.

$C_X = \frac{(\Sigma X)^2}{N}$ $\quad C_Y = \frac{(\Sigma Y)^2}{N}$ $\quad C_{XY} = \frac{\Sigma X \Sigma Y}{N}$

Step 2: Computation of total sum of squares (total SS)

$SS_X = \Sigma X^2 - C_X$
$SS_Y = \Sigma Y^2 - C_Y$
$SS_{XY} = \Sigma XY - C_{XY}$

Step 3: Computation of Sum of Squares (SS) among the means of the groups

$$\text{SS among means for X} = \frac{\Sigma X_1^2}{N_1} + \frac{\Sigma X_2^2}{N_2} + \frac{\Sigma X_3^2}{N_3} \ldots\ldots\ldots - C_X$$

$$\text{SS among means for Y} = \frac{\Sigma Y_1^2}{N_1} + \frac{\Sigma Y_2^2}{N_2} + \frac{\Sigma Y_3^2}{N_3} \ldots\ldots\ldots - C_Y$$

$$\text{SS among means for XY} = \frac{\Sigma X_1 Y_1}{N_1} + \frac{\Sigma X_2 Y_2}{N_2} + \frac{\Sigma X_3 Y_3}{N_3} + \ldots\ldots - C_{XY}$$

Step 4: Computation of sum of squares (SS) within Groups

Within group SS for X = SS_X (total) – SS among means for X
Within group SS for Y = SS_Y (total) – SS among means for Y
Within group SS for XY = SS_{XY} (total) – SS among means for XY

Step 5: Calculation of the number of degrees of freedom
df (Among means) = K-1, where K is the number of groups
df (within groups) = N-K, where N is the total sample

Step 6: Analysis of Variance of X and Y scores taken separately
Let F_X and F_Y be the F-ratios of X and Y scores taken separately. These can be calculated using the formula

$$F_X = \frac{MS_X\,(among\text{-}groups)}{MS_X\,(within\text{-}groups)} \quad \text{and} \quad F_Y = \frac{MS_Y\,(among\text{-}groups)}{MS_Y\,(within\text{-}groups)}$$

$$\text{Where, } MS_X = \frac{SS_X}{df} \quad \text{and} \quad MS_Y = \frac{SS_Y}{df}$$

The computed values of F for X and Y scores are tested for significance at both levels.

Step 7: Computation of adjusted sum of squares for Y (SS_{YX})
The initial differences in the groups X scores may cause variability in their final scores measured after giving the treatment. Hence for controlling this variability in the final scores, necessary adjustments are made in the sum of squares for Y by using the formula given below.

$$SS_{YX} = SS_Y - \frac{(SS_{XY})^2}{SS_X}$$

$$SS_{YX}\ (\text{total}) = SS_Y\,(total) - \frac{total\ (SS_{XY})^2}{SS_X\,(total)}$$

$$SS_{YX}\ (\text{within means}) = SS_Y\,(within) - \frac{within\ (SS_{XY})^2}{SS_X\,(within)}$$

$$SS_{YX}\,(among\ means) = SS_{YX}\,(total) - SS_{YX}\,(within)$$

Step 8: Computation of Analysis of Covariance
The analysis of Covariance F_{YX} is calculated using the formula

$$F_{YX} = \frac{MS_{YX}(among)}{MS_{YX}(within)} \quad \text{where} \quad MS_{YX} = \frac{SS_{YX}}{df}$$

The computed value of F is tested for significance at both the levels of significance. If there exists significant difference among the means of final scores, it can be concluded that one treatment is better than others. In such cases, further steps can be outlined finding the better treatment among others.

Step 9: Computation of regression coefficient for within groups

An unbiased estimate of the regression of Y on X can be computed by using the formula,

$$b = \frac{SS_{XY}\,(within)}{SS_X\,(within)}$$

Step 10: Computation of adjusted Y means
Let M_{YX} be the means of the final score after ruling out the initial differences in X scores. This can be calculated using the formula
$M_{YX} = M_Y - b\,(M_X - GM_X)$
Where,

M_Y - Unadjusted means for Y scores

M_X - Means for X scores

b - Regression coefficient for within groups

GM_X - General mean for X

Step 11: Significance of difference among adjusted Y means

The significance of differences among adjusted Y means is examined by applying the 't' test for testing the separate differences between group means. This can be computed by using the formula

$$t = \frac{MD}{SE_D}$$

Where, MD is the difference between two means and SE_D is the standard error of difference between two adjusted means. It can be computed by the formula

$$SE_D = SD_{YX}\sqrt{\left[\frac{1}{N_1}+\frac{1}{N_2}\right]}$$

Where, SD_{YX} is the standard deviation of the adjusted Y scores. It can be computed

by the formula,

$$SD_{YX} = \sqrt{V_{YX}}(within)$$

IV. RESULTS AND DISCUSSION

4.1 *Comparison of the Pre-test and Post-test scores of Experimental and Control groups on components of Visual Thinking in Mathematics*

