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SOME CONTRBUTIONS ON LOCALLY CONVEX SPACES

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ABSTRACT

This paper manages a few parts of the theory of locally convex projective limits. Since the start of the theory of locally convex spaces, a fundamental gadget was to lessen inquiries in general spaces to the inquiries in easier or better-referred to spaces, for example, Banach or Frechet-spaces. This methodology might be productive if the space under thought was developed out of those spaces. There is likewise a valuable theory managing last locally convex topologies that originate from topologies on certain convex subsets of a given vector space. It is discovered that total locally convex spaces are those where a subspace of the double is feeble and closed if every one of its crossing points with equicontinuous set are frail and closed.

Keywords: *Inductive and Projective limit, Convex spaces, Locally Convex Space.*

I. Introdction

If L is a locally angled space and S is a subspace, by then S can be seen as locally brought spaces up in its own one of a kind right, just by using the repressions of the semi standards on L. For example, if V is an open subset of the incredible plane, we can consider K(V), the space of holomorphic chips away at V, as a locally raised space as a subspace of D(V).(We are verifiably using the space of complex-regarded reliable limits on V).If S is, moreover, closed in L, by then we can regard the remainder space L/S as a locally bended space by using the method which is used to give a remainder of a normed space with a standard. By the day's end, if p is a semi-norm on L we portray a semi standard $\check{p}$ on L/S by putting

$$\tilde{p}(x) = \inf\{p(x') : x' \in E \quad \pi(x') = x\}$$

where π indicates the characteristic mapping structure L onto L/S.

We will habitually utilize the straightforward reality that if (L,S) resp. (E_1, S_1) are locally arched spaces with F resp. F_1 a closed subspace of E resp. E_1 and T is a continuous linear mapping from E into E_1 which maps F into F_1, then T lifts to a continuous linear mapping $\tilde{T}$ from E/F into E_1/F_1.

Limited items: If (E_i, S_i) (i = 1, . . . , n) is a limited group of locally arched spaces, at that point the item structure on $E = \prod_{i=1}^{n} E_i$ is that one which is characterized by the seminorms of the structure

$$(x_1, \ldots, x_n) \mapsto p_1(x_1) + \cdots + p_n(x_n)$$

where p_1, . . . ,p_n is any decision of seminorms, whereby $p_i \in S_i$. The relating topology on E is essentially the Cartesian item topology.

Projective structures: Assume that E is a vector space and that (E_α, S_α) is a recorded group of locally arched spaces. Assume that for each α ∈ Awe are given a straight mapping T_α from E into Eα and that these mappings separate E for example on the off chance that x in E is non-zero, at that point there is a α so that T_α , $T_\alpha(x) \neq 0.$ At that point the group of all semi-norms of the structure $x \mapsto p_\alpha(T_\alpha(x))$

where α ∈ A and p_α is in S_α, portray a locally angled structure on E called the projective structure provoked by the T_α. The looking at topology on E is none other than the hidden topology provoked by the T_α. Consequently a linear mapping from a locally bended space F into E is consistent accepting and if, for each α, the blend T_α F from F into E_α is relentless.

For example, if S is a locally moderate space, by then the locally raised structure on C(S) is effectively the projective structure provoked by the control mappings from C(S) into the Banach spaces C(K) as K experiences the family K(S).

Projective limits: Let (E_α, S_α) be a gathering of locally bended spaces requested by a planned set A so that for each α ∈ A there is a constant straight mapping πβα: $E_\beta \to E_\alpha$ and the going with closeness conditions are fulfilled:

- For each α ∈A, παα is the character mapping on Eα;
- On the remote possibility that α ≤ β ≤ γ, by then πβα ∘ πγβ = πγα.

At that point

$$E_0 = \{x = (x_\alpha) \in \prod E_\alpha : \pi_{\beta\alpha}(x_\beta) = x_\alpha \text{ for } \alpha \leq \beta\}$$

is a subspace of the thing space, in sureness a shut subspace. We call E0,with the locally raised structure impelled from the thing, the projective farthest reaches of the E_α. We furthermore make πα for the imprisonment of the projection from the thing onto E_α to E_0.

