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# ZAGREB INDICES OF LOLLIPOP GRAPH

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*Abstract:* (m, n)-Lollipop graph is a special type of graph consisting Of Complete graph on m vertices for  $m \ge 3$  and a path graph on n vertices for  $n \ge 2$ , connected with a bridge. In this chapter we are going to find the Zagreb indices and also find their corresponding polynomials.

Keywords – Lollipop Graph, Zagreb indices.

# I. Introduction

The concepts of connectivity in Chemical Graph Theory, which define relationships between the structure of a molecule and its properties. One important parameter, topological index, which characterizesmolecular graph and remain invariant under graph automorphism are called Zagreb Indices. These parameters introduced by Gutman I [39] are defined by using sum and product of degrees of vertices joining an edge.

Consider a subset of E(G) denoted as  $E_{a,b} = \{(u,v) \in E(G) / d(u) = a \text{ and } d(v) = b\}$ . Partitioning the edge set E(G) into disjoint sets  $E_{a,b}$  with all possible choices of pairs a,b we can determine the Zagreb and hyper Zagreb indices and their corresponding polynomials for first and second kind.

The Zagreb indices, hyper Zagreb indices and their corresponding polynomials we use definitions given by Gutman [39] stated as follows:

$$M_1(G) = \sum_{E_{uv \in E(G)}} [d_G(u) + d_G(v)]$$
 ------ (1)

$$M_2(G) = \sum_{uv \in E(G)} [d_G(u)d_G(v)] - \dots$$
 (2)

$$M_1(G, x) = \sum_{uv \in E(G)} x^{[d_G(u) + d_G(v)]}$$
(3)

$$M_2(G, x) = \sum_{uv \in E(G)} x^{[d_G(u)d_G(v)]}$$
(4)

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## **II Main Results:**

**Definition 2.1:** The graph obtained by Joining Complete graph  $K_m$   $(m \ge 3)$  to path  $P_n(n \ge 2)$  with bridge called Lollipop graph. Lollipop is denoted by  $L_{m,n}$ . Fig (1)



Figure 1: L<sub>5.4</sub> Lollipop graph

#### 2.2 Zagreb indices of Complete Graph

A complete graph is a graph in which each pair of graph vertices is connected by an edge. The complete graph with m vertices is denoted  $K_m$  and has  $\frac{m(m-1)}{2}$  edges. Vertex and edge partition of complete graphs  $K_m$  where  $m \ge 3$ We have only the one vertex set V(G)

$$V_{m-1} = \{v \in V(G); d_G(v) = m - 1\}; |V_{m-1}| = m$$

We have only the edge set E(G)

 $E_{m-1,m-1} = \{uv \in E(G): d_G(u) = d_G(v) = m-1\}; \left|E_{m-1,m-1}\right| = \frac{m(m-1)}{2}.$ 

**Corollary 2.3:** The first and second Zagreb indices and there polynomials of a complete graph  $K_m$  is

$$M_{1}(K_{m}) = m(m-1)^{2}$$

$$M_{2}(K_{m}) = \frac{m(m-1)^{3}}{2}$$

$$M_{1}(K_{m}, x) = \frac{m(m-1)}{2} [x^{[2(m-1)]}]$$

$$M_{2}(K_{m}, x) = \frac{m(m-1)}{2} [x^{(m-1)^{2}}]$$

**Proof:** The first Zagreb indices of Complete graph and there polynomial is

$$\begin{split} M_{1}(K_{m}) &= \sum_{E_{m-1,m-1}} [d_{G}(u) + d_{G}(v)] \\ &= \frac{m(m-1)}{2} [m-1+m-1] \\ M_{1}(K_{m}) &= m(m-1)^{2} \\ M_{1}(K_{m},x) &= \sum_{uv \in E(G)} x^{[d_{G}(u)d_{G}(v)]} \\ M_{1}(K_{m},x) &= \sum_{E_{m-1,m-1}} x^{[d_{G}(u)+d_{G}(v)]} \\ &= \frac{m(m-1)}{2} [x^{[m-1+m-1]}] \\ &= \frac{m(m-1)}{2} [x^{[2(m-1)]}] \end{split}$$

