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A NOTE ON SOLUTION OF LINEAR EQUATION AND ITS APPLICATIONS

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Abstract

This paper is discussed about gauss elimination method and gauss seidel iteration method in linear equation and solved problem used to this method.

Keywords: Linear equation, Gauss Elimination method and Gauss Seidel Iteration method.

1. Introduction

Systems of linear equations Arouse in Europe with the introduction in 1637 [5] by Rene Descartes[4]of coordinate in geometry. In fact in this new geometry, now called Cartesian geometry, lines and plane are represented by linear equations, and computing their intersections amount to solving of linear equations.

Linear algebra is the branch of mathematics concerning linear equations such as; in three dimensional Euclidean space these three planes represent solutions to linear equations, and their intersection represents the set of common solutions in this case a unique point.

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b$$

Linear algebra is central to almost all areas of mathematics. For instance, linear algebra is fundamental in modern presentations of geometry, including for defining basic objects, such as lines, planes and rotations. Also functional analysis is the application of linear algebra to spaces of functions.

Linear algebra is also used in most sciences and fields of engineering, because it allows modeling many natural phenomena, and computing efficiently with such models for non linear systems which cannot be modeled with linear algebra, it is often used for dealing with first order approximations, using the fact that the differential of a multivariate functions at a point is the linear map that best approximate the function near that point.

2. Preliminaries

Definition 2.1. [5] An equation in which the variable or variables have an exponent of 1. Graphs of two variable linear equations are lines.

Example 2.3

$$2x+3y=4$$
$$y=-2x+10$$

Definition 2.4. [1] A system of linear equations is a collection of one or more linear equations involving the same set of variables. Or linear systems are systems of equations in which the variables are never multiplied with each other but only the constants and then summed up.

A linear equation in variables $a_1x_1+a_2x_2+a_3x_3+....a_nx_n=b$

Where $a_1,a_2,a_3,....a_n$ and b are constants of real and complex numbers. The constant a_i is called the coefficient of x_i and b is called the constant term of the equation.

Definition 2.5. [3] Gauss elimination method is a direct method which consists of transforming the given system of simultaneous equations to an equivalent upper triangular system. From this transformed system the required solution can be obtained by the method of back substitution.

Consider the system of n equations in n unknowns given by AX=B where A is the coefficient matrix.

The augmented matrix is

$$(A,B)=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & :b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & :b_2 \\ .... & .... & .... & .... & ... \\ .... & .... & .... & .... & ... \\ a_{n1} & a_{n2} & .... & a_{nn} & b_n \end{pmatrix}$$

To transform the system to an equivalent upper triangular system, we use the following row operations.

The row operation $R_i \to R_i - \frac{a_{i1}}{a_{11}} R_1; i=2,3,...n$

makes all the entries $a_{21},a_{31},....a_{n1}$ in the first column zero.

Here the first equation is the pivotal equation. $a_{11} \neq 0$ is called pivot and $-\frac{a_{i1}}{a_{11}}$ for i=2,3,…. are called multipliers for first elimination. If $a_{11} = 0,$ we interchange the first row with another suitable row so as to have $a_{11} \neq 0.$

Next we do the row operation $R_i \rightarrow R_i - \frac{a_{2i}}{a_{22}} R_2;\ i = 3,4,...,n.$

This makes all entire $a_{32}, a_{42}, ..., a_{n2}$ in the second column zero.

In makes all entries $a_{32}, a_{42}, ...a_{n2}$ in the second column zero.

In general the row operation

$$R_i \rightarrow R_{i-1} - \frac{a_{ik}}{a_{kk}} R_k;\ i = k+1, k = 2,...,n$$

will make all the entries $a_{k+1}, k, a_{k+2}, ..., a_{nk}$ in the k^{th} column zero.

Hence the given system of equations is reduced to the form UX=D where U is an upper triangular matrix. The required solution can be obtained by the method of back substitution.

Definition 2.6. [3] Gauss Seidel iteration method is a refinement of Gauss-Jacobi method.

As in the Jacobi iteration method let

We start with the initial values $x_2^{(0)}, x_3^{(0)}, ..., x_n^{(0)}$ and we get from (1)

$$x_1^{(1)} = \frac{1}{a_{11}} \left[c_1 - a_{12} x_2^{(0)} - a_{13} x_3^{(0)} - .... - a_{1n} x_n^{(0)} \right]$$

In the second equation we use $x_1^{(1)}$ for x_1 and $x_3^{(0)}$ for x_3 etc. and $x_n^{(0)}$ for x_n .

