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SINGLE STEP METHOD SOLUTIONS OF FERMATEAN FUZZY DIFFERENTIAL EQUATIONS WITH INITIAL VALUE PROBLEMS

R.Anand[1] & G.Thavaselvan[2]

[1,2] **Assistant professor, Department of Mathematics, CARE College of Engineering , Thayanur, Tiruchirappalli – 620009,Tamilnadu, India.**

Abstract: Cauchy Problem for Fuzzy differential Equations defined by O. Kaleva [10] .In this article, we study one step method numerical solution of fermatean fuzzy differential equations with initial value problem. This new numerical method is used for finding the numerical solution of the cubic membership functions problems. The accuracy and efficiency of the proposed method is illustrated by solving suitable example in fermatean fuzzy initial value problem.

Keywords: fermatean fuzzy set, differential equation, fuzzy initial value ,continous, indeterminacy, fuzzy convex, Runge-kutta method.

1.Introduction: The topic of fuzzy differential equations (FDEs) have been rapidly growing in recent years. The concept of fuzzy derivative was first introduced by Zadeh [16], it was followed up by Dubois and Prade [7] by using the extension principle in their approach. Other methods have been discussed by Puri and Ralescu [13] and Goetschel and Voxman [8]. Kandel and Byatt [10] applied the concept of fuzzy differential equation (FDE) to the analysis of fuzzy dynamical problems. The FDE and the initial value problem (Cauhy problem) were rigorously treated by Kaleva [[9],[11]], Seikkala [14] and other researchers.The numerical methods for solving fuzzy differential equations are introduced in [[1], [2], [4]]. Buckley and Feuring [[3],[4]] introduced two analytical methods for solving N^{th}-order linear differential equations with fuzzy initial value conditions. Their first method of solution was to fuzzy if the crisp solution and then check to see if it satisfies the differential equation with fuzzy initial value conditions; and the second method was the reverse of the first method, they first solved the fuzzy initial value condition and the checked to see if it defined a fuzzy function.

The idea of intuitionistic fuzzy set was first published by Atanassov [3], as a generalization of the notion of fuzzy set. Atanassov [2] explored the concept of fuzzy set theory [16] by intuitionistic fuzzy set (IFS) theory. The existence and uniqueness of the solution of a differential equation with intuitionistic fuzzy data has been studied by several authors [2]. Numerical solution of Intuitionistic Fuzzy differential Equations By Euler and Taylor Methods has been introduced in [4]. Intuitionistic fuzzy cauchy problem is solved numerically by Runge–Kutta of order four method based on [6] and establish that this method is better than Euler method. Yager [15] explored a typical division of these collections known as q-rung orthopair uncertainty collection in which the aggregate of the q^{th} power of the help for and the q^{th} power of the help against is limited by one. He explained that as 'q' builds the space of truth able orthopairs increments and therefore gives the user more opportunity in communicating their conviction about value of membership. At the point when q = 3, Senapati and Yager [15] have evoked q-rung orthopair uncertainty collection as fermatean uncertainty sets (FUSs).Pythagorean uncertainty collections have studied the concentration of many researchers within a short period of time. For example, Yager [15] has derived up a helpful decision technique in view of Pythagorean uncertainty aggregation operators to deal with Pythagorean uncertainty MCDM issues. In this article, we study one step method numerical solution of fermatean fuzzy differential equations with initial value problem. This new numerical method is used for find the numerical solution of the cubic membership functions problems. The accuracy and efficiency of the proposed method is illustrated by solving suitable example in fermatean fuzzy initial value problem.

2. Preliminaries

In this section, we consider the initial value problem for the fermatean uncertainty differential equation.