The second Zagreb indices of Complete graph and there polynomial is

$$\begin{split} M_2(K_m) &= \sum_{E_{m-1,m-1}} [d_G(u)d_G(v)] \\ &= \frac{m(m-1)}{2} [(m-1)(m-1)] \\ &= \frac{m(m-1)^3}{2} \\ M_2(K_m, x) &= \sum_{uv \in E(G)} x^{[d_G(u)d_G(v)]} \\ &= \sum_{E_{m-1,m-1}} x^{[d_G(u)d_G(v)]} \\ &= \frac{m(m-1)}{2} [x^{[(m-1)(m-1)]}] \\ &= \frac{m(m-1)}{2} [x^{(m-1)^2}] \end{split}$$

# 2.4 Zagreb indices of Path Graph

A path in a graph is a finite or infinite sequence of edges which joins a sequence of vertices.  $n (n \ge 2)$ vertices and (n - 1) edges.

**Case 1:** Vertex and edge partition of path graphs  $P_n$  where n = 2

We have only the one vertex set V(G)

$$V_1 = \{v \in V(G); d_G(v) = m - 1\}; |V_1| = 2$$

We have only the edge set E(G)

$$E_{1,1} = \{uv \in E(G): d_G(u) = d_G(v) = 1\}; |E_{1,1}| = 1.$$

**Case 2:** Vertex and edge partition of path graphs  $P_n$  where  $n \ge 2$ 

We have the following two partitions of the vertex set V(G)

 $V_1 = \{v \in V(G); d_G(v) = 1\}; |V_1| = 2$ 

$$V_2 = \{v \in V(G); d_G(v) = 2\}; |V_2| = n - 2$$

We have the following two partitions of the edge set E(G)

$$E_{2,2} = \{uv \in E(G): d_G(u) = d_G(v) = 2\}; |E_{2,2}| = n - 3$$

$$E_{2,1} = \{uv \in E(G): d_G(u) = 2, d_G(v) = 1\}; |E_{2,1}| = 2.$$

**Corollary 2.5:** The first and second Zagreb indices and there polynomials of a Path graph P<sub>n</sub>

$$M_{1}(P_{n}) = \begin{cases} 2 & \text{where } n = 2 \\ 4n - 6 & \text{where } n > 2 \end{cases}$$

$$M_{1}(P_{n}, x) = \begin{cases} x^{2} & \text{where } n = 2 \\ (n - 3)x^{4} + 2x^{3} & \text{where } n > 2 \end{cases}$$

$$M_{2}(P_{n}) = \begin{cases} 1 & \text{where } n = 2 \\ 4n - 8 & \text{where } n > 2 \end{cases}$$

$$M_2(P_n, x) = \begin{cases} x^2 & \text{where } n = 2\\ (n-3)x^4 + 2x^2 & \text{where } n > 2 \end{cases}$$