In the Jacobi method we use $x_1^{(0)}$ for x_1. Thus we get

$$x_2^{(1)} = \frac{1}{a_{22}} \left[c_2 - a_{11} x_1^{(1)} - a_{13} x_3^{(0)} - ... - a_{2n} x_n^{(0)} \right].$$

Proceeding like this we find the first iteration values as

$x_1^{(1)}, x_2^{(1)}, ..., x_n^{(1)}$

In general if the values of the variable in the r^{th} iteration are $x_1^{(r)}, x_2^{(r)}, ...x_n^{(r)}$ then the values in the $(r+1)^{th}$ iteration are given by

$$x_1^{(r+1)} = \frac{1}{a_{11}}\left[c_1 - a_{12}x_2^{(r)} - a_{13}x_3^{(r)} - ..... - a_{1n}x_n^{(r)}\right]$$

$$x_2^{(r+1)} = \frac{1}{a_{22}}\left[c_2 - a_{11}x_1^{(r+1)} - a_{13}x_3^{(r)} - ..... - a_{1n}x_n^{(r)}\right]$$

....

$$x_n^{(r+1)} = \frac{1}{a_{nn}}\left[c_n - a_{n1}x_2^{(r+1)} - a_{n2}x_2^{(r+1)} - .... - a_{n,n-1}x_{n-1}^{(r+1)}\right]$$

3. Gauss elimination method

The purpose of this article is to describe how the solutions to a linear system are actually found the fundamental idea is to add multiples of one equation to the others in order to eliminate a variable and to continue this process until only one variable is left. Once this final variable is determined, its value is substituted back into the other equations in order to evaluate the remaining unknowns. This method, characterized by step-by-step elimination of the variables, is called Gaussian elimination.

Example 3.1

Solve the following system of equations using gauss elimination method.

X-Y+Z=1

-3X+2Y-3Z=-6

2X-5Y+4Z=5

Solution

The given set of equation can be written as

$$\begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -3 \\ 2 & -5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -6 \\ 5 \end{pmatrix}$$

The augmented matrix is $(A,B) = \begin{pmatrix} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{pmatrix}$$

Apply $R_2 \rightarrow R_2 - 2R_1$

Apply $R_3 \rightarrow R_3 - 3R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & \frac{12}{5} & 12 \end{pmatrix} R_3 \to R_3 + \frac{1}{5}R_1$$

The given systems of equations reduces to systems

$\frac{12}{5}$Z=12

-5Y+2Z=-5

X+Y+Z=9

Now by back substitution we obtain solution

X=1, Y=3 and Z=5

Example 3.2

Solve the equations X+Y=2 and 2X+3Y=5 by gauss elimination method.

Solution

The given set of equation can be written as

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

The augmented matrix is (A,B) $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \end{pmatrix}$

We note a11=1$\neq$0 is the pivot. The first equation is the pivot equation and -$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is the multiplier for the second equation.

$$(A,B) = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

Apply $R_2 \to R_1$

Apply $R_2 \to R_2 - 2R_1$

The given set of equation is

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

X+Y=2

Y=1

By back substitution we get Y=1 and X=1.

4. Gauss Seidel iteration method

In numerical linear algebra, the Gauss Seidel method, also known as the Liebmann method or the method of successive displacement, is an iterative method used to solve a system of linear equations.

Example 4.1

Solve 2X+Y=3, 2X+3Y=5 Gauss Seidel iteration method.

Solution

Clearly the coefficient matrix $\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$ is diagonally dominant and hence Gauss Seidel iteration method can be applied.

The given equations can be written as

$$x = \frac{1}{2}(3 - y) \rightarrow (1)$$

$$y = \frac{1}{3}(5 - 2x) \rightarrow (2)$$

Putting $y = 0$ in (1) we get, $x = 1.5$

$$x = \frac{1}{2}(3 - 0)$$

$$x = \frac{3}{2} = 1.5$$

Using $x = 1.5$ in (2) we get, $y = 0.6667$

Putting $y = 0.6667$ in (1) we get, $x = 1.6667$

Putting $x = 1.6667$ in (2) we get, $y = 0.8889$

Putting $y = 0.8889$ in (1) we get, $x = 1.0556$

Putting $x = 1.0556$ in (2) we get, $y = 0.9629$

Putting $y = 0.9629$ in (1) we get, $x = 1.0186$

Putting $x = 1.0186$ in (2) we get, $y = 0.9876$

Putting $y = 0.9876$ in (1) we get, $x = 1.0062$

$x = 1.0062$ in (2) we get, $y = 0.9959$

y =0.9959 in (1) we get, x =1.0021

x =1.0021 in (2) we get, y =0.9986

y =0.9986 in (1) we get, x =1.0007

x =1.0007 in (2) we get, y =0.9995

y =0.9995 in (1) we get, x =1.0001

x =1.0001 in (2) we get, y =0.9999

Using y =0.9999 in (1) we get, x =1

Using x =1 in (2) we get, y =1

Hence x =1, y =1 is the solution of the two equations.

