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Fermatean Quadripartitioned Neutrosophic Set

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Abstract: In this paper, we have introduced the concept of Fermatean Quadripartitioned Neutrosophic Set and its properties were discussed. Also the interior and closure of its topological spaces are also studied.

***Index Terms* – neutrosophic set,neutrosophic pythagorean set, quadripartitioned neutrosophic set,fermatean quadripartitioned neutrosophic set**

I. INTRODUCTION

Fuzzy sets were introduced by Zadeh in 1965 that permits the membership perform valued within the interval [0,1] and set theory it's an extension of classical pure mathematics. Fuzzy set helps to deal the thought of uncertainty, unclearness and impreciseness that isn't attainable within the cantorian set. As Associate in Nursing extension of Zadeh's fuzzy set theory intuitionistic fuzzy set(IFS) was introduced by Atanassov in 1986, that consists of degree of membership and degree of non membership and lies within the interval of [0,1]. IFS theory wide utilized in the areas of logic programming, decision-making issues, medical diagnosis , clustering analysis etc.

Florentine Smarandache introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of]−0 1+[. Since neutrosophic set deals the indeterminateness effectively it plays an very important role in several applications areas embrace info technology, decision web, electronic database systems, diagnosis, multicriteria higher cognitive process issues etc.,

To method the unfinished data or imperfect data to unclearness a brand new mathematical approach i.e., To deal the important world issues, Wang (2010) introduced the idea of single valued neutrosophic sets(SVNS) that is additionally referred to as an extension of intuitionistic fuzzy sets and it became a really new hot analysis topic currently.

Further, R. Radha and A. Stanis Arul Mary outlined a brand new hybrid model of Pentapartitioned Neutrosophic Pythagorean sets (PNPS) and Quadripartitioned neutrosophic pythagorean sets in 2021.

II. PRELIMINARIES

2.1 Definition [17]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where T_A , $I_A, F_A: U \rightarrow [0,1]$ *and* $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

Here, $T_A(x)$ and $F_A(x)$are dependent neutrosophic components and $I_A(x)$ is an independent component.

2.2 Definition [10]

Let R be a universe. A Neutrosophic pythagorean set A with T and F as dependent Neutrosophic Pythagorean components and U as independent component for A on R is an object of the form

$$A = \{< x, T_A, U_A, F_A >: r \in R\}$$

Where $(T_A)^2 + (F_A)^2 \leq 1$ and

$$(T_A)^2 + (U_A)^2 + (F_A)^2 \leq 2$$

Here, $T_A(x)$ is the truth membership, $U_A(x)$ is indeterminancy membership and $F_A(x)$ is the false membership .
Remark: When T and F as dependent Neutrosophic Components, then T + F ≤ 1.

2.3 Definition[10]

The complement of a Neutrosophic Pythagorean set $A = \{< x, T_A, U_A, F_A >: r \in R\}$ with dependent Neutrosophic Pythagorean components is
$A^C = \{< x, F_A, 1 - U_A, T_A >: r \in R\}$.

2.4 Definition[10]

Let $A = \{< x, T_A, U_A, F_A >: r \in R\}$ and $B = \{< x, T_B, U_B, F_B >: r \in R\}$ are two Neutrosophic Pythagorean sets with dependent Neutrosophic Pythagorean components on the universe R. Then the union and intersection of two sets can be defined by
$A \cup B$ = {max (T_A, T_B), min (U_A, U_B),min (F_A, F_B)},
$A \cap B$ = {min (T_A, T_B), max (U_A, U_B),max (F_A, F_B)}.

2.5 Example[10]

Let R = {a, b} and A ={ (a, 0.4,0.6,0.3),(b,0.5,0.6,0.2)}.
Then τ = {0,1, A} is a topology on R. Then A is a Neutrosophic Pythagorean set.

2.6 Example[10]

Let R = {a, b} and A ={ (a, 0.7,0.6,0.7),(b,0.7,0.6,0.7)}.
Then τ = {0,1, A} is a topology on R. Since $T_A + F_A > 1$,then A is a Neutrosophic Pythagorean set with dependent neutrosophic pythagorean components but not dependent neutrosophic components.
But $(T_A)^2 + (U_A)^2 + (F_A)^2 \leq 2$. Hence A is a Pythagorean Neutrosophic set.