**Proof:** The first Zagreb indices of Path graph and there polynomial is  $P_n$ JCR

 $M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$ 

Case (i): n = 2

$$M_{1}(P_{n}) = \sum_{E_{1,1}} [d_{G}(u) + d_{G}(v)] = 1(1+1) = 2$$
  
$$M_{1}(P_{n}, x) = \sum_{E_{1,1}} x^{[d_{G}(u) + d_{G}(v)]} = x^{1+1} = x^{2}$$

Case (ii): n > 2

$$\begin{split} \mathsf{M}_{1}(\mathsf{P}_{n}) &= \sum_{\mathsf{E}_{2,2}} [\mathsf{d}_{\mathsf{G}}(\mathsf{u}) + \mathsf{d}_{\mathsf{G}}(\mathsf{v})] + \sum_{\mathsf{E}_{2,1}} [\mathsf{d}_{\mathsf{G}}(\mathsf{u}) + \mathsf{d}_{\mathsf{G}}(\mathsf{v})] \\ &= \mathsf{n} - 3[2+2] + 2[2+1], = 4(\mathsf{n} - 3) + 6 = 4\mathsf{n} - 6 \\ \mathsf{M}_{1}(\mathsf{P}_{n}) &= 4\mathsf{n} - 6 \\ \mathsf{M}_{1}(\mathsf{P}_{n}, \mathsf{x}) &= \sum_{\mathsf{E}_{2,2}} [\mathsf{x}^{[\mathsf{d}_{\mathsf{G}}(\mathsf{u}) + \mathsf{d}_{\mathsf{G}}(\mathsf{v})]}] + \sum_{\mathsf{E}_{2,1}} [\mathsf{x}^{[\mathsf{d}_{\mathsf{G}}(\mathsf{u}) + \mathsf{d}_{\mathsf{G}}(\mathsf{v})]}] \\ &= (n-3)\mathsf{x}^{4} + 2\mathsf{x}^{3} \end{split}$$

The second Zagreb indices of Path graph and there polynomial is

Case (iii): n = 2 $M_2(P_n) = \sum_{E_{1,1}} [d_G(u)d_G(v)] = 1$  $M_2(P_n, x) = \sum_{E_{1,1}} x^{[d_G(u)d_G(v)]} = x$ **Case** (iv): n > 2  $M_2(P_n) = \sum_{E_{2,2}} [d_G(u)d_G(v)] + \sum_{E_{2,1}} [d_G(u)d_G(v)]$  $= (n-3)[2 \times 2] + 2[2 \times 1] = 4n - 8$  $M_2(P_{n,x}) = \sum_{E_{2,2}} [x^{d_G(u)d_G(v)}] + \sum_{E_{2,1}} [x^{d_G(u)d_G(v)}]$ 

$$= (n-3)x^4 + 2x^2$$

#### 2.6 Zagreb indices of Lollipop Graph L<sub>m,n</sub>

Vertex and edge partition of Lollipop graphs  $L_{m,n}$  where  $m \ge 3$  and  $n \ge 2$ We have the following four partitions of the vertex set V(G)

.

$$V_{1} = \{v \in V(G); d_{G}(v) = 1\}; |V_{1}| = 1$$
  

$$V_{2} = \{v \in V(G); d_{G}(v) = 2\}; |V_{2}| = n - 1$$
  

$$V_{m-1} = \{v \in V(G); d_{G}(v) = 2\}; |V_{m-1}| = m - 1$$
  

$$V_{m} = \{v \in V(G); d_{G}(v) = 2\}; |V_{m}| = 1$$

We have the following 5 partitions of the edge set E(G)

$$E_{2,1} = \{uv \in E(G): d_G(u) = 2, d_G(v) = 1\}; |E_{2,1}| = 1\}$$

$$E_{2,2} = \{uv \in E(G): d_G(u) = d_G(v) = 2\}; |E_{2,2}| = \begin{cases} n-2 & \text{where } n > 2\\ 0 & \text{where } n = 2 \end{cases}$$

$$E_{m-1m-1} = \{uv \in E(G): d_G(u) = d_G(v) = m-1\}; |E_{m-1,m-1}| = \frac{m(m-1)}{2} - (m-1).$$
$$E_{m,m-1} = \{uv \in E(G): d_G(u) = m, d_G(v) = m-1\}; |E_{m,m-1}| = m-1$$

$$E_{m,m-1} = \{uv \in E(G): d_G(u) = m, d_G(v) = m - 1\}; |E_{m,m-1}| = m$$
$$E_{m,2} = \{uv \in E(G): d_G(u) = m, d_G(v) = 2\}; |E_{m,2}| = 1$$