Components	Groups	Mean	Median	Mode	Standard deviation	Skewness	Kurtosis
Visual Discrimination Skill	Experimental Pre-test	1.896	2	2	0.859	0.570	-0.466
	Experimental Post-test	3.5	3.5	4	1.106	0.266	-0.553
	Control Pre-test	2.33	2	2	0.994	0.609	0.434
	Control Post-test	2.37	2	2	1.181	0.712	-0.005
Scientific Deduction skill	Experimental pre-test	1.84	2	2	0.553	-0.097	0.352
	Experimental post-test	2.088	2	1	1.114	0.83	-0.079
	Control pre-test	1.94	2	2	0.97	1.011	0.33
	Control post test	1.112	2	1	0.995	1.195	0.48
Visual reading skill	Experimental pre-test	2.5	2	1	1.367	0.927	0.791
	Experimental post-test	3.67	3	3	1.45	0.71	-0.39
	Control pre-test	2.48	2	3	0.995	0.372	0.053
	Control post-test	3	3	3	1.65	0.416	-0.626
Skill of analysing and interpreting geometrical shapes	Experimental pre-test	2.12	2	1	1.201	1.475	3.185
	Experimental post-test	3.07	3	1	0.985	0.216	-1.17
	Control pre-test	2.42	2	2	1.057	0.678	0.611
	Control post-test	2.18	2.5	2	1.24	0	-1.29

Table 4.1: Descriptive Statics of Pre-test and Post-test scores of the Experimental and Control groups with respect to the components of Visual Thinking

Table 4.1 displayed mean, median, mode, standard deviation, skewness and kurtosis of the pre-test and post-test scores of the Experimental and groups when the components of Visual Thinking were considered for the purpose of study. The descriptive statistics indicated that the mean values of Experimental and Control groups when considered with respect to components of Visual Thinking (Visual Discrimination Skill, Scientific Deduction Skill, Visual Reading Skill and Skill of analyzing and interpreting the

geometrical shape), when subjected to pre-test and post-test were 1.896, 3.5, 2.33, 2.37, 1.84, 2.088, 1.94, 1.112, 2.5, 3.67, 2.48, 3, 2.12, 3.07, 2.42 and 2.18 respectively.
The standard deviations for each variable indicated that data were widely spread around their respective means.
The descriptive statistics from Table 4.1 showed that the values were normally distributed about their mean and variance.

Figures and Tables

Table 1

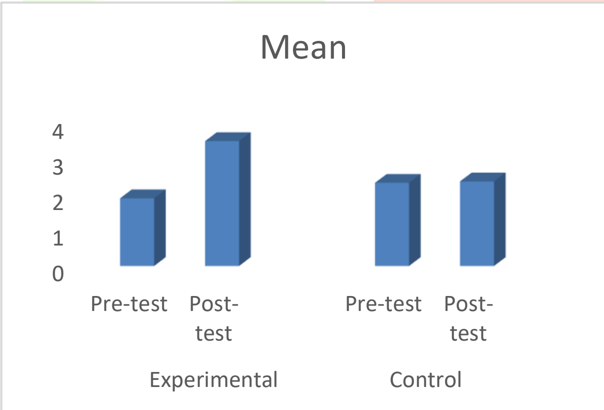
Descriptive Statistics of pre-test and post-test scores of Experimental and Control groups on Visual Discrimination Skill

Groups		Mean	Median	Mode	Standard deviation	Skewness	Kurtosis
Experimental	Pre-test	1.896	2	2	0.859	0.57	-0.466
	Post-test	3.5	3.5	4	1.106	0.266	-0.553
Control	Pre-test	2.33	2	2	0.994	0.609	0.434
	Post-test	2.37	2	2	1.181	0.712	-0.005

From the Table 1, it is clear that the arithmetic mean, median and mode for the pre-test scores of Control group and Experimental group are almost the same. Standard deviation values also indicate that the sample is almost homogeneous with regard to the pre-test and post-test scores on Visual Discrimination Skill. The distribution is positively skewed for pre-test of Experimental group, pre-test of Control group, post-test of Experimental group and post-test of Control group. This means that the scores are massed at the lower end of the distribution. The kurtosis value is greater than 0.263 for pre-test of Control group. Hence the distribution is platykurtic. The kurtosis value is less than 0.263 for pre-test of Experimental, post-test of Experimental and post-test of Control groups meaning that the distribution is leptokurtic.

Figure 1

Bar diagram representing the mean values of pre-test and post-test scores of Experimental and Control groups on Visual Discrimination Skill



Analysis of genuineness of the difference in visual thinking in mathematics of experimental and control groups with respect to the component- visual discrimination skill

The statistical procedure of ANCOVA as given by Garret (1981) is given under the following sub-heads:

- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Discrimination Skill using Analysis of Variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Discrimination Skill using Analysis of Co-variance

- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Discrimination Skill using Adjusted Means. The details of analysis carried out under the component Visual Discrimination Skill are given below:

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Discrimination Skill using Analysis of Variance

In this part of the analysis, the Total Sum of Squares, Mean Square Variances and F-ratios for the pre-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component Visual Discrimination Skill were computed. The Analysis of Variance Table for the pre-test scores (x) and post-test scores (y) of students taught using Augmented Reality Content and Activity-Oriented Method are presented below in Table 2.

Table 2

Summary of Analysis of Variance of the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component Visual Discrimination Skill

Source of variation	df	SSx	SSy	MSx (Vx)	MSy (Vy)	Fx	Fy
Among Means	1	1.67	209.1	1.67	209.07	0.60	31.34
Within Groups	58	160.07	386.9	2.76	6.67		
Total	59	161.73	595.9				

From the results obtained in Table 2, it is clear that the values of Fx are not significant at 0.05 level of significance since these are less than the table values required (Fx = 4.00 with df (1,58)) for significance. It shows that there is no significant difference between pre-test scores of students in the experimental and control groups for the Visual Thinking Test under the component Visual Discrimination Skill.

