IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

2-ODD LABELING ON SOME GRAPHS

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ABSTRACT: A graph G is said to be 2- odd labeling of graph if the vertices V of the graph G can be labeled with integer that should be distinct for any two vertices which are adjacent , then their modulus difference is labelled as either an odd integer (or) exactly 2. In this paper, we investigate 2-odd labeling for some classes of graphs.

KEYWORDS: 2-odd labeling, triple wheel graph, gear graph, Mobius ladder, friendship graph.

1. INTRODUCTION

In this paper, we consider all graphs which are finite, simple, planar connected, connected and undirected graphs. For a graph G (V, E), or simple G, we mean a graph G with its vertex set V and edge set as E. According to J. D. Laison et al. A graph G is 2- odd, if there exists a one-to-one labeling h: V (G) →Z (which has set of all integers) such that for any two vertices u and v which are adjacent, then its modulus difference is either an odd integer or exactly 2. Which is |h (u) – h (v)| is either an odd integer (or) exactly 2. This is also defined as h (uv) = |h (u)-h (v)| and so called h is a 2-odd labeling of graph G. So G is a 2-odd graph if and only if there exists a 2-odd labeling of graph G. We can notice that in 2-odd labeling of graph G, the vertex label of G must be distinct but the edge label should not have like that. Therefore, by this definition h (uv) must be either 2 (or) odd if uv is not an edge of G. We have already seen 2-odd labeling on wheel graph, double wheel graph, helm graph, umbrella graph and butterfly graph here, in this paper, we can see some more graphs which also have 2-odd labeling by our investigation.

Note 1: A 2- odd labeling of a graph G is not unique.

2. MAIN RESULTS

In this part, we prove 2-odd labeling of some new graphs such as triple wheel graph, gear graph, Mobius ladder graph and friendship graph.

THEOREM 1:

1. All triangle-free graphs are 2-odd.
2. Every tree is 2-odd.
3. All grid graphs are 2-odd.

Definition 1: Triple wheel graph

A triple wheel graph TW_n of size n is composed of $2C_n \Lambda K_1$ .TW_n consists of three cycles each of size n where the vertices of the three cycles are all connected to a common hub represented by k_1.

THEOREM 2:

The Triple wheel graph TW_n admits a 2-odd labeling for n≥3.

Proof:

Let TW_n be the given Triple wheel graph with n≥3.

We label the central vertex as v_0.

The inner cycle vertices as $v_1, v_2, \ldots, v_n$.

The middle cycle vertices as $u_1, u_2, \ldots, u_n$.

The outer cycle vertices as $w_1, w_2, \ldots, w_n$.

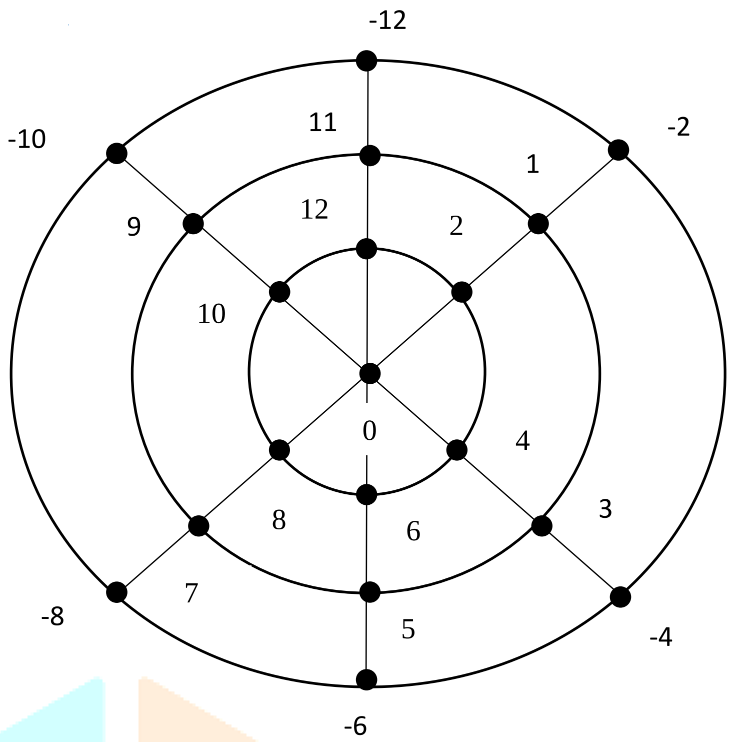
We define: one-to-one labeling $h: v(TW_n) \to Z$. Let, $h(v_0)$ as center.

Let $h(v_0)=0$, $h(v_1)=2$ and $h(v_2)=4$

$h(v_i)=h(v_{i-1})+2 \quad 3 \leq i \leq n$

$h(u_i)=(h(v_i)-1) \quad 1 \leq i \leq n$

$h(w_i)= -h(v_i) \quad 1 \leq i \leq n$



2-odd labeling of triple wheel graph

DEFINITION 2: A GEAR GRAPH

A gear graph, denoted G_n is a graph obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graph W_n.

Thus, G_n has 2n+1 vertices and 3n edges.

THEOREM 3:

A gear graph G_n admits 2-odd labeling of graph $n \leq 5$.

Proof:

Let G_n be denoted as a gear graph.

Let G_0 be the center vertex and 3n edges.