2.7 Example[10]

Let R = {a, b} and A ={ (a, 0.8,0.5,0.7),(b,0.7,0.5,0.7)}.
Then τ = {0,1, A} is a topology on R. Since $T_A + F_A > 1$, $(T_A)^2 + (F_A)^2 > 1$,then A is not a Neutrosophic Pythagorean set with dependent neutrosophic pythagorean components and dependent neutrosophic components.
But $(T_A)^2 + (U_A)^2 + (F_A)^2 \leq 2$. Hence A is a Pythagorean Neutrosophic set.

2.8 Definition[1]

Let X be a universe. A Fermatean neutrosophic set (FN)A on X is an object of the form
$A = \{< x, T_A, C_A, U_A, F_A >: x \in X\}$

$$(T_A)^3 + (I_A)^3 + (F_A)^3 \leq 2$$

Here, $T_A(x)$ is the truth membership,
$I_A(x)$ is indeterminancy membership,
$F_A(x)$ is the false membership

III. FERMATEAN QUADRIPARTITIONED NEUTROSOPHIC SET

3.1 Definition

Let X be a universe. A Fermatean quadripartitioned neutrosophic set (FQN)A on X is an object of the form
$A = \{< x, T_A, C_A, U_A, F_A >: x \in X\}$

$$(T_A)^3 + (C_A)^3 + (U_A)^3 + (F_A)^3 \leq 2$$

Here, $T_A(x)$ is the truth membership,
$C_A(x)$ is contradiction membership,
$U_A(x)$ is ignorance membership,
$F_A(x)$ is the false membership

3.2Example

Let R= {a,b,c},and A= {(a,0.3,0.6,0.4,0.7),(b,0.5,0.2,0.5,0.3),(c,0.2,0.4,0.5,0.4)}. Then A is a Fermatean quadripartitioned neutrosophic set on R.

3.3 Definition

A Fermatean quadripartitioned neutrosophic set A is contained in another Fermatean quadripartitioned neutrosophic set B (i.e) A$\subseteq B$ if $T_A \leq T_B, C_A \leq C_B, I_A \geq I_B, U_A \geq U_B$ and $F_A \geq F_B$

3.4 Example

Let R= {a,b,c},and A= {(a,0.3,0.6,0.4,0.7),(b,0.5,0.2,0.5,0.3),(c,0.2,0.4,0.5,0.4)} and B ={(a,0.5, 0.6,0.2,0.5),(b,0.6,0.4,0.3,0.2),(c, 0.5,0.6,0.3,0.2)} are Fermatean quadripartitioned neutrosophic sets on R. Then A ⊆ B.

3.5 Definition

The complement of a Fermatean quadripartitioned neutrosophic A on X denoted by $(A)^c$ and is defined as

A^C(x)= $\{< x, F_A, U_A , C_A, T_A >: x \in X\}$

3.6 Example

Let X = {a,b,c},and A= {(a,0.3,0.6,0.4,0.7),(b,0.5,0.2,0.5,0.3),(c,0.2,0.4,0.5,0.4)} is a Fermatean quadripartitioned neutrosophic sets on R. Then A^C = {(a,0.7,0.4,0.6,0.3),(b,0.3,0.5,0.4,0.6),(c,0.2,0.3,0.4,0.2)}

3.7 Definition

Let X be a non-empty set, A = $< x, T_A, C_A, U_A, F_A >$ and B = $< x, T_B, C_B, U_B, F_B >$ are two Fermatean quadripartitioned neutrosophic sets. Then

A∪B = < x, $max(T_A, T_B), max(C_A, C_B), min(U_A, U_B), min(F_A, F_B)$>

A∩B = $< x, min(T_A, T_B), min(C_A, C_B), max(U_A, U_B), max(F_A, F_B)$>

3.8 Example

Let R= {a,b,c},and A= {(a,0.2,0.5,0.3,0.6),(b,0.4,0.1,0.4,0.2),(c,0.1,0.3,0.4,0.3)} and B ={(a,0.4,0.5,0.1,0.4),(b,0.5,0.3,0.2,0.1),(c, 0.4,0.5,0.2,0.1)} are Fermatean quadripartitioned neutrosophic sets on R. Then