The table values of F required for significance at 0.01 level with df (1,58) is 7.08. The obtained Fy value, Fy =31.34 is significant at 0.01 level since the obtained value is greater than table value for the respective df. The significant Fy value indicate that the experimental and control groups differ significantly in their post-test scores on Visual Thinking in Mathematics under the component Visual Discrimination Skill

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Discrimination Skill using Analysis of Co-Variance

The above computation has to be carried out for the purpose of correcting the post-test (y) scores for the difference in the pre-test (x) scores. So ANCOVA was adopted for further computation. The results of the analysis are presented in Table 3.

Table 3

Summary of Analysis of Co-Variance for the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component Visual Discrimination Skill

Source of variation	df	SSx	SSy	SSxy	SSyx	MSy (Vy)	SDy(x)
Among Means	1	1.67	209.07	-18.67	211.51	211.51	
Within Groups	57	160.07	386.9	211.5	384.42	6.74	2.60
Total	58	161.73	595.93	192.84	595.93		Fyx=31.36

The Fyx ratio as indicated in Table 3 is significant for the Visual Thinking Test (Fyx = 7.08, df (1,57)) since the obtained value is greater than table value required for significance at 0.01 level with the corresponding df. The significant ratios for the adjusted post-test scores show that the final Mean scores of students on Visual Thinking in Mathematics under the component Visual Discrimination Skill in the experimental and control groups differ significantly after they were adjusted for the difference in the pre-test scores for the Visual Thinking Test. The significant F-ratio necessitates proceeding to test the difference separately by the test of significance of difference between means.

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Discrimination Skill using adjusted Means

The adjusted means for the post-test scores of students in the experimental and control groups were calculated using regression coefficients. The data and results are shown in Table 4 below.

Table 4

Data for Adjusted Means of post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component-Visual Discrimination Skill

Groups	N	Mx	My	Mxy	SEm	t-value
Experimental	30	7.57	16.5	16.72	0.67	
Control	30	7.90	12.8	12.55		6.22^{**}
Total	60	7.73	14.63			

$^{**}p<0.01$

From the table t for degree of freedom 57

t at 0.05 level = 2.00   t at 0.01 level= 2.66

The obtained 't' value is significant at 0.01 level (t= 6.22; p<0.01) for the Visual Thinking Test under the component Visual Discrimination Skill. This implies that Augmented Reality Content is better than Activity-Oriented Method in enhancing Visual Thinking in Mathematics with special reference to Visual Discrimination Skill for the secondary school students.

Figure 4

Graphical Representation of the Pre-test and Adjusted Post-test Means on Visual Thinking in Mathematics of Students in the Experimental and Control groups under the component-Visual Discrimination Skill

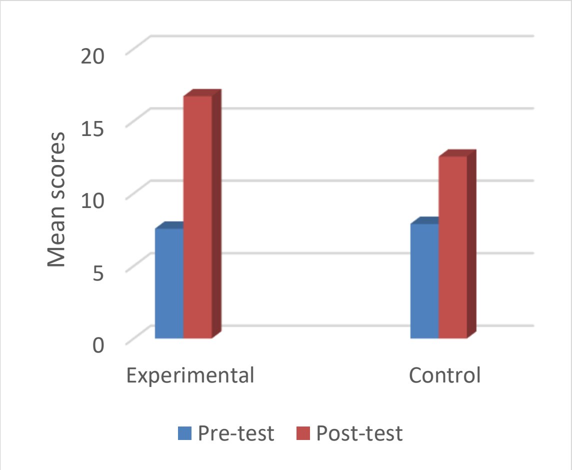


Table 5

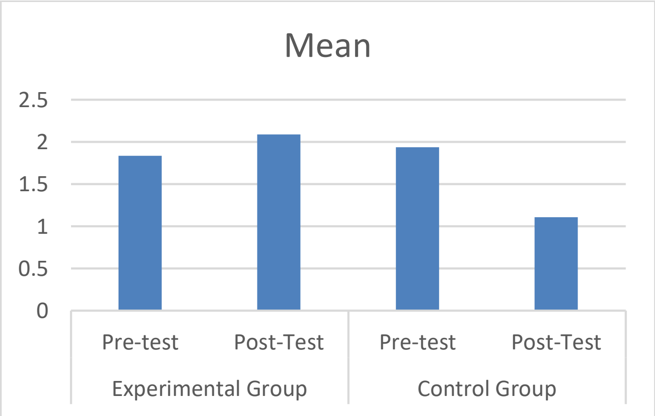
Descriptive Statistics of pre-test and post-test scores of Experimental and Control groups on Scientific Deduction Skill

Groups		Mean	Median	Mode	Standard deviation	Skewness	Kurtosis
Experimental	Pre-test	1.84	2	2	0.553	-0.097	0.352
	Post-test	2.088	2	1	1.114	0.830	-0.079
Control	Pre-test	1.939	2	2	0.966	1.011	0.329
	Post-test	1.112	2	1	0.995	1.195	0.480

From the Table 5, it is clear that the arithmetic mean, median and mode for the pre-test scores of Control group and Experimental group are almost the same. Standard deviation values also indicate that the sample is almost homogeneous with regard to the pre-test and post-test scores on Scientific Deduction Skill. The distribution is positively skewed for post-test of Experimental group, pre-test of Control group and post-test of Control group. This means that the scores are massed at the lower end of the distribution. The kurtosis value is greater than 0.263 for pre-test of Control group. Hence the distribution is platykurtic. The kurtosis value is less than 0.263 for post-test of Experimental, meaning that the distribution is leptokurtic.