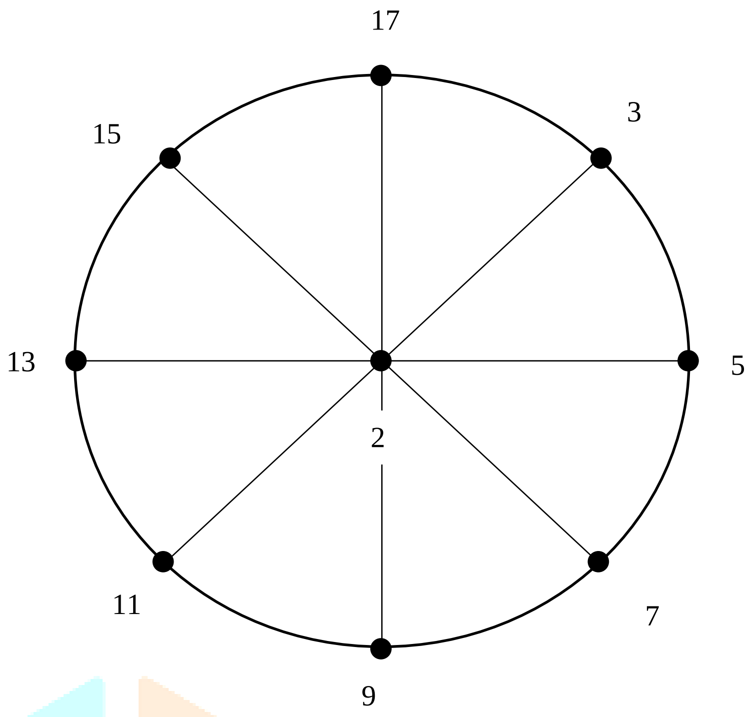
Define a one-to-one labeling h: V (G_n)→ Z without loss of generosity.

G_n is a planar connected graph

Let G_0 = 2 $n \leq 5$

H (G_n) = 2n+1

We can observe that h induces required 2-odd labeling of Gn.



A gear graph G_8 admits 2-odd labeling

Definition 3: Mobius ladder

The Mobius ladder M_n, for even number n, is formed from an n-cycle by adding edges, connecting opposite pairs of vertices in the cycle.

THEOREM 4:

Mobius ladder graph admits 2-odd labeling n≤6.

Proof:

Let M_n be the given Mobius graph on $n \leq 6$ vertices.

It is clear to see that there are 2n vertices.

$v_1, v_2 \ldots v_{2n}$ and the central vertex v_0.

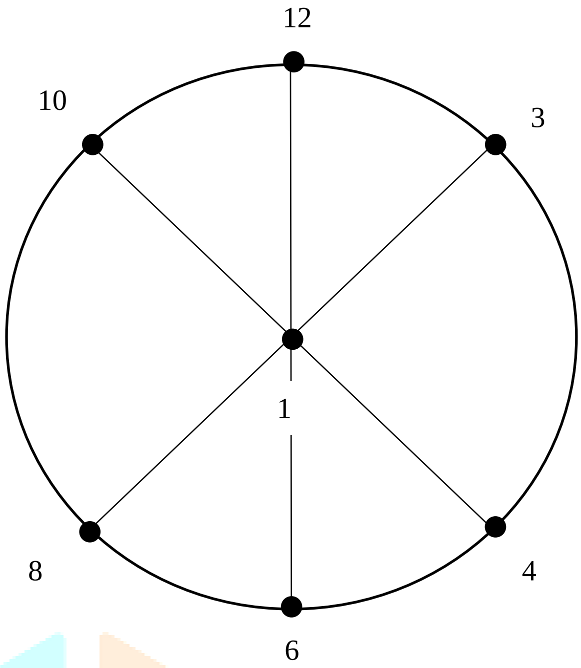
Now we define an injective function or one-to-one function f: V (M_n) →Z without loss of generosity.

$f(V_0)=1$, $f(V_1)=3$, $f(V_2)=4$.

$f(V_i)=f(V_{i-1})+2 \quad 3 \leq i \leq 2n$

f is the required 2-odd labeling of M_n.

$|f(V_i)-f(V_{i-1})|=2$ for $2 \leq i \leq 2n$



Mobius ladder M_6 admits 2-odd labeling

Definition 4: Friendship graph

A friendships graph F_n is a graph which consists of n triangle with a common vertex.

THEOREM 5:

A friendship graph F_n admits 2-odd labeling

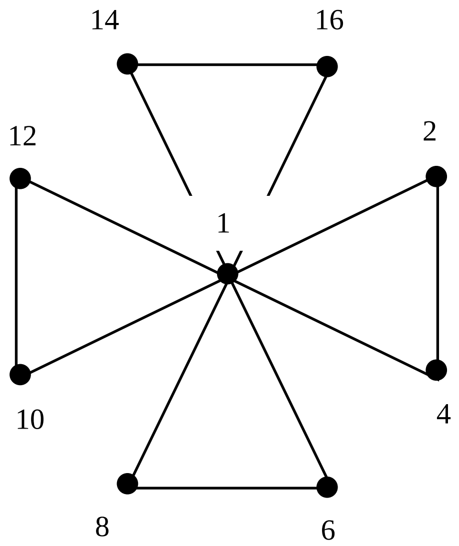
Proof:

Let V (F_n) = {V_1, V_2,…,V_{2n+1}} with V_0 as center vertex let f(V_0)=1 and f(V1)=2

We define: one –to –one labeling f: V (F_n) →Z

F (V_i) = V_{i-1} +2 for $2 \leq i \leq 2n + 1$

We can observe that f induces required 2-odd labeling of F_n.



A Friendship graph F_8 admits 2-odd labeling

CONCLUSION:

The 2-odd labeling of some classes of graph such as triple wheel graph, gear graph, Mobius ladder and friendship graph are investigated. By investigating of 2-odd labeling of some other classes of graph and finding the prime distance of 2-odd labeling of graphs are still open and for future work. One can also explore the exclusive application of 2-odd labeling in real life problems.

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