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MINIMUM NEIGHBORHOOD DOMINATION OF GRAPHS

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Abstract: In this paper,we define a new domination parameter called minimum neighbourhood domination. Also we define and study the minimum neighborhood domination number of some class of graphs

Keywords: Dominating set,minimumneighborhood dominating set,minimumneighborhood dominating number.

I. INTRODUCTION

Let G = (V , E) be a non-trivial simple graph with $|V| = p$, $|E| = q$. Let D be the subset of V is a dominating set if every vertex in $V - D$ is adjacent to atleast one element in D. The minimum cardinality of dominating set is called domination number and it is denoted by $\gamma(G)$.[5] There was need for minimum neighbourhood dominating set in the case that, management has to pass information to each and every student of the institution. For that, management uses the concept of dominating set and chooses a set of students as dominating set. But it left some students unaware of information because some students were dominated by many students and those dominat students lavishly thought other will pass the information. To reduce this problem we introduce a concept called minimum neighbourhood domination by imposing a condition on dominating set.

II. Main Results

Definition 2.1 Minimum neighbourhood dominating set

Let $G = (V, E)$ be a non-trivial simple graph. A subset $D \subseteq V(G)$ is a minimum neighbourhood dominating set if D is a dominating set and if for every $v_i \in D, |\cap_{i=1}^{n} N(v_i)| < \delta(G)$ holds.

Definition 2.2 Minimum neighbourhood dominating number

The minimum cardinality of minimum neighbourhood dominating set of a graph G is called as minimum neighbourhood dominating number and it is denoted by $\gamma_{mN}(G)$

Theorem 2.3 *For a path graph* P_n, $\gamma_{mN}(P_n) = \begin{cases} 2; & if \ \ n = 2, \\ \frac{n}{3}; & if \ \ n = 3k, k = 2,3\ldots, \\ \lfloor\frac{n}{3}\rfloor + 1; & if \ \ n \neq 3k, k = 2,3\ldots \end{cases}$

Proof. Let P_n be a path graph with vertex set $V = \{v_1, v_2, \ldots v_n\}$. Let $D \subseteq V(G)$ be a minimum neighbourhood dominating set of the path graph. To compute minimum neighbourhood dominating number, we consider the following cases:

Case 1: Let n=2 and the vertex set of P_2 be $\{v_1, v_2\}$. Here minimum neighbourhood dominating set is $D = \{v_1, v_2\}$. Therefore $\gamma_{mN}(P_2) = 2$.

Case 2: $n = 3k, k = 2,3\ldots,$

Let k=2 , n=6 and the vertex set of P_6 be $\{v_1, v_2, \ldots v_6\}$. Here every vertex not in the set $D = \{v_2, v_5\}$ is adjacent to at least one element of D and $|N(v_2) \cap N(v_5)| < \delta(P_6)$.So that $D = \{v_2, v_5\}$ is a minimum neighbourhood dominating set of P_6. Therefore $\gamma_{mN}(P_6) = 2$. As proceeding for any $n = 3k, k = 2,3\ldots,$ every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor - 1$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)})\ldots \cap N(v_{1+3i})| < \delta(P_{3k}), i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor - 1$. So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor - 1$ is a minimum neighbourhood dominating set of P_{3k}. Therefore $\gamma_{mN}(P_{n=3k}) = \frac{n}{3}$.

Case 3: $n \neq 3k, k = 2,3\ldots,$

Let n=3 and the vertex set of P_3 be $\{v_1, v_2, v_3\}$. Here every vertex not in the set $D = \{v_1, v_2\}$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_2)| < \delta(P_3)$. So that $D = \{v_1, v_2\}$ is a minimum neighbourhood dominating set of P_3. Therefore $\gamma_{mN}(P_3) = 2$. Similarly for any $n \neq 3k, k = 2,3\ldots,$ every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)})\ldots(v_{1+3i})| < \delta(P_{n\neq 3k}), i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor$.So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor$ is a minimum neighbourhood dominating set of $P_{n\neq 3k}$. Therefore $\gamma_{mN}(P_{n\neq 3k}) = \lfloor\frac{n}{3}\rfloor + 1$.

Theorem 2.4 *For a cycle graph* C_n, $\gamma_{mN}(C_n) = \begin{cases} \frac{n}{3}; & if \ \ n = 3k, k = 2,3\ldots, \\ \lfloor\frac{n}{3}\rfloor + 1; & if \ \ n \neq 3k, k = 2,3\ldots \end{cases}$

Proof. Let C_n be a cycle graph with vertex set $V = \{v_1, v_2, \ldots v_n\}$. Let $D \subseteq V(G)$ be a minimum neighbourhood dominating set of the cycle graph. To compute minimum neighbourhood dominating number, we consider the following cases:

Case 1: $n = 3k, k = 2,3\ldots,$

Let k=2,n=6 and the vertex set of C_6 be $\{v_1, v_2, \ldots v_6\}$. Here every vertex not in the set $D = \{v_1, v_4\}$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_4)| < \delta(C_6)$. So that $D = \{v_1, v_4\}$ is a minimum neighbourhood dominating set of C_6. Therefore $\gamma_{mN}(C_6) = 2$.

As proceeding for any $n = 3k, k = 2,3\ldots,,$ every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor - 1$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)})\ldots \cap N(v_{1+3i})| < \delta(C_{3k}), i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor - 1$. So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor - 1$ is a minimum neighbourhood dominating set of C_{3k}. Therefore $\gamma_{mN}(C_{n=3k}) = \frac{n}{3}$.