Figure 5

Bar diagram representing Pre-test and Post-test scores of Experimental and Control Groups on Scientific Deduction Skill



Analysis of genuineness of the difference in visual thinking in mathematics of experimental and control groups with respect to the component- scientific deduction skill

The statistical procedure of ANCOVA as given by Garret (1981) is given under the following sub-heads:

- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Scientific Deduction Skill using Analysis of Variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Scientific Deduction Skill using Analysis of Co-variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Scientific Deduction Skill using Adjusted Means.

The details of analysis carried out under the component Scientific Deduction Skill are given below:

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Scientific Deduction Skill using Analysis of Variance

In this part of the analysis, the Total Sum of Squares, Mean Square Variances and F-ratios for the pre-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component Scientific Deduction Skill were computed. The Analysis of Variance Table for the pre-test scores (x) and post-test scores (y) of students taught using Augmented Reality Content and Activity-Oriented Method are presented below in Table 6

Table 6

Summary of Analysis of Variance of the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component Scientific Deduction Skill

Source of variation	df	SSx	SSy	MSx (Vx)	MSy (Vy)	Fx	Fy
Among Means	1	2.40	224.3	2.40	224.27	0.79	35.79
Within Groups	58	176.53	363.5	3.04	6.27		
Total	59	178.93	587.7				

From the results obtained in Table 6, it is clear that the values of Fx are not significant at 0.05 level of significance since these are less than the table values required (Fx=4.00 with df (1,58)) for significance. It shows that there is no significant difference between pre-test scores of students in the experimental and control groups for the Visual Thinking Test when the component Scientific Deduction Skill is considered.

The table values of F required for significance at 0.01 level with df (1,58) is 7.08. The obtained Fy value, Fy =35.79 is significant at 0.01 level since the obtained value is greater than table value for the respective df. The significant Fy value indicate that the experimental and control groups differ significantly in their post-test scores on Visual Thinking in Mathematics under the component Scientific Deduction Skill

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Scientific Deduction Skill using Analysis of Co-Variance

The above computation has to be carried out for the purpose of correcting the post-test (y) scores for the difference in the pre-test (x) scores. So ANCOVA was adopted for further computation. The results of the analysis are presented in Table 7

Table 7

Summary of Analysis of Co-Variance for the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component – Scientific Deduction Skill

Source of variation	df	SSx	SSy	SSxy	SSyx	MSy (Vy)	SDy(x)
Among Means	1	2.40	224.27	-23.20	232.21	232.21	
Within Groups	57	176.53	363.5	232.2	353.60	6.20	2.49
Total	58	178.93	587.73	209.01	585.81		Fyx=37.43

The Fyx ratio as indicated in Table 7 is significant for the Visual Thinking Test (Fyx = 7.08, df (1,57)) since the obtained value is greater than table value required for significance at 0.01 level with the corresponding df. The significant ratios for the adjusted post-test scores show that the final Mean scores of students on Visual Thinking in Mathematics under the component Scientific Deduction Skill in the experimental and control groups differ significantly after they were adjusted for the difference in the pre-test scores for the Visual Thinking Test. The significant F-ratio necessitates proceeding to test the difference separately by the test of significance of difference between means.

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Scientific Deduction Skill using adjusted Means

The adjusted means for the post-test scores of students in the experimental and control groups were calculated using regression coefficients. The data and results are shown in Table 8 below.

Table 8

Data for Adjusted Means of post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component-Scientific Deduction Skill

Groups	N	Mx	My	Mxy	SEm	t-value
Experimental	30	6.33	16.7	16.93	0.64	
Control	30	6.73	12.8	12.54		6.83
Total	60	6.53	14.73			

From Table 8, it is clear that the obtained 't' value is significant at 0.01 level (t= 6.83; p<0.01) for the Visual Thinking Test under the component Scientific Deduction Skill. This implies that Augmented Reality Content is better than Activity-Oriented Method in enhancing Visual Thinking in Mathematics with special reference to Scientific Deduction Skill for the secondary school students.

Figure 8

Graphical Representation of the Pre-test and Adjusted Post-test Means on Visual Thinking in Mathematics of Students in the Experimental and Control groups under the component- Scientific Deduction Skill

