IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Generation of prime graphs using corona product of cycles and wheels with complete graphs

M. Thamothara Pandian[1], S. Senthil[2], and K. Rajendran[3][1]Research Scholar

Department of Mathematics

Vels Institute of Science Technology and Advanced StudiesChennai, India

[2]Assistant Professor

Department of Mathematics

Vels Institute of Science Technology and Advanced StudiesChennai, India

[3]Assistant Professor

Department of Mathematics

Vels Institute of Science Technology and Advanced StudiesChennai, India

Abstract

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots, |V|$ such that for each edge xy the labels assigned to x and y are relatively prime. In this paper, we prove that graphs$C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even.

Mathematics Subject Classification: 05C78;05C05

Keywords: **p**rime labeling; prime graphs; corona product of graphs;

1 Introduction

Graphs we considered here are finite, simple and undirected graphs. For the graph theoretic terminologies, we refer the book [7]. Rosa [5] introduced various graph labeling and after that many researchers introduced various graph labeling for to decompose graphs. One such graph labeling is prime labeling introduced by Entringer. A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots, |V|$ such that for each edge xy the labels assigned to x and y arerelatively prime. In the year 1980, Entringer conjectured that all trees have a prime labeling. Seoud et.al., [6] proved the necessary and sufficient conditions for a graph to be prime. They also gave a procedure to determine whether or not a graph is prime. Deretsky et al., [1] proved that cycles and disjoint union of cycles are prime. Lee et. al. [4] proved that complete graph does not have a prime labeling for $n \geq 4$ and wheel graphs W_n are prime if and only if n is even. For an exhaustive survey on Graceful Tree Conjecture, refer the excellent survey by Gallian [3]. In this paper, we prove that graphs

$C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even.

2 Prime labeling of $C_n \odot K_1$ and $W_n \odot K_1$

From the literature of prime labelings, we know that cycle graphs C_n are prime graphs. In this section, we prove that the corona product of cycles with the complete graph on one vertex allow prime labeling.

Theorem 1. *$C_n \odot K_1$ admit prime labeling for any $n \geq 3$.*

Proof. Consider a cycle C_n with vertices as $v_1, v_2, \ldots, v_n$ along with its prime labeling given by the function $f(v_i) = i$ for any $1 \leq i \leq n$. It is clear that, the function f gives the prime labeling for the cycle C_n. In the generation of graph $C_n \odot K_1$, let u_i be the corresponding vertex copy of the vertex $v_i \in V(C_n)$. By the definition of corona product of graphs, $C_n \odot K_1$ has $2n$ vertices and $2n$ edges. Assume that those $2n$ verticesare $v_1, v_2, \ldots v_n, u_1, u_2, \ldots, u_n$. Now, let us define the prime labeling g : $V(C_n \odot K_1) \to$

$\{1, 2, 3, \ldots, 2n\}$ for $C_n \odot K_1$ using the prime labeling of cycles C_n. Define $g(v_i) = 2f(v_i)-1$for $1 \leq i \leq n$ and $g(u_i) = 2f(v_i)$ for $1 \leq i \leq n$. It is clear that from the definition of g, labels of the vertices of $C_n \odot K_1$ are distinct and from the set $\{1, 2, 3, \ldots, 2n\}$. Since the labels of the vertices v_i are odd by the definition of the function g, their vertex labels are

relatively prime. Since the labels of the vertices u_i are even and their unique adjacent vertex v_i receive an odd label by the definition of the function g, which proves the prime labeling of the graph $C_n \odot K_1$ for any n ≥ 3. □

Theorem 2. *$W_n \odot K_1$ admit prime labeling for any even n.*

Proof. Consider a wheel W_n with unique central vertex as u and spokes vertices as$v_1, v_2, \ldots, v_n$ along with its prime labeling given by the function $f(v) = 1$ and $f(v_i) = i+1$for any $1 \leq i \leq n$. It is clear that, the function f gives the prime labeling for the wheel W_n when n is even. In the generation of graph $W_n \odot K_1$, let u_i be the corresponding vertex copy of the vertex $v_i \in V(W_n)$. Similarly, let u be the corresponding vertex copy of the vertex v for the corona product between W_n and K_1. By the definition of corona product of graphs, $W_n \odot K_1$ has $2n + 2$ vertices and $3n + 1$ edges. Assume that those$2n + 2$ vertices are $v, v_1, v_2, \ldots v_n, u, u_1, u_2, \ldots, u_n$. Now, let us define the prime labeling$g : V(W_n \odot K_1) \to \{1, 2, 3, \ldots, 2n+2\}$ for $W_n \odot K_1$ using the prime labeling of wheel W_n.Define $g(v_i) = f(v_i)$ for $1 \leq i \leq n$, $g(v) = f(v) = 1$, $g(u_i) = f(v_i)+n+1$ for $1 \leq i \leq n-1$, $g(u_n) = f(v_n) + 1$ and $g(u) = 2n + 1$. It is clear that from the definition of g, labels of the vertices of $W_n \odot K_1$ are distinct and from the set $\{1, 2, 3, \ldots, 2n + 2\}$. Since the labelsof the vertices v_i are same as the labels of the vertices of the corresponding wheel graph, their vertex labels are relatively prime. Further, wheel graphs are prime graph and the labeling of g are defined such that the labels of adjacent vertices in $W_n \odot K_1$ are relative prime, it is clear that the graph $W_n \odot K_1$ admits the prime labeling for any even n. □

3 Conclusion

In this paper, we prove that graphs $C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even. Main objective of the paper is to generate prime graphs using the corona product of graphs. In this direction, we raise a question of how to generate prime graphs using some other binary products apart from corona product of two graphs.

References

[1] T. Deretsky, S. M. Lee, and J. Mitchem, *On vertex prime labelings of graphs*, in Graph Theory, Combinatorics and Applications Vol. 1, J. Alavi, G. Chartrand, O. Oellerman, and A. Schwenk, eds., Proceedings 6th International Conference Theory and Applications of Graphs (Wiley, New York, 1991) 359-369.

[2] Farrugia R. and Harary F, *On the corona of two graphs*, Aequationes Math, 4, (1970),322-325.

[3] Gallian J.A., *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Com- binatorics, 18, (2015), #DS6.

[4] S. M. Lee, I. Wui and J. Yeh, *On the amalgamation of prime graphs*, Bull. Malaysian Math. Soc. (Second Series), 11 (1988) 59-67.

[5] Rosa A, *On certain valuations of the vertices of a graph*, Theory of graphs, (Interna- tional Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris, (1967), 349-355.

[6] M. A. Seoud, A. El Sonbaty, and A. E. A. Mahran, *On prime graphs*, Ars Com- bin.,104 (2012) 241-260.

[7] West D.B., *Introduction to Graph Theory*, Prentice Hall of India, 2nd Edition, 2001.