A∪B = {(a,1.5,0.5,0.1,0.1),(b,0.5,0.3,0.2,0.1),(c, ,0.4,0.5,0.2,0.1)}

3.9 Definition

A Fermatean quadripartitioned neutrosophic set A over the universe X is said to be empty Fermatean quadripartitioned neutrosophic set ∅ with respect to the parameter A if $T_A = 0, C_A = 0, U_A = 1, F_A = 1, \forall x \in X, \forall e \in A$. It is denoted by ∅ or 0

3.10 Definition

A Fermatean quadripartitioned neutrosophic set A over the universe X is said to be Δ universe Fermatean quadripartitioned neutrosophic set with respect to the parameter A if

$T_A = 1, C_A = 1, U_A = 0, F_A=0$

It is denoted by Δ or 1

3.11 Definition

Let A and B be two Fermatean quadripartitioned neutrosophic sets on X then A\B may be defined as

A\B = $< x, min(T_A, F_B), min(C_A, U_B), max(U_A, C_B), max(F_A, T_B) >$

3.12 Theorem

Let K(A),L(B) and M© are three Fermatean quadripartitioned neutrosophic sets over the universe X. Then the following propertied holds true.

- Commutative la

a) A ∪ B = B ∪ A

b) A ∩ B = B ∩ A

- Associative law

c) (A ∪ B) ∪ C = A ∪ (B ∪ C)

d) (A ∩ B) ∩ C = A ∩ (B ∩ C)

- Distributive law

e) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

f) A ∩ (B ∪C) = (A ∩ B) ∪ (A ∩ C)

- Absorption law

g) A ∪ (A ∩ C) = A

f) A ∩ (A ∪ C) = A

- Involution law

i) $(A^C)^C = A$

- Law of contradiction

j) $A \cap A^C = \emptyset$

- De Morgan's law

k) $(A \cup B)^C = A^C \cap B^C$

l) $(A \cap B)^C = A^C \cup B^C$

3.12 Theorem

Let K and L are two Fermatean quadripartitioned neutrosophic sets over the universe X. Then the following are true.
(i) K ⊆ L iff K ∩ L = K
(ii) K ⊆ L iff K ∪ L = L

3.13 Theorem

Let K be Fermatean quadripartitioned neutrosophic set over the universe X. Then the following are true.

(i) $(\emptyset)^c = X$
(ii) $(X)^c = \emptyset$
(iii) K ∪ ∅ = K
(iv) (ii) K ∪ X = X
(v) (i) K ∩ ∅ = ∅
(vi) (ii) K ∩ X = K, A

Proof: It is obvious

3.14 Definition

A Fermatean quadripartitioned neutrosophic topology on a non-empty set X is a τ of Fermatean quadripartitioned neutrosophic sets satisfying the following axioms.

i) $0_M, 1_M \in \tau$

ii) The union of the elements of any sub collection of τ is in τ

iii) The intersection of the elements of any finite sub collection τ is in τ

The pair (X, τ) is called an Fermatean quadripartitioned neutrosophic Topological Space over X.

Note :

1. Every member of τ is called a FQN open set in X.
2. The set $A_M$is called a FQN closed set in X if $A_M \in \tau^c$, where $\tau^c = \{A_M{}^c : A_M \in \tau\}$.

3.15 Example :

Let M = {b₁, b₂} and Let A_M, B_M, C_M be Fermatean quadripartitioned neutrosophic sets where
$A_M = \{< b_1, 0.5,0.1,0.7,0.2 >< b_2, 0.7,0.5,0.2,0.1 > < b_3, 0.6,0.5,0.4,0.3 >\}$
$B_M = \{< b_1, 0.6,0.7,0.1,0.2 >< b_2, 0.2,0.3,0.4,0.7 > < b_3, 0.5,0.6,0.1,0.3 >\}$
$C_M = \{< b_1, 0.6,0.7,0.1,0.2 >< b_2, 0.7,0.5,0.2,0.1 > < b_3, 0.6,0.6,0.1,0.3 >\}$

$\tau = \{A_M, B_M, C_M, 0_M, 1_M\}$ is an Fermatean quadripartitioned neutrosophic topology on M.

3.16 Preposition

Let (M, τ_1) and (M, τ_2) be two Fermatean quadripartitioned neutrosophic topological space on M, Then $\tau_1 \cap \tau_2$ is an Fermatean quadripartitioned neutrosophic topology on M where

$\tau_1 \cap \tau_2 = \{A_M : A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$

Proof :

Obviously $0_M, 1_M \in \tau$.