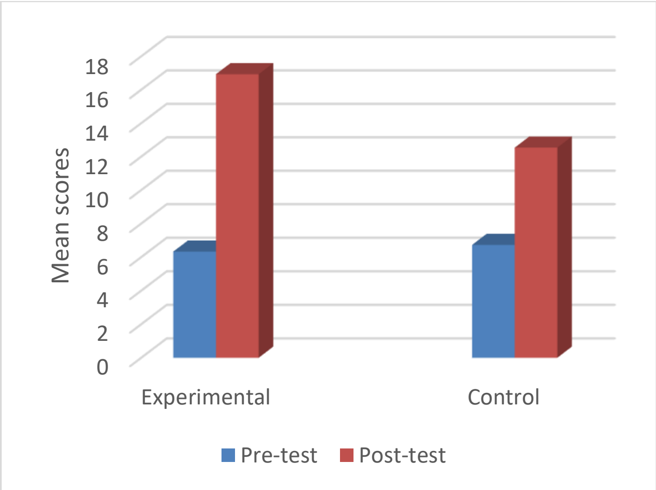


Table 9

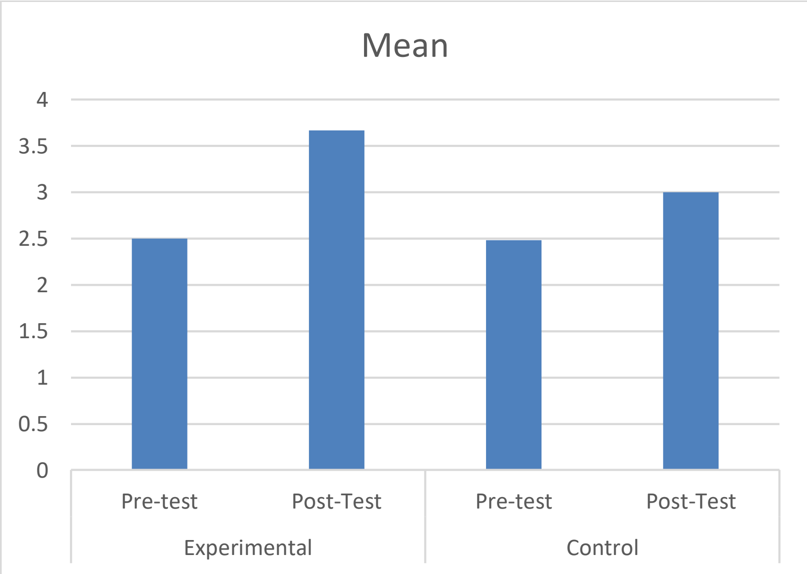
Descriptive statistics of pre-test and post-test scores of experimental and control groups on Visual Reading Skill

Groups		Mean	Median	Mode	Standard deviation	Skewness	Kurtosis
Experimental	Pre-test	2.5	2	1	1.37	0.93	0.791
	Post-test	3.67	3	3	1.45	0.71	-0.39
Control	Pre-test	2.48	2	3	0.995	0.372	0.053
	Post-test	3	3	3	1.65	0.416	-0.626

From the Table 9, it is clear that the arithmetic mean, median and mode for the pre-test scores of Control group and Experimental group are almost the same. Standard deviation values also indicate that the sample is almost homogeneous with regard to the pre-test and post-test scores on Visual Reading Skill. The distribution is positively skewed for post-test of Experimental group, pre-test of Control group and post-test of Control group. This means that the scores are massed at the lower end of the distribution. The kurtosis value is greater than 0.263 for pre-test of Experimental group. Hence the distribution is platykurtic. The kurtosis value is less than 0.263 for post-test of Experimental, meaning that the distribution is leptokurtic.

Figure 9

Bar diagram representing the pre-test and post-test scores of Experimental and Control groups on Visual Reading Skill



Analysis of genuineness of the difference in visual thinking in mathematics of experimental and control groups with respect to the component-visual reading skill

The statistical procedure of ANCOVA as given by Garret (1981) is given under the following sub-heads:

- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Reading Skill using Analysis of Variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Reading Skill using Analysis of Co-variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Reading Skill using Adjusted Means. The details of analysis carried out under the component Visual Reading Skill are given below:

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Reading Skill using Analysis of Variance

In this part of the analysis, the Total Sum of Squares, Mean Square Variances and F-ratios for the pre-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component Visual Reading Skill were computed. The Analysis of Variance Table for the pre-test scores (x) and post-test scores (y) of students taught using Augmented Reality Content and Activity-Oriented Method are presented below in Table 10

Table 10

Summary of Analysis of Variance of the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component- Visual Reading Skill

Source of variation	df	SSx	SSy	MSx (Vx)	MSy (Vy)	Fx	Fy
Among Means	1	0.15	198.0	0.15	198.02	0.05	45.04
Within Groups	58	168.03	255.0	2.90	4.40		
Total	59	168.18	453.0				

From the results obtained in Table 10, it is clear that the values of Fx are not significant at 0.05 level of significance since these are less than the table values required (Fx=4.00 with df (1,58)) for significance. It shows that there is no significant difference between pre-test scores of students in the experimental and control groups for the Visual Thinking Test when the component Visual Reading Skill is considered.

The table values of F required for significance at 0.01 level with df (1,58) is 7.08. The obtained Fy value, Fy =45.04 is significant at 0.01 level since the obtained value is greater than table value for the respective df. The significant Fy value indicate that the experimental and control groups differ significantly in their post-test scores on Visual Thinking in Mathematics under the component Visual Reading Skill

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Reading Skill using Analysis of Co-Variance

The above computation has to be carried out for the purpose of correcting the post-test (y) scores for the difference in the pre-test (x) scores. So ANCOVA was adopted for further computation. The results of the analysis are presented in Table 11

Table 11

Summary of Analysis of Co-Variance for the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component – Visual Reading Skill

Source of variation	df	SSx	SSy	SSxy	SSyx	MSy (Vy)	SDy(x)
Among Means	1	0.15	198.02	-5.45	200.61	200.61	
Within Groups	57	168.03	255.0	200.6	244.18	4.28	2.07
Total	58	168.18	452.98	195.16	444.79		Fyx=46.83

The Fyx ratio as indicated in Table 11 is significant for the Visual Thinking Test (Fyx = 7.08, df (1,57)) since the obtained value is greater than table value required for significance at 0.01 level with the corresponding df. The significant ratios for the adjusted post-test scores show that the final Mean scores of students on Visual Thinking in Mathematics under the component Visual Reading Skill in the experimental and control groups differ significantly after they were adjusted for the difference in the pre-test scores for the Visual Thinking Test. The significant F-ratio necessitates proceeding to test the difference separately by the test of significance of difference between means.

