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Paranormed Structure Of Certain Spaces

[1]Tawheeda Rasool and [1]Abdul Hamid Ganie

[1]Govt. College of Education, IASE M.A Road Srinagar, 190001, Kashmir-India.

[2]Department of Applied Sciences & Humanities, SSM College of Engineering Parihaspora, Pattan-193121

Abstract: The spaces of Riesz type have been discussed by many authors. The basic notion of this manuscript is to bringout the new aspects of Riesz spaces by employing the concept of modulus function. Also, some inclusion relation will be established.

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1. Introduction

By Λ represent the set of all sequences with complex terms and note that any subspace of Λ is said to be as a *sequence space.* By symbols N and C, we represent the set of non-negative integers and the set of complex numbers, respectively. As in [7, 15, 16], we denote by ℓ_∞, c and c_0, respectively, the space of all bounded sequences, the space of convergent sequences and the sequences converging to zero. Also, by ℓ_1, $\ell(p)$, cs and bs we denote the spaces of all absolutely, p-absolutely convergent, convergent and bounded series, respectively, as can be seen in [8, 18, 44].

As in [9]-[27], for an infinite matrix $T = (t_{i,j})$ and $\nu = (\nu_k) \in \Lambda$, the T-transform of ν is $T\nu = \{(T\nu)_i\}$ provided it exists $\forall\ i \in \mathbb{N}$, where $(T\nu)_i = \sum_{j=0}^{\infty} t_{i,j}\nu_j$.

For an infinite matrix $T = (t_{i,j})$, the set G_T, where

$$G_T = \{\nu = (\nu_j) \in \Lambda : T\nu \in G\}, \qquad (1)$$

is known as the matrix domain of T in G [30, 31, 40].

A infinite matrix $G = (\varrho_{nk})$ is said to be regular if and only if the following conditions hold:

$$(i)\ \lim_{n\to\infty}\sum_{k=0}^{\infty}\varrho_{nk}=1,$$

$$(ii)\ \lim_{n\to\infty}\varrho_{nk}=0,\ (k=0,1,2,...),$$

$$(iii)\ \sum_{k=0}^{\infty}|\varrho_{nk}|<M,\ (M>0,\ n=0,1,2,...)$$

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Let (q_k) be a sequence of positive numbers and let us write, $Q_n = \sum q_k$

$k=0$

for $n \in N$. Then the matrix $R^q = (r_{nk}^q)$ of the Riesz mean (R,q_n) is given by

$$r_{nk}^q = \begin{cases} \frac{q_k}{Q_n}, & \text{if } 0 \le k \le n, \\ 0 & \text{if } k > n \end{cases}$$

The Riesz mean (R,q_n) is regular if and only if $Q_n \to \infty$ as $n \to \infty$ as can be seen in [35, 42].

Quite recently, in [38], the author has introduced the following:

$$r^q(u,p)=\left\{\zeta=(\zeta_k)\in\Lambda:\sum_k\left|\frac{1}{Q_k}\sum_{j=0}^{k}u_jq_j\zeta_j\right|^{p_k}<\infty\right\}$$

where, $0 < p_k \le H < \infty$.

In [29], the author introduced the following difference sequence spaces $W(\triangle)$:

$W(\triangle) = \{\zeta = (\zeta_k) \in \Lambda : (\triangle\zeta_k) \in W\}$,

where, $W \in \{\ell_\infty, c, c_0\}$ and $\triangle\zeta_k = \zeta_k - \zeta_{k+1}$.

In [2], the author has studied the sequence space as

$$bv_p=\left\{\zeta=(\zeta_k)\in\Lambda:\sum_k|\Delta x_k|^p<\infty\right\},$$

where $1 \le p < \infty$. With the notation of (1), the space bv_p can be redefined as

$bv_p = (l_p)_\triangle, 1 \le p < \infty$

where, $\triangle$ denotes the matrix $\triangle = (\triangle_{nk})$ defined as

$$\triangle_{nk} = \begin{cases} (-1)^{n-k}, & \text{if } n-1 \le k \le n, \\ 0, & \text{if } k < n-1 \text{ or } k > n. \end{cases}$$

In [32], the author introduced the concept of modulus function. We call a function $F : [0,\infty) \to [0,\infty)$ to be modulus function if

(i) $F(\zeta) = 0$ if and only if $\zeta = 0$,

(ii) $F(\zeta + \eta) \leq \{(\zeta) + \{(\eta)\ \forall\ \zeta \geq 0, \eta \geq 0$

(iii) F is increasing, *and*

(iv) F is continuous from the right at 0.

One can easily see that if F_1 and F_2 are modulus functions then so is $F_1 + F_2$; and the function F^j ($j \in \mathbf{N}$), the composition of a modulus function F with itself j times is also modulus function.

Recently, in [36] the new space was introduced by using notion of modulus function as follows:

$$L(\mathcal{F}) = \left\{ \zeta = (\zeta_r) : \sum_r |\mathcal{F}(\zeta_r|)| < \infty \right\}$$

The approach of constructing a new sequence space by means of matrix domain of a particular limitation method has been studied by several authors. $(\ell_\infty)_{N_q}$ and c_{N_q}(see, [43]), $(\ell_p)_{C_1} = X_p$ and $(\ell_\infty)_{C_1} = X_\infty$(see, [34]), $(\ell_\infty)_{R^t} = r^t_\infty$, $(c)_{R^t} = r^t_c$ and $(c_o)_{R^t} = r^t_0$(see, [28]), $(\ell_p)_{R^t} = r^t_p$ (see, [1]), $(c_0)_{A^r} = a^r_0$ and $c_{A^r} = a^r_c$(see, [3]), $[c_0(u,p)]_{A^r} = a^r_0(u,p)$ and $[c(u,p)]_{A^r} = a^r_c(u,p)$ (see, [4], $r^q(u,p) = \{l(p)\}_{R_{uq}}$ (see, [38])and etc.

2. The sequence space $r_F