Inductive structures: Let E be a vector space, (E_α) α∈A a family of locally convex spaces and, for each α ∈A, T_α a linear mapping from Eα into E. We suppose further that the union of the ranges $T_\alpha(E_\alpha)$ of these mappings is E. For each set γ = (p_α) of semi-norms indexed by A, whereby pα ∈Sα, we define a semi-norm p_γ on E by putting

$$p_\gamma(x) = \inf\{\sum_{\alpha \in J} p_\alpha(x_\alpha)\}$$

the infimum being taken over the representations of x as a finite sum $\sum_{\alpha\in J} T_\alpha(x_\alpha)$. The set S of all such semi-norms does not as a rule separate E. Thus we may must have response to the trap referenced above and utilize these semi-norms to characterize a locally curved structure on the remainder space E/N_S. (Were mark that there are neurotic models where it is important to take this remainder space in any case, this won't be the situation for the spaces which we will build as such). The structure on E has the property that a direct mapping from E/N_S into a locally curved space F is persistent if and just if $T_\alpha \circ \pi S \circ T$ is ceaseless from E_α into F for each $\alpha \in A$. Further, a totally raised subset U of E/N_S is a τS-neighborhood of zero if and just if $(\pi S \circ T_\alpha)^{-1}(U)$is an area of zero in E_α for each α. The cases wherein will be intrigued are the accompanying:

II. PREMILINARIES AND DEFINITIONS

Projective limits: Let (E_α, S_α) be a group of locally curved spaces ordered by a coordinated set A so that for each $\alpha \in A$ there is a nonstop straight mapping $\pi\beta\alpha$: $E_\beta \to E_\alpha$ and the accompanying similarity conditions are satisfied:

- For each $\alpha \in A$, $\pi\alpha\alpha$ is the character mapping on E_α;
- On the off chance that $\alpha \leq \beta \leq \gamma$, at that point $\pi\beta\alpha \circ \pi\gamma\beta = \pi\gamma\alpha$.

At that point

$$E_0 = \{x = (x_\alpha) \in \prod E_\alpha : \pi_{\beta\alpha}(x_\beta) = x_\alpha \text{ for } \alpha \leq \beta\}$$

is a subspace of the item space, in certainty a closed subspace. We call E_0,with the locally raised structure actuated from the item, the projective furthest reaches of the Eα. We additionally compose πα for the confinement of the projection from the item onto E_α to E_0.

The space which we have quite recently built is described by the accompanying all inclusive property: if F is a locally arched space and (T_α) α∈A is a group of nonstop direct mappings, whereby T_α maps F into Eα and we have $T_\alpha = \pi\beta\alpha \circ T_\beta$ at whatever point $\alpha \leq \beta$, at that point there is an interesting persistent straight mapping T from F into E so that $T_\alpha = \pi a \circ T$ for each a. Then again, every such mapping from F into E emerges along these lines.The proof is easy. If $x$is an element of F, we simply define Tx to be the thread (Tα(x))α∈A.The following are examples of projective limits:

The product $\prod_{\alpha\in A} E_\alpha$ can be regarded as the projective limit of the family of finite products $\{\prod_{\alpha\in H} E_\alpha : J \in \mathcal{F}(\mathcal{S})\}$.here F (A) is directed by in clusion and the linking mappings πJ′,J(J ⊂J′) are the natural projections.

Intersections: Let X be a vector space and (E_α)α∈A an ordered group of vector subspaces, furnished with locally curved structures so that on the off chance that $\alpha \leq \beta$, at that point $E_a \subset E_b$ and the regular infusion from E_α into E_β is persistent. At that point the projective furthest reaches of the family (E_α) can be recognized (as a vector space) with the convergence of the subspaces thus the last has a characteristic locally raised structure. It is known as the (locally arched) crossing point of the E_α. We will be particularly inspired by the accompanying instances of this development:

The space of consistent capacities. Give S a chance to be a locally reduced space and direct K(S), the group of compacta of S, by consideration. on the off chance that $K \subset K'$, $\rho K',K$ means the confinement administrator from $C(K')$ into $C(K)$.Then $\{C(K), \rho K',K\}$ is a projective framework and its projective point of confinement can beidentified with (C(S), SK).

Direct sums: As usual, (E_α) $\alpha \in A$ is an indexed family of locally convex spaces. $\bigoplus_{\alpha\in A} E_\alpha$denotes the vector space direct Q sum i.e. the subspace of $\prod_{\alpha\in A} E_\alpha$ consisting of those vectors (x_α) for which at most finitely many of the $x\alpha$ are non-zero. Now if $J \in \mathcal{J}(\mathcal{A})$, there is a natural injection from $\prod_{\alpha\in J} E_\alpha$ into $\bigoplus_{\alpha\in A} E_\alpha$. hence we can provide the latter with the corresponding inductive locally convex structure. It is then called the (locally convex) direct sum of the E_α. (Note that this is finer than the structure induced from the Cartesian product in particular, it is not necessary to take a quotient space in the construction of the inductive structure).