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Visual Reading Skill using adjusted Means

The adjusted means for the post-test scores of students in the experimental and control groups were calculated using regression coefficients. The data and results are shown in Table 12 below:

Table 12

Data for Adjusted Means of post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component-Visual Reading Skill

Groups	N	Mx	My	Mxy	SEm	t-value
Experimental	30	6.83	16.8	16.89	0.53	
Control	30	6.93	13.2	13.14		7.02
Total	60	6.88	15.02			

From Table 12, it is clear that the obtained 't' value is significant at 0.01 level (t= 7.02; p<0.01) for the Visual Thinking Test under the component Visual Reading Skill. This implies that Augmented Reality Content is better than Activity-Oriented Method in enhancing Visual Thinking in Mathematics with special reference to Visual Reading Skill for the secondary school students.

Figure 12

Graphical Representation of the Pre-test and Adjusted Post-test Means on Visual Thinking in Mathematics of Students in the Experimental and Control groups under the component- Visual Reading Skill

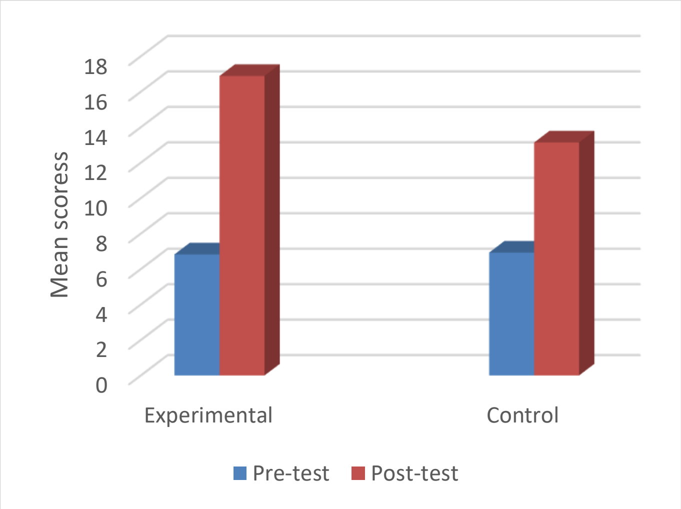


Table 13

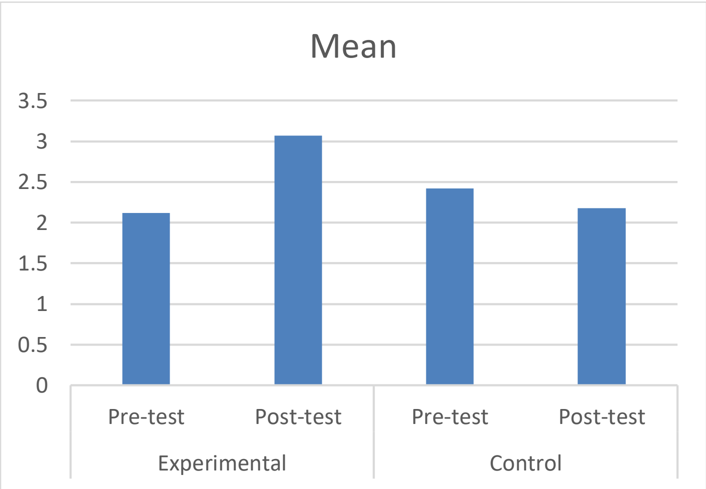
Descriptive statistics of pre-test and post-test scores of experimental and control groups on skill of analyzing and interpreting the visual shape

Groups		Mean	Median	Mode	Standard deviation	Skewness	Kurtosis
Experimental	Pre-test	2.12	2	1	1.201	1.475	3.185
	Post-test	3.07	3	1	0.985	0.216	-1.17
Control	Pre-test	2.42	2	2	1.06	0.678	0.611
	Post-test	2.18	2.5	2	1.24	0	-1.290

From the Table 13, it is clear that the arithmetic mean, median and mode for the pre-test scores of Control group and Experimental group are almost the same. Standard deviation values also indicate that the sample is almost homogeneous with regard to the pre-test and post-test scores on Skill of analyzing and interpreting the visual shape. The distribution is positively skewed for post-test of Experimental group, pre-test of Control group and post-test of Experimental group. This means that the scores are massed at the lower end of the distribution. Skewness is zero for post-test of control group, meaning that the distribution is normal. The kurtosis value is greater than 0.263 for pre-test of Experimental and Control groups. Hence the distribution is platykurtic. The kurtosis value is less than 0.263 for post-test of Experimental, meaning that the distribution is leptokurtic.