Definition -2.1: [Senapati and Yager, 2019a] Let 'X' be a universe of discourse A. Fermatean uncertainty set "F" in X is an object having the form $F = \{\langle x, m_F(x),\ n_F(x)\rangle / x \in X\}$,
where $m_F(x): X \to [0,1]$ and$n_F(x): X \to [0,1]$,including the condition
$0 \le \left(m_F(x)\right)^3 + \left(n_F(x)\right)^3 \le 1$, for all x ∈ XThe numbers $m_F(x)$signifies the level (degree) ofmembership and $n_F(x)$indicate the non-membership of the element 'x' in the set F.All through this paper, we will indicate a fermatean fuzzy set is FFS.

For any FFS 'F' and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - \left(m_F(x)\right)^2 - \left(n_F(x)\right)^2}$ is to find out as the degree of indeterminacy of 'x' to F. For convenience, senapathi and yager called $\left(m_F(x),\ n_F(x)\right)$ a fermatean fuzzynumber (FFN) denoted by $F = (m_F,\ n_F)$.

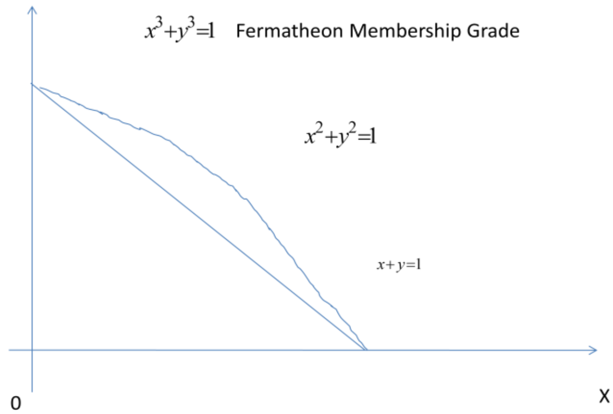
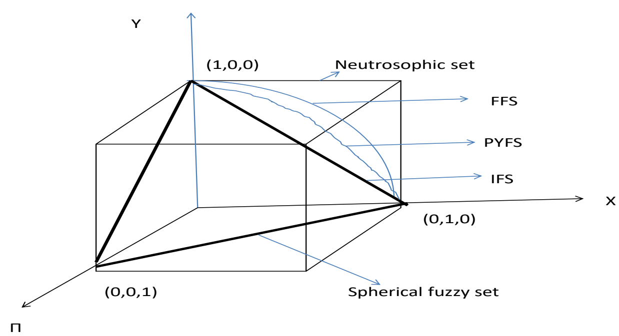


Fig-1

We will explain the membership grades (MG's) related Fermatean uncertainty collections as Fermatean membership grades.

Theorem 2.2:[Senapati and Yager, 2019a] The collections of FMG's is higher than the set of pythagorean membership grades (PMG's) and bi –fuzzy membership grades (BMG's).

Proof: This improvement can be evidently approved in the following figure.



Here we find that BMG's are all points beneath the line $x + y \leq 1$, the PMG's are all points with $x^2 + y^2 \leq 1$. We see that the BMG's enable the presentation of a bigger body of non-standard membership grades than BMG's and PMG's. Based on fermatean fuzzy membership grades,

we study interval-valued fermatean fuzzy soft set in matrix aspects.

Throughout this paper, fermatean fuzzy set is abbreviated by FFS and fermatean fuzzy numbr by FFI.

Definition 2.3: Let D be a non-empty crisp set and $\gamma_1 = ({\alpha_{D_1}}^3(m), {\beta_{D_1}}^3(m))$ and

$\wp_2 = ({\alpha_{D_2}}^3(m), {\beta_{D_2}}^3(m))$ be FFSs on D. Then

(i) $\wp_1 = \wp_2$ if and only if ${\alpha_{D_1}}^3(m) \leq {\alpha_{D_2}}^3(m)$and ${\beta_{D_1}}^3(m)) \geq {\beta_{D_2}}^3(m)$.

(ii) $\wp_1 = \wp_2$if and only if $\wp_1 \subset \wp_2$ and $\wp_2 \subset \wp_1$.