Inductive limits: There is a construction dual to that of projective limits which we now describe (E_α) is a family of locally convex spaces indexed by a directed set A and for each $\alpha \leq \beta$ there is a continuous linear mapping $i_{\alpha\beta} : E_\alpha \to E_\beta$ such that the following conditions are satisfied:

- for each α, $i_{\alpha\alpha}$ is the identity
- If $\alpha \leq \beta \leq \gamma$, then $i_{\beta\gamma} \circ i_{\alpha\beta} = i_{\alpha\gamma}$.

Let N be the closed subspace of the direct sum $\bigoplus_{\alpha\in A} E_\alpha$,E_α which is generated by elements of the form

$$\{i_\alpha(x) - i_\beta \circ i_{\alpha\beta}(x) : \alpha \leq \beta, x \in E_\alpha\}$$

where $i\alpha$ denotes the injection from $E\alpha$ into the direct sum. Then $\bigoplus_{\alpha\in A} E_\alpha/N$, with the quotient structure inherited from the direct sum, is called the in-ductive limit of the spectrum $(E_\alpha, i_{\alpha\beta}$. It is characterised by the following universal property. For each space F and every family (T_α) of ceaseless direct mappings (where $T\alpha$ maps $E\alpha$ into F) which fulfills the conditions $T_\alpha = T_\beta \circ i_{\alpha\beta}$ $(\alpha \leq \beta)$, there is a one of a kind persistent straight mapping T from E into F so that $T_\alpha = T \circ \pi N \circ I$ for each α.

Models: I. Associations: X is a vector space, (E_α) $\alpha \in A$ an ordered arrangement of subspaces, each with a locally arched structure so that on the off chance that $\alpha \leq \beta$, at that point $E_\alpha \subset E_\beta$ and the consideration is nonstop. Assume that there is a Hausdorff topology τ on X whose limitation to each $E\alpha$ is coarser than $\tau S\_$. At that point as far as possible can be related to the association and the presence ot τ guarantees that the comparing group of semi standards on E isolates focuses. E, with the inductive structure, is the (locally raised) association of the E_α. We will be keen on the accompanying specific models.

We presently consider the Hahn-Banach hypothesis for locally arched spaces. As we have seen, an appropriate type of this outcome is valid for every single such space. This guarantees they can be outfitted with a tasteful duality hypothesis, a reality which is of specific result in the Schwartzman conveyance hypothesis. We start with an elective evidence of the Hahn-Banach hypothesis which delineates the kind of contention which can regularly be utilized to diminish results on locally raised spaces to the relating ones for nor prescription spaces.

We review that the double of a locally raised space, which we signify by E′, is the space of all constant straight structures on E (for example the persistent straight mappings from E into the authoritative one-dimensional space R or C).

Suggestion 1 Let f be a direct structure on the locally raised space (E, S).

At that point coming up next are proportionate:

- F is consistent;
- There exists a p in S so that $|f(x)| \leq 1$ if $x \in Up'$
- There is a $p \in S$ so that $|f| \leq p$;
- $|f|$ is a consistent semi-norm on E;
- There is a p in S and a $\tilde{f} \in E_p{}'$ with the goal that f factorizes over ˜ f for example $f = \tilde{f} \circ \omega p$;
- Ker f is closed.In a similar way we can demonstrate that on the off chance that M is a lot of direct structures on E, at that point coming up next are comparable:
- M is equicontinuous at zero;
- M is equicontinuous on E;
- There is a p in S so that $|f| \leq p$ for every $f \in M$;
- There is a p in S and a standard limited subset of $E_p{}'$ $\widetilde{M}$ so that $M = \widetilde{M} \circ \omega p$.

We would now be able to express the Hahn-Banach hypothesis for locally raised spaces. Note that the quantitative perspective is supplanted by the way that equicontinuous sets of functionals can be lifted at the same time to equicontinuous families.

Suggestion 2 Proposition (Hahn-Banach hypothesis) Let M be an equicontinuous group of straight structures on a subspace F of the locally arched space E. At that point there is an equicontinuous family M_1 of E′ with the goal that M is the arrangement of limitations of the individuals from M_1 to F.