Figure 13

Bar diagram representing the pre-test and post-test scores of Experimental and Control groups on the Skill of analyzing and interpreting the visual shape



Analysis of genuineness of the difference in visual thinking in mathematics of experimental and control groups with respect to the component- skill of analyzing and interpreting the visual shape

The statistical procedure of ANCOVA as given by Garret (1981) is given under the following sub-heads:

- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component skill of analyzing and interpreting the visual shape using Analysis of Variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component skill of analyzing and interpreting the visual shape using Analysis of Co-variance
- Comparison of scores on Visual Thinking in Mathematics of experimental and control groups under the component skill of analyzing and interpreting the visual shape using Adjusted Means

The details of analysis carried out under each sub heads are given below:

Table 14

Summary of Analysis of Variance of the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups under the component- Skill of analyzing and interpreting the visual shape

Source of variation	df	SSx	SSy	MSx (Vx)	MSy (Vy)	Fx	Fy
Among Means	1	4.27	190.8	4.27	190.82	1.58	37.62
Within Groups	58	157.07	294.2	2.71	5.07		
Total	59	161.33	485.0				

From the results obtained in Table 14 , it is clear that the values of Fx are not significant at 0.05 level of significance since these are less than the table values required (Fx=4.00 with df (1,58)) for significance. It shows that there is no significant difference between pre-test scores of students in the experimental and control groups for the Visual Thinking Test when the component Skill of analyzing and interpreting the visual shape is considered.

The table values of F required for significance at 0.01 level with df (1,58) is 7.08. The obtained Fy value, Fy =37.62 is significant at 0.01 level since the obtained value is greater than table value for the respective df. The significant Fy value indicate that the experimental and control groups differ significantly in their post-test scores on Visual Thinking in Mathematics under the component Skill of analyzing and interpreting the visual shape

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Skill of analyzing and interpreting the visual shape using Analysis of Co-Variance

The above computation has to be carried out for the purpose of correcting the post-test (y) scores for the difference in the pre-test (x) scores. So ANCOVA was adopted for further computation. The results of the analysis are presented in Table 15

Table 15

Summary of Analysis of Co-Variance for the pre-test and post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component – Skill of analyzing and interpreting the visual shape

Source of variation	df	SSx	SSy	SSxy	SSyx	MSy (Vy)	SDy(x)
Among Means	1	4.27	190.82	-28.53	199.16	199.16	
Within Groups	57	157.07	294.2	199.2	285.36		2.24
Total	58	161.33	484.98	170.63	484.52		Fyx=39.78

The Fyx ratio as indicated in Table 15 is significant for the Visual Thinking Test (Fyx = 7.08, df (1,57)) since the obtained value is greater than table value required for significance at 0.01 level with the corresponding df. The significant ratios for the adjusted post-test scores show that the final Mean scores of students on Visual Thinking in Mathematics under the component Skill of analyzing and interpreting the visual shape in the experimental and control groups differ significantly after they were adjusted for the difference in the pre-test scores for the Visual Thinking Test. The significant F-ratio necessitates proceeding to test the difference separately by the test of significance of difference between means.

Comparison of the scores on Visual Thinking in Mathematics of experimental and control groups under the component Skill of analyzing and interpreting the visual shape using adjusted Means

The adjusted means for the post-test scores of students in the experimental and control groups were calculated using regression coefficients. The data and results are shown in Table 16 below

Table 16

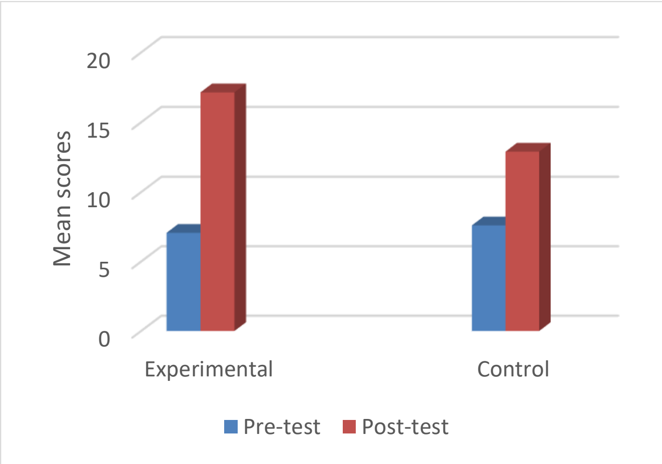
Data for Adjusted Means of post-test scores on Visual Thinking in Mathematics of the experimental and control groups for the Visual Thinking Test under the component- Skill of analyzing and interpreting the visual shape

Groups	N	Mx	My	Mxy	SEm	t-value
Experimental	30	7.07	16.8	17.14	0.58	7.34
Control	30	7.60	13.2	12.90		
Total	60	7.33	15.02			

From Table 16 , it is clear that the obtained 't' value is significant at 0.01 level (t= 7.34; p<0.01) for the Visual Thinking Test under the component Visual Reading Skill. This implies that Augmented Reality Content is better than Activity-Oriented Method in enhancing Visual Thinking in Mathematics with special reference to Skill of analyzing and interpreting the visual shape for the secondary school students.

Figure 16

Graphical Representation of the Pre-test and Adjusted Post-test Means on Visual Thinking in Mathematics of Students in the Experimental and Control groups under the component- Skill of analyzing and interpreting the visual shape



Conclusion

This research aimed at identifying the impact of the use of the Augmented Reality Content when teaching Mathematics in the development of Visual Thinking and in particular, components of Visual Thinking , among secondary school students, using the experimental approach and a pre-test post-test non-equivalent group design. When analyzing results, it became clear that the students of the experimental group exceeded the students of the control group in Visual Thinking when the components of Visual Thinking as Visual Discrimination Skill, Scientific Deduction Skill, Visual Reading Skill and Skill of analyzing and interpreting the visual shape were considered as the Augmented Reality Technology adapted the actual reality through adding digital features to improve the perception of the learner.