(iii) ${\wp_1}^c = ({\beta_{D_1}}^3(m), {\alpha_{D_1}}^3(m))$.

(iv) [ ] $\wp_1 = ({\alpha_{D_1}}^3(m)\ , 1 - {\alpha_{D_2}}^3(m)\}$, <> $\wp_1 = (1 - {\beta_{D_2}}^3(m), {\beta_{D_1}}^3(m))$.

Definition 2.4: Let **{**$\wp$i} be an arbitrary family of FFS's in C, where $\wp$i = ((α_D^{3}(m), β_D^{3}(m)) for each i ∈ J.

Thenμ

(i) $\cap \varpi_{i} =$ **(** $\wedge \alpha_D{}^3$(mi), $\wedge \beta_D{}^3$(mi))

(ii) $\cup \varpi_{i} =$ ($\vee \alpha_D{}^3$(mi), $\vee \beta_D{}^3$(mi)).

Remark 2.5: Clearly, in the case of ordinary fuzzy sets, $\pi_A(x) = 0$ for every x ε X.

we denote by FFI = { (α ,β)→ R $[0; 1]^2$; for all x ε R ≤ α(x) + β(x) ≤ 1}

The collection of all fermatean fuzzy number by FFI. An element (α ,β) of FFI is called fermatean fuzzy number if it satisfies the following conditions:

(i) is normal, i.e., there x_0, x_1 ε R such that $\alpha(x_0) = 1$ and $\beta(x_1) = 1$.

(ii) α is fuzzy convex and β is fuzzy concave.

(iii) α is upper semi-continuous and β is lower semi-continuous.

(iv)supp(α) = cl{x ε R : β(x) < 1 } is bounded.

Definition 2.6: If (α , β) a fuzzy number , so we can see $[(\alpha, \beta)]^r$ as $[\alpha]^r$ and $[(\alpha, \beta)]^r$ as $[1\text{-}\beta]^r$.

Definition 2.7: Let < α , β > an element of FFI and r ε [0,1], we define the following sets;

$[<\alpha, \beta>]_L^+ (r) = \min \{ x \in R / \alpha(x) \geq r \}$,

$[<\alpha, \beta>]_R^+ (r) = \max \{ x \in R / \alpha(x) \geq r \}$,

$[<\alpha, \beta>]_L^- (r) = \min \{ x \in R / \beta(x) \leq 1 - r \}$,

$[<\alpha, \beta>]_R^- (r) = \max \{ x \in R / \beta(x) \leq 1 - r \}$,

Definition 2.8 (Operations on Fermatean fuzzy number):

Let $[<\alpha, \beta>]_r = [[<\alpha, \beta>_L{}^+](r), [<\alpha, \beta>_R{}^+](r)]$,

Let $[<\alpha, \beta>]^r = [[<\alpha, \beta>_L{}^+](r), [<\alpha, \beta>_R{}^+](r)]$

we define the following operations by ;

$[<\alpha, \beta> \oplus <x, y>]^r = [<\alpha, \beta>]^r + [<x, y>]^r$,

$[\lambda <\alpha, \beta>]^r = \lambda [<\alpha, \beta>]^r$,

$[<\alpha, \beta> \oplus <x, y>]_r = [<\alpha, \beta>]_r + [<x, y>]_r$,

$[\lambda <\alpha, \beta>]_r = \lambda [<\alpha, \beta>]_r$,

Where < α , β >, < x, y > ε FFI and λ ε R.

Definition 2.10 : Let Ω: [a,b] $\rightarrow$ FFI be fermatean fuzzy valued mapping and x_o ε [a, b]. Then Ω is called fermatean fuzzy continous in x_o if and only if for all ($\varepsilon > 0$) (there exist $\vartheta > 0$) (for all t ε [a, b]) tel que $|x - x_o| < \Omega \rightarrow dp(\Omega(x), \Omega(x_o)) \leq \varepsilon$.