Examples:

1) . Of course, normed spaces are examples of locally convex space, where we use the single norm to generate a locally convex structure.
2) If E is a normed space and F is a separating subspace of its dual E ′ , then the latter induces a family S of semi-norms, namely those of the form $px : x \to |f(x)|$ for $f \in F$. This induces a locally convex structure on E which we denote by $S_w(F)$. The corresponding topology σ(E, F) is called the weak topology induced by F. The important cases are where F = E′ , respectively where E is the dual G′ of a normed space and F is G (regarded as a subspace of E′ = G′′).
3) The fine locally convex structure: If E is a vector space, then the set of all seminorms on E defines a locally convex structure on E which we call the fine structure for obvious reasons.
4) The space of continuous functions: If S is a completely regular space, we denote by K(S) or simply by K, the family of all compact subsets of S. If K ∈ K, then

$$p_K(x) = \sup\{|x(t)| : t \in K\}$$

is a semi-norm on C(S), the space of continuous functions from S into R. The family of all such semi-norms defines a locally convex structure SK on C(S) the corresponding topology is that of compact convergence i.e. uniform convergence on the compacta of S.

5). Differentiable functions. If k is a positive integer, C k (R) denotes the family of all k-times continuously differentiable functions on R. For each r ≤ k and K in K(R), the mapping

$$p_K^r : x \mapsto \sup\{|x^{(r)}(t)| : t \in K\}$$

is a seminorm on C_k (R). The family of all such seminorms defines a locally convex structure on C_k(R).

6). Spaces of operators: Let H be a Hilbert space. On the operator space L(H), we consider the following semi-norms:

$$p_x : T \mapsto \|Tx\|$$

$$p_x^* : T \mapsto \|T^*x\|$$

$$p_{x,y} : T \mapsto |(Tx|y)|$$

for x and y in H. The family of all semi-norms of the first type define the bf strong locally convex structure on L(H), while those of the first two type define the strong *-structure. Finally, those of the third type define the weak operator structure.

IV. Dual pairs. We have seen that the duality between a normed space and its dual can be used to define weak topologies on E and E′ . For our purposes, a more symmetrical framework for such duality is desirable. Hence we consider two vector spaces E and F, together with a bilinear form (x, y) → hx, yi from E × F into R, which is separating i.e. such that

• if y ∈ F is such that hx, yi = 0 for each x in E, then y = 0;;

• if x ∈ E is such that hx, yi = 0 for each y in F, then x = 0.

Then we can regard F as a subspace of E^*, the algebraic dual of E, by associating to each y in F the linear functional

$$x \mapsto \langle x, y \rangle$$

Similarly, E can be regarded as a subspace of F^*(E, F) is then said to be a dual pair. The typical example is that of a normed space, together with its dual or, more generally, a subspace of its dual which separates E. For each y ∈ F, the mapping py : x → |hx, yi| is a semi-norm on E and the family of all such semi-norms generates a locally convex structure which we denote by S_w(F) – the weak structure generated by F.

A subset B of F is said to be bounded for the duality if for each x in E,

$$\sup\{|\langle x, y\rangle| : y \in B\} < \infty.$$

In this case, the mapping

$$p_B : x \mapsto \sup\{p_y(x) : y \in B\}$$

is a semi-norm on E. Let B denote a family of bounded subsets of F whose union is the whole of F. Then the family {pB : B ∈ B} generates a locally convex structure S_B on E, that of uniform convergence on the subsets of B.

Thus if B consists of the singletons of F, we rediscover the weak structure. If B is taken to be the family of those absolutely convex subsets of F which are compact for the topology defined by $S_w(E)$ on F, then S_B is called the Mackey structure and the corresponding topology (which is denoted by τ (E, F)) is called the Mackey topology. Finally, if we take for B the family of all bounded subsets of F, then we have the strong structure–the corresponding topology is called the strong topology.

A rich source of dual pairs is provided by the so-called sequence spaces. These are, by definition, subspaces of the space ω = R N i.e. the family of all real-valued sequences which contain φ, the spaces of those sequences with finite support (i.e. φ = {x ∈ω :ξ_n = 0 except for finitely many n}.

φ and ω are regarded as locally convex spaces, ω with the structure defined by the semi-normspn : x ⟶ |ξ_n| and φ with the fine structure.