The use of the Augmented Reality Technology when presenting the skill side by side with its details achieved better learning than the conventional way and enhanced the development of the visual thinking skills**.**

REFERENCES

[1] Ahmad, N.N. (2020). Augmented Reality for Learning Mathematics: A Systematic Literature Review. International Journal of Emerging Technologies in Learning, 15(16), 106-122. https://doi.org/10.3991/ijet.v15i16.14961

[2] Al-Ghamdi, I. (2020). The effectiveness of the augmented reality-based teaching strategy in developing mathematical proficiency and visual thinking among middle school students. The Journal of Educational Sciences: King Saud University- College of Education, 32(3),485-511**.**

[3] Arnheim, R. (1969). Visual Thinking. United States of America: University of California Press Ltd. ISBN: 0-520-01871-0

[4] Arthur, J., & Waring, M. (Eds.). (2012). Research Methods & Methodologies in Education (1st ed.). New Delhi: Sage Publications India Pvt Ltd.

[5] Aveyard, H. (2010). Doing a literature review in health and social care: A practical guide (2 nd ed.). Berkshire, England: Open University Press

[6] Azuma, R.T. (1997). A survey of augmented reality Presence: Teleoperators & Virtual Environments, 6(4), 355-385. https://doi.org/10.1162/pres.1997.6.4.355.

[7] Bartle, R.A. (2003). Designing Virtual World. Berkeley, CA: New Riders Publishing.

[8] Bergin, T. (2018). An Introduction to Data Analysis- Quantitative, Qualitative and Mixed Methods. Thousand Oaks, CA: Sage

[**9**] Best, J.W., & Khan, J.V. (2007). Research in Education (9th ed.). New Delhi: Prentice Hall

[10] Bloom, B., Engelhart, M., Furst, E., Hill, W., & Krathwohl, D. (1956). Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain. New York, Toronto: Longmans, Green

[11] Borg, Walter R. & Gall, M.D. (2003). Educational Research- An Introduction (7th ed.). Los Angeles, CA: Allyn & Bacon

[12] Cai, S., Liu, E., Shen, Y., Liu, C., Li, S., & Shen, Y. (2020). Probability learning in mathematics using augmented reality: impact on student's learning gains and attitudes. Interactive Learning Environments, 28(5), 560-573. https://doi.org/10.1080/10494820.2019.1696839.

[13] Campbell, K., Collis, K. & Watson, J. (1995). Visual Processing during Mathematical Problem Solving. Educational Studies in Mathematics, 28(2) , 177-194. https://doi.org/10.1007/BF01295792.

[14] Daniela, L. (Ed.). (2020). New Perspectives on Virtual and Augmented Reality: Finding New Ways to Teach in a Transformed Learning Environment. New York: Routledge Taylor & Francis Group. ISBN: 978-1-003-00187-4 (ebk)

[15] Dubin, R. (1978). Theory building (Rev. ed.). New York: Free Press

[16] Dunleavy, M., & Dede, C. (2014). Augmented reality teaching and learning. In Handbook of research on educational communications and technology (pp. 735-745). New York, NY: Springer.

[**17**] Duran, M. & Bekdemir, M. (2013). Evaluation of visual math literacy self -efficacy perception with visual mathematics accomplishment. *Pegam Journal of Education and Instruction, 3*(3), 27-40.

[**18**] Estapa, A. & Nadolny, L. (2015). The Effect of an Augmented Reality Enhanced Mathematics Lesson on Student Achievement and Motivation. Journal of STEM Education: Innovations & Research, 16(3), 40-48.

[**19**] Fishman, B., & Dede, C. (2016). Handbook of Research on Teaching (5th ed.). American Educational Research Association

[**20**] Fraser, M. (1994). 'Quality in higher education: an international perspective' in Green, D. (Ed.), What is Quality in Higher Education? Buckingham: Open University Press.

[**21**] Furht, B. (Ed.). (2011). Handbook of Augmented Reality. Heidelberg, London: Springer. e-ISBN 978-1-4614-0064-6

[**22**] Giaquinto, M. (2007). Visual Thinking in Mathematics: An epistemological study. New York: Oxford University Press Inc.

[**23**] NCTM (2000). Principles and Standards for School Mathematics. Reston VA: NCTM.

[**24**] Orozo, C., Esteban, P., & Trefftz, H. (2006). Collaborative and distributed augmented reality in teaching multi-variate calculus.

[**25]** Osamah, M., Zyad, A., Ali, B., & Wafa, A. (2019). Effect of augmented reality and simulation on the achievement of mathematics and visual thinking among students. International Journal of Emerging Technologies in Learning (IJET), 14(18), 164-185. https://doi.org/10.3991/ijet.v14i18.10748.

[**26]** Stuessy, C. (1984) Path Analysis: A model for the development of scientific reasoning abilities in adolescents. *Journal of Research in Science Teaching*, *26*(1), 41-53.

[**27]** Sun, K.T., & Chen, M.H. (2019). Utilizing free augmented reality app for learning geometry at elementary school in Taiwan: Take volumetric measurement of compound body for example. International Journal of Distance Education Technologies (IJDET), 17(4), 36-53. https://doi:10.4018/IJDET.2019100103.