Definition 2.11: Ω is called fermatean fuzzy continous if and only if is fermatean continous in every point of [a, b].

3. Fermatean fuzzy differential Equations

In this section, we consider the initial value problem for the fermatean fuzzy differential equation.

$$y'(t) = \mathscr{J}(t, y(t)),\ t \in T \quad \text{-------------------------------} \quad (1)$$

$y(to) = (\alpha t_o, \beta t_o)$ ε FFI

where x ε FFI is unknown I = [t_o, T] and $\mathscr{J}$: I x FFI $\rightarrow$ FFI. $y(t_o)$ is fermatean fuzzy number.

Denote the r-level set

$[y(t)]_r = [y(t)]_L^{+}, \ y(t)]_R^{+}]$

$[y(t)]^r = [y(t)]_L^{-}, \ y(t)]_R^{-}]$

$[y(t_o)]_r = [y(t_o)]_L^{+}(r), y(t_o)]_R^{+}(r)]$

$[y(t_o)]^r = [y(t_o)]_L^{-}(r), y(t_o)]_R^{-}(r)]$

$[\mathscr{J}(t, y(t))]_r = [\mathscr{J}_1+(t, y(t); r, \mathscr{J}_2+(t, y(t); r]$

$[\mathscr{J}(t, y(t))]^r = [\mathscr{J}_3^-(t, y(t); r, \mathscr{J}_4^-(t, y(t); r]$

where

$$\left.\begin{aligned}
\mathscr{J}_1+(t, y(t); r) &= \inf \{\mathscr{J}(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\} \\
\mathscr{J}_2+(t, y(t); r) &= \sup \{\mathscr{J}(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\} \\
\mathscr{J}_3+(t, y(t); r) &= \inf \{\mathscr{J}(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\} \\
\mathscr{J}_4+(t, y(t); r) &= \sup \{\mathscr{J}(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\}
\end{aligned}\right\} \text{-----------} \quad (2)$$

Denote

$$\left.\begin{aligned}
\mathscr{J}_1+(t, y(t); r) &= A\,\{f(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\} \\
\mathscr{J}_2+(t, y(t); r) &= B\,\{f(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\} \\
\mathscr{J}_3+(t, y(t); r) &= C\,\{f(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\} \\
\mathscr{J}_4+(t, y(t); r) &= D\,\{f(t, \alpha) / \alpha \in [y(t)]_L^{+}(r), [y(t)]_R^{+}(r)\}
\end{aligned}\right\} \text{-----------} \quad (3)$$

Denote by G (I, FFI) the set of all continous mappings from I to FFI.

Defining the metric

$M(\mathscr{J}, \mathscr{H}) = \max d_\alpha (\mathscr{J}_1 t, \mathscr{J}_2 t), (\mathscr{H}_1 t, \mathscr{H}_2 t))$ where t ε I. Here $\mathscr{J}(t) = (\mathscr{J}_i t, \mathscr{J}_2 t)$ and $\mathscr{H}(t) = (\mathscr{H}_1 t, \mathscr{H}_2 t)$.

4. Proposed Method

Let $[\ Y(t)\ ]_{\alpha} = [\ Y(t_n)]_L^{+}(r)\ ,\ [Y(t_n)]_R^{+}\ (r)\ ]$

$[\ Y(t)\ ]^{\alpha} = [\ Y(t_n)]_L^{-}(r)\ ,\ [Y(t_n)]_R^{-}\ (r)\ ]$ be the exact solution of (1) and

$[\ Y(t)\ ]_{\alpha} = [\ Y(t_n)]_L^{+}(r)\ ,\ [Y(t_n)]_R^{+}\ (r)$

$[\ Y(t)\ ]^{\alpha} = [\ Y(t_n)]_L^{-}(r)\ ,\ [Y(t_n)]_R^{-}(r)$

Be approximate solutions at t_n , $0 \leq n \leq N$. The solutions are calculated by grid points at

$to < t_1 < t_2 < \ldots\ldots\ldots\ t_n\ = T\ ,\ \ h = T - t_o\ /N\ ,\ \ t_n = t_o + nh\ ,\ n = 0,1,2\ \ldots\ldots\ldots$ (4)

we recall that equation (3).