III. RESULT AND DISCUSSION

We currently consider the Hahn-Banach theorem for locally convex spaces. As we have seen, an appropriate type of this outcome is valid for every such space. This guarantees they can be furnished with an acceptable duality hypothesis, a reality which is of specific result in the Schwartzian dissemination hypothesis. We start with an elective evidence of the Hahn-Banach theorem which shows the kind of contention which can regularly be utilized to lessen results on locally convex spaces to the relating ones for normed spaces. We review that the double of a locally convex space, which we indicate by E′ , is the space of all continuous linear structures on E (for example the continuous linear mappings from E into the accepted one-dimensional space R or C).

Theorem II.1 Let f be a linear form on the locally convex space (E,S). Then the following are equivalent:

- f is continuous;
- there exists a p in S so that |f(x)| ≤ 1 if x ∈ Up'
- there is a p ∈ S so that |f| ≤ p;
- |f| is a continuous semi-norm on E;
- there is a p in S and an $\tilde{f}$∈ E_p' so that f factorises over $\tilde{f}$ i.e. f = $\tilde{f}$ ◦ ωp;
- Ker f is closed.

In the same way we can show that if M is a set of linear forms on E, then the following are equivalent:

- M is equicontinuous at zero
- M is equicontinuous on E;

there is a p in S so that $|f| \leq p$ for each $f \in M$;

• there is a p in S and a norm bounded subset of E′ p $\widetilde{M}$ so that M = $\widetilde{M} \circ \omega p$.

We can now state the Hahn-Banach theorem for locally convex spaces. Note that the quantitative aspect is replaced by the fact that equicontinuous sets of functionals can be lifted simultaneously to equicontinuous families.

Theorem II.2 (Hahn-Banach theorem) Let M be an equicontinuous family of linear forms on a subspace F of the locally convex space E. Then there is an equicontinuous family M_1 of E′ so that M is the set of restrictions of the members of M_1 to F.

Proof. We choose a semi-norm p as in (4) above and apply the Hahn-Banach theorem for normed spaces to find a bounded family $\widetilde{M_1}$which extends the subset $\widetilde{M}$of the dual of Fp. Then $M_1 = \widetilde{M_1} \circ \omega p$ has the required property.

Exactly as in the case of normed spaces, this result has a number of corollaries which we list without proofs:

- a linear form f in the dual of F can be lifted to one in the dual of E;
- if x_0 is an element of E and G is a closed subspace of E which does not contain x0, then there exists an f in E′ so that f (0) on G and $f(x_0) = 1$;
- if $x_0 \in E$ and p is a continuous semi-norm on E so that $p(x_0) = 0$, then there is a continuous linear form f on E so that $f(x_0) = 1$ and $p(x_0)f(x) \leq p(x)$ for each $x \in E$;
- let $x_1, \dots, x_n$ be linearly independent elements of a locally convex space E. Then there exists elements $f_1, \dots, f_n$ in E′ so that $f_i(x_i) = 1$ for each i‘ and $f_i(x_j) = 0$ for each distinct pair i, j;
- let A be a closed, absolutely convex subset of a locally convex space which does not contain the point x_0. Then there is a continuous linear form f on E so that $f(x_0) > 1$ and $|f(x)| \leq 1$ for each x in A. In other words, a point x lies in the closed, absolutely convex hull of a set B if and only if for each continuous linear form f on E which is less than one in absolute value on B, we have $|f(x)| \leq 1$.

We now consider some topological and uniform concepts (such as completeness, compactness etc.) in the context of locally convex spaces and show how they may be characterised using the canonical representation of the space E as a projective limit of Banach spaces:

A subset M of a locally convex space (E,S) is said to be S-complete (or simply complete) if it is complete for the uniform structure induced by S. It is S-bounded if each semi-norm p in S is bounded on M and S-compact (resp. relatively compact) if it is compact (relatively) for the topology τS. Finally it is S-pre compact if it is relatively compact in the completion Eˆ of E. of course, we shall omit the prefix S in the above notation unless it is not clear from the context which locally convex structure we are dealing with.

The following comments on these definitions are obvious:

- In the definition of boundedness, it is sufficient to check that each p in a generating family of semi-norms is bounded on M;
- we have the implications

compact ⇒ relatively compact ⇒pre compact⇒ bounded

and compact ⇒ complete.

• if M is complete, then it is τS closed. On the other hand, every closed subset of a complete set is complete;

• if we denote by B_S the collection of bounded subsets of E, then this family has the following stability properties: if B, C ∈ BS and λ > 0, then $\lambda B \in B_S$, $B + C \in B_S$ and $\Gamma(B) \in B_S$ (where Γ(B) denotes the closed, convex hull of B).

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