**Theorem 2.7:** The first Zagreb indices and their polynomial for Lollipop graph  $L_{m,n}$ 

$$M_{1}(L_{m,n}) = \begin{cases} 3 + \frac{m(m-1)}{2} - (m-1).[2m-2] + (m-1)(2m-1) + m + 2 \text{ where } n = 2\\ 3 + 4(n-2) + \left[\frac{m(m-1)}{2} - (m-1)\right](2m-2) + (m-1)(2m-1) + m \text{ where } n > 2 \end{cases}$$
$$M_{1}(L_{m,n}, x) = \begin{cases} x^{3} + (n-2)x^{4} + \frac{m^{2}-3m+2}{2}(x^{2m-2}) + (m-1)x^{2m-1} + x^{m+2} \text{ where } n = 2\\ x^{3} + (n-2)x^{4} + \frac{m^{2}-3m+2}{2}(x^{2m-2}) + (m-1)x^{2m-1} + x^{m+2} \text{ where } n > 2 \end{cases}$$

**Proof:** The first Zagreb indices and their polynomial of  $L_{m,n}$  are defined as

**Proof:** The second Zagreb indices of  $L_{m,n}$  are defined as

$$\begin{split} \mathsf{M}_{2}(\mathsf{G}) &= \sum_{uv \in \mathsf{E}(\mathsf{G})} [\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)] \\ &= \sum_{E_{2,1}} [\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)] + \sum_{E_{2,2}} [\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)] + \sum_{E_{m-1,m-1}} [\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)] \\ &\sum_{E_{m,m-1}} [\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)] + \sum_{E_{2,2}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} + \sum_{E_{m-1,m-1}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} \\ &= \sum_{E_{2,1}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} + \sum_{E_{2,2}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} + \sum_{E_{m-1,m-1}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} \\ &\sum_{E_{m,m-1}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} + \sum_{E_{m,2}} x^{[\mathsf{d}_{\mathsf{G}}(u) \mathsf{d}_{\mathsf{G}}(v)]} \\ & \mathsf{Case}(\mathbf{i}): n = 2 \\ \mathsf{M}_{2}(\mathsf{L}_{m,n}) &= 1[2 \times 1] + 0[2 + 2] + \left[\frac{\mathsf{m}(\mathsf{m}-1)}{2} - (\mathsf{m}-1)\right] [(m-1)(m-1)] + \\ & (m-1)[(m(m-1)] + 1[m \times 2] \\ \mathsf{M}_{2}(\mathsf{L}_{m,n}, x) &= x^{2} + \frac{\mathsf{m}^{2-3m+2}}{2} x^{(\mathsf{m}-1)^{2}} + (\mathsf{m}-1) x^{\mathsf{m}(\mathsf{m}-1)} + 2\mathsf{m} \\ \mathsf{Case}(\mathbf{ii}): n > 2 \\ \mathsf{M}_{2}(\mathsf{L}_{m,n}) &= 1[2 \times 1] + (n-2)[2 \times 2] + \left[\frac{\mathsf{m}(\mathsf{m}-1)}{2} - (\mathsf{m}-1)\right] [(m-1)(m-1)] + \\ 1[m(m-1)] + 1[m \times 2] \\ \mathsf{M}_{2}(\mathsf{L}_{m,n}) &= x^{2} + (\mathsf{n}-2) x^{4} \frac{\mathsf{m}^{2} - 3\mathsf{m} + 2}{2} x^{(\mathsf{m}-1)^{2}} + (\mathsf{m}-1) x^{\mathsf{m}(\mathsf{m}-1)} + 2\mathsf{m} \\ \mathsf{III} \mathsf{Results:} \\ 1. \mathsf{M}_{1}[\mathsf{L}_{\mathsf{m},n}] &= \mathsf{M}_{1}(\mathsf{K}_{\mathsf{m}}) + \mathsf{M}_{1}(\mathsf{P}_{\mathsf{n}}) + 2(\mathsf{m}+1) \\ 2. \mathsf{M}_{2}[\mathsf{L}_{\mathsf{m},n}] &= \mathsf{mM}_{2}(\mathsf{K}_{\mathsf{m}}) + (\mathsf{n}-27\mathsf{M}_{2}(\mathsf{P}_{\mathsf{n}}) \end{split}$$

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