Let $A_M, B_M \in \tau_1 \cap \tau_2$

Then $A_M, B_M \in \tau_1$ and $A_M, B_M \in \tau_2$

We know that τ_1 and τ_2 are two Fermatean quadripartitioned neutrosophic topological space M.

Then $A_M \cap B_M \in \tau_1$ and $A_M \cap B_M \in \tau_2$

Hence $A_M \cap B_M \in \tau_1 \cap \tau_2$.

Let τ_1 and τ_2 are two Fermatean quadripartitioned neutrosophic topological spaces on X.

Denote $\tau_1 \vee \tau_2 = \{A_M \cup B_M : A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$

$\tau_1 \wedge \tau_2 = \{A_M \cap B_M : A_M \in \tau_1 \text{ and } A_M \in \tau_2\}$

3.17 Example

Let A_M *and* B_M be two Fermatean quadripartitioned neutrosophic topological space on X.

Define $\tau_1 = \{0_M, 1_M, A_M\}$

$\tau_2 = \{0_M, 1_M, B_M\}$

Then $\tau_1 \cap \tau_2 = \{0_M, 1_M\}$ is a Fermatean quadripartitioned neutrosophic topological space on M.

But $\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M\}$,

$\tau_1 \vee \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cup B_M\}$ *and*

$\tau_1 \wedge \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cap B_M\}$ are not Fermatean quadripartitioned neutrosophic topological space on X.

3.18 Definition

Let (M, τ) be a Fermatean quadripartitioned neutrosophic topological space on M and let A_M belongs to Fermatean quadripartitioned neutrosophic set on M. Then the interior of A_M is denoted as FQNInt (A_M). It is defined by FQNInt (A_M) = $\cup \{B_M \in \tau : B_M \subseteq A_m\}$

3.19 Definition

Let (M, τ) be a Fermatean quadripartitioned neutrosophic topological space on M and let A_M belongs to Fermatean quadripartitioned neutrosophic set M. Then the closure of A_M is denoted as FQNCl (A_M). It is defined by FQNCl (A_M) = $\cap \{B_M \in \tau^C : A_M \subseteq B_m$

3.20 Theorem

Let (M, τ) be a Fermatean quadripartitioned neutrosophic topological space over M. Then the following properties are hold.

i) 0_M *and* 1_M are Fermatean quadripartitioned neutrosophic closed sets over M

ii) The intersection of any number of Fermatean quadripartitioned neutrosophic closed set is a Fermatean quadripartitioned neutrosophic closed set over M.

iii) The union of any two Fermatean quadripartitioned neutrosophic closed set is an Fermatean quadripartitioned neutrosophic set over M.

Proof

It is obviously true.

3.21 Theorem

Let (M, τ) be a be a Fermatean quadripartitioned neutrosophic topological space over M and Let $A_M \in$ Fermatean quadripartitioned neutrosophic topological space. Then the following properties hold.

(i) FQNInt $(A_M) \subseteq A_M$

(ii) $A_M \subseteq B_M$ implies FQNInt $(A_M) \subseteq$ FQNInt (B_M) .

(iii) FQNInt $(A_M) \in \tau$.

(iv) A_M is a FQN open set implies FQNInt $(A_M) = A_M$.

(v) FQNInt (FQNInt (A_M)) = FQNInt (A_M)

(vi) FQNInt $(0_M) = 0_M$, FQNInt $(1_M) = 1_M$.

3.22 Theorem

Let (M, τ) be a be a Fermatean quadripartitioned neutrosophic topological space over M and Let A_M is in the Fermatean quadripartitioned neutrosophic topological space. Then the following properties hold.

(i) $A_M \subseteq$ FQNCl(A_M)

(ii) $A_M \subseteq B_M$ implies FQNCl $(A_M) \subseteq$ FQNCl (B_M).

(iii) FQNCl $(A_M)^c \in \tau$.

(iv) A_M is a FQN closed set implies FQNCl $(A_M) = A_M$.

(v) FQNCl (FQNCl (A_M)) = FQNCl (A_M)

(vi) FQNCl $(0_M) = 0_M$, FQNCl $(1_M) = 1_M$.

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