Example 4.2

Solve the following system of equation using Gauss Seidel iteration method.

10X+2Y+Z=9

X+10Y-Z=-22

-2X+3Y+10Z=22.

Solution

Clearly the given system of equations is diagonally dominant. Hence it can be solved by gauss Seidel iteration method.

The given system of equations can be written as

$$x = \frac{1}{10}(9 - 2y - z) \rightarrow (1)$$

$$y = \frac{1}{10}(-22 - x + z) \rightarrow (2)$$

$$z = \frac{1}{10}(22 + 2x - 3y) \rightarrow (3)$$

First iteration:

Putting y =0 and z =0 in (1) we get, x =0.9

Putting x =0.9 and z =0 in (2) we get, $y = \frac{1}{10}(-22 - 0.9)$

y =-2.29

Putting x =0.9 and y =-2.29 in (3) we get, $z=\frac{1}{10}(22+2(0.9)-3(-2.29))$

z =3.067

Second iteration:

Putting y =-2.29 and z =3.067 in (1) we get, $x=\frac{1}{10}(9-2(-2.29)-3.067)$

x =1.0513

Putting x =1.0513 and z =3.067 in (2) we get, $y=\frac{1}{10}(-22-1.0513+3.067)$

y =-1.9984

Putting x =1.0513 and y =-1.9984 in (3) we get, $z=\frac{1}{10}(22+2(1.0513)-3(-1.9984))$

z =3.0098

Third iteration:

Putting y =-1.9984 and z =3.0098 in (1) we get, $x=\frac{1}{10}(9-2(-1.9984)-3.0098)$

x =0.9987

Putting x =0.9987 and z =3.0098 in (2) we get, $y=\frac{1}{10}(-22-0+.9987+3.0098)$

y =-1.9989

Putting x =0.9987 and y =-1.9989 in (3) we get, $z=\frac{1}{10}(22+2(0.9987)-3(-1.9989))$

z =2.9994

Fourth iteration:

Putting y =-1.9989 and z =2.9994 in (1) we get, $x=\frac{1}{10}(9-2(-1.9989)-2.9994)$

x =0.9998

Putting x =0.9998 and z =2.99994 in (2) we get, $y=\frac{1}{10}(-22-0.9998+2.9994)$

y =-2

Putting x =0.9998 and y =-2 in (3) we get, $z=\frac{1}{10}(22+2(0.9998)-3(-2))$

$z=3$

Proceeding like this in the next iteration we get,

$x=1$, $y=-2$ and $z=3$.

5. Applications of Linear Equations

The applications of linear equations are vast and are applicable in numerous real-life situations. To handle real-life situations using algebra, we change the given situation into mathematical statements. So that it clearly illustrates the relationship between the unknown variables and the known information. The following are the steps involved to reiterate a situation into a mathematical statement,

- Convert the real problem into a mathematical statement and frame it in the form of an algebraic expression that clearly defines the problem situation.
- Identify the unknowns in the situation and assign variables of these unknown quantities.
- Read the situation clearly a number of times and cite the data, phrases, and keywords. Sequentially organize the obtained information.
- Write an equation using the algebraic expression and the provided data in the statement and solve it using systematic equation solving techniques.
- Reframe the solution to the problem statement and analyze if it exactly suits the problem. Using these steps, the applications of word problems can be solved easily.

5.1 Applications of linear equations in real life

The following are some of the example in which applications of linear equations are used in real life.

- If can be used to solve age related problems.
- It is used to calculated speed, distance and time of a moving object.
- Geometry related problems can be solved.
- It is used to calculate money and percentage related problems.
- Work, time and wages problems can be solved.
- Problems based on force and pressure can be solved.

6. Conclusion

Based on the readings and observation, we have to concluded from this paper that the gauss elimination method and gauss seidel iteration method are used to solved numerical problems.

We also discussed the properties of gauss elimination method and gauss seidel iteration method, some applications of linear equations in real life.

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