The Runge-kutta method of order four that calculates the value of the function in four intermediate points as follows;

$[y(t_n)]_L^{+} = [\ y(t_n)]_L^{+}\ (r) + 1/6\ [\ K_1 + 2K_2 + 2K_3 + K_4]$ ………………. (5)

where

$K_1 = h\ A\ (\ t_n\ ,[\ y(t_n)]_L^{+}(r)\ ,\ [y(t_n)]_R^{+}\ (r))\ )$

$K_2 = h\ B\ (\ t_n + h/2\ \ ,[\ y(t_n)]_L^{+}(r) + k/_12\ [y(t^n)]_R^{+}\ (r)) + k_1/2\ )$

$K_3 = h\ C\ (\ t_n + h/2\ \ ,[\ y(t_n)]_L^{+}(r) + k_2/2\ [y(t_n)]_R\ (r))\ _1^{+} + k_2/2\ )$

$K_4 = h\ D\ (\ t_n + h\ [\ y(n)]_L^{+}(r) + k_{3,}\ \ [y(t_n)]_R^{+}\ (r)) + k_3)$

and

$[y(t_n+1)]_R^{+} = [\ y(tn)]_R^{+}\ (r) + 1/6\ [\ K_1 + 2K_2 + 2K_3 + K_4]$ ………………. (6)

Where

$K_1 = h\ B\ (\ t_n\ ,[\ y(t_n)]_L^{+}(r)\ ,\ [y(t_n)]_R^{+}\ (r))\ )$

$K_2 = h\ B\ (\ t_n + h/2\ [\ y(t_n)]_L^{+}(r) + k_1/2,\ [y(t_n)]_R^{+}\ (r)) + k1/2\ )$

$K_3 = h\ B\ (\ t_n + h/2\ [\ y(t_n)]_L^{+}(r) + k_2/2,\ [y(t_n)]_R^{+}\ (r)) + k2/2\ )$

$K_4 = h\ B\ (\ t_n + h\ [\ y(t_n)]_L^{+}(r) + k_{3,}\ [y(t_n)]_R^{+}(\ r)) + k_3)$

And

$[y(t_n+1)]L- = [\ y(t_n)]_L^{+}\ (r) + 1/6\ [\ K_1 + 2K_2 + 2K_3 + K_4]$ ………………. (7)

Where

$K_1 = h\ C\ (\ t_n\ ,[\ y(t_n)]_L^{-}(r)\ ,\ [y(t_n)]_R\text{-}\ (r))\ )$

$K_2 = h\ C\ (\ t_n + h/2\ [\ y(t_n)]_L^{-}(r) + k_1/2,\ [y(t_n)]_R^{-}\ (r)) + k_1/2\ )$

$K_3 = h\ C\ (\ t_n + h/2\ [\ y(t_n)]_L^{-}(r) + k_2/2,\ [y(t_n)]_R^{-}\ (r)) + k_2/2\ )$

$K_4 = h\ C\ (\ t_n + h\ [\ y(t_n)]_L^{-}(r) + k_3,\ \ [y(t_n)]_R^{-}(r)) + k_3)$

And

$[y(t_n+1)]_R^{-} = [\ y(t_n)]_R^{-}\ (r) + 1/6\ [\ K_1 + 2K_2 + 2K_3 + K_4]$ ………………. (8)

$K_1 = h\ D\ (\ t_n\ ,[\ y(t_n)]_L^{-}(r)\ ,\ [y(tn)]_R^{-}\ (r))\ )$

$K_2 = h\ D\ (\ t_n + h/2\ \ ,[\ y(t_n)]_L^{-}(r) + k_1/2\ [y(t_n)]_R^{-}\ (r)) + k_1/2\ )$

K_3 =h D (t_n + h/2 ,[$y(t_n)]_L^-(r)$ + $k_2/2$ $[y(t_n)]_R^-$ (r)) + $k_2/2$)

K_4 =h D (t_n + h ,[$y(t_n)]_L^-(r)$ + k_3 $[y(t_n)]_R^-$ (r)) + k_3).