Case 2: $n \neq 3k, k = 2,3\ldots,$

Let n=3 and the vertex set of C_3 be $\{v_1, v_2, v_3\}$. Here every vertex not in the set $D = \{v_1, v_2\}$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_2)| < \delta(C_3)$.So that $D = \{v_1, v_2\}$ is a minimum neighbourhood dominating set of C_3. Therefore $\gamma_{mN}(C_3) = 2$. Similarly for any $n \neq 3k, k = 2,3\ldots,$ every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)})\ldots \cap N(v_{1+3i})| < \delta(C_{n\neq 3k}), i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor$.So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)}, \ldots v_{1+3i}\}, i = 0,1,2,3\ldots\lfloor\frac{n}{3}\rfloor$ is a minimum neighbourhood dominating set of $C_{n\neq 3k}$. Therefore $\gamma_{mN}(C_{n\neq 3k}) = \lfloor\frac{n}{3}\rfloor + 1$.

Theorem 2.5 *Let G be a graph, then $\gamma_{mN}(G) = 2$. When G is (i) Complete graph K_n, (ii)Complete bipartite graph $K_{m,n}$, (iii) Crown graph $H_{n,n}$, (iv)Wheel graph $W_{1,n}$, (v) t-fold wheel graph $W_{t,n}, t \geq 1$.*

Proof. Let K_n be a complete graph with vertex set $V = \{v_1, v_2, \ldots v_n\}$. Let $D \subseteq V(G)$ be a minimum neighbourhood dominating set of the complete graph.

Let n=2 and the vertex set of K_2 be $\{v_1, v_2\}$. Here $D = \{v_1, v_2\}$. Therefore $\gamma_{mN}(K_2) = 2$.

As proceeding for any n, every vertex not in the set $D = \{v_1, v_2,\}$ is adjacent to at least one element of D and $|N(v_1) \cap N(v_2)| < \delta(K_n) = n - 1$.So that $D = \{v_1, v_2\}$ is a minimum neighbourhood dominating set of K_n. Therefore $\gamma_{mN}(K_n) = 2$.

In similar way we obtain minimum neighbourhood dominating number of (ii)complete bipartite graph $K_{m,n}$, (iii) crown graph $H_{n,n}$, (iv)wheel graph $W_{1,n}$, (v) t-fold wheel graph $W_{t,n}, t \geq 1$

Theorem 2.6 *Let G be a graph, then $\gamma_{mN}(G) = 2$. When G is (i)Flower graph $Fl_{1,n,n}$,(ii)Friendship graph F_m, (iii)Barbell graph B_n, (iv) Fan graph $F_{1,n}$, (v)Double Fan graph $F_{2,n}$, (vi) Generalized Fan graph $F_{m,n}$, (vii)Windmill graph $Wd_{m,n}$, (viii)Shell graph $C(n, n-3)$, (ix)Shell Flower graph $[C(n, n-3) \cup K_2]^k$ (x)Jewel graph J_n.*

Proof. Let $Fl_{1,n,n}$ be a Flower graph with vertex set $V = \{u_1, v_1, v_2, \ldots v_n, w_1, w_2, \ldots w_n\}$. Let $D \subseteq V(G)$ be a minimum neighbourhood dominating set of the Flower graph.

Let n=3 and the vertex set of $Fl_{1,3,3}$ be $\{u_1, v_1, v_2, v_3, , w_1, w_2, w_3\}$. Here every vertex not in the set $D = \{u_1, w_1\}$ is adjacent to at least one element of D and $|N(u_1) \cap N(w_1)| < \delta(Fl_{1,3,3})$.So that $D = \{u_1, w_1\}$ is a minimum neighbourhood dominating set of $Fl_{1,3,3}$. Therefore $\gamma_{mN}(Fl_{1,3,3}) = 2$.

Let n=4 and the vertex set of $Fl_{1,4,4}$ be $\{u_1, v_1, v_2, v_3, v_4, w_1, w_2, \ldots w_4\}$. Here every vertex not in the set $D = \{u_1, w_1\}$ is adjacent to at least one element of D and $|N(u_1) \cap N(w_1)| < \delta(Fl_{1,4,4})$.So that $D = \{u_1, w_1\}$ is a minimum neighbourhood dominating set of $Fl_{1,4,4}$. Therefore $\gamma_{mN}(Fl_{1,4,4}) = 2$

As proceeding for any n, every vertex not in the set $D = \{u_1, w_1,\}$ is adjacent to at least one element of D and $|N(u_1) \cap N(w_1)| < \delta(Fl_{1,n,n})$.So that $D = \{u_1, w_1\}$ is a minimum neighbourhood dominating set of $Fl_{1,n,n}$. Therefore $\gamma_{mN}(Fl_{1,n,n}) = 2$.

In similar way we obtain minimum neighbourhood dominating number of (ii)Friendship graph F_m, (iii)Barbell graph B_n, (iv) Fan graph $F_{1,n}$, (v)Double Fan graph $F_{2,n}$, (vi) Generalized Fan graph $F_{m,n}$, (vii)Windmill graph $Wd_{m,n}$, (viii)Shell graph $C(n, n-3)$, (ix)Shell Flower graph $[C(n, n-3) \cup K_2]^k$ (x)Jewel graph J_n.

3 Conclusion

A new domination parameter called minimum neighbourhood domination was introduced. Minimum neighborhood dominating number was defined and minimum neighborhood dominating number of some class of graphs are found.

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