Let A (t, α^+, β^+), B (t, α^+,β^+), C (t, α^+, β^+) and D (t, α^+, β^+) be the functions of (3), where α^+, β^+,α^-,β^- are the constants and $\alpha^+ \leq \beta^+$ and $\alpha^- \leq \beta^-$.

5.Numerical Example

Consider the fermatean fuzzy initial value problem,

$y'(t) + y(t) = \vartheta(t)$, for all $t \geq 0$ …………………………… (1)

y_o = (-1,1,0,-1.5,1.5) and ϑ (t) = 2 exp (-t) y_o.

Applying the method of solution proposed in [11] we get,

$[y(t)]_L^+ (r) = (\alpha -1) \exp(-t) (1 +2t)$

$[y(t)]_R^+ (r) = (1- \alpha) \exp(-t) (1 +2t)$

$[y(t)]_L^- (r) = (3t + 1.5)\ \ (r -1)\exp(-t)$

$[y(t)]_R^- (r) = (3t +1.5)\ \ (1- r) \exp(-t)$

Therefore the exact solution is given by

$[Y(t)]_r$ = [(r-1) exp (-t) (1+2t), (1-r) exp (-t) (1+2t)]

$[Y(t)]^r$ = [(3t+1.5) (r-1) exp (-t), (3t +3/2), (1-r) exp (-t)] which at t = 0.1 are

$[Y(0.1)]_r$ = [(r-1) exp (-0.1) (1.2), (1-r) exp (-0.1) (1.2)]

$[Y(0.1)]^r$ = [(0.2+0.5) (r-1) exp (-0.1), (0.2 +0.5), (1-r) exp (-0.1)]

Applying the Runge-kutta method proposed we get;

$[y(t_n+1)]_L^+ (r) = [y(t_n)]_L^+ (r) + 1/6 [K_1 +2K_2+2K_3+K_4]$,

where

$K_1 = h [\vartheta(t_n)]_L^+(r) – [y(t_n)]_L^+ - k_1 /2]$

$K_2 = h [\vartheta(t_n + h/2)]_L^+(r) – [y(t_n)]_L^+ - k_2 /2]$

$K_3 = h [\vartheta(t_n + h/2)]_L^+(r) – [y(t_n)]_L^+ - k_3 /2]$

$K_4 = h [\vartheta(t_n + h)]_L^+(r) – [y(t_n)]_L^+ - k_3]$

And

$[y(t_n+1)]_R^+ = [y(t_n)]_R^+ (r) + 1/6 [K_1 +2K_2+2K_3+K_4]$ where n = 0,1,2 ….. N and h = 1/N.

The exact solutions and approximate solutions by R.K method compared with other single step methods like Eulers , modified methods and Taylor's approach are plotted at t =0.1 and h = 1/4

Euler's method	Modified Euler's method	Runge-kutta method
1.345	1.894	0.0231
1.098	0.986	0.235
1.967	1.679	0.1384
1.873	1.943	0.2000

Table-1 Comparison among Euler's, modified Euler's ,Taylor's and R.K method.

Conclusion: In this work, our proposed method give the least and beat solution among various one step methods. we have focused the proposed 4th order R-K method to find the numerical solutions of fermatean fuzzy differential equations with some initial value problem will attain minimum values which is suitable than others..

Future work: One can obtain the similar results by the way of multi-step methods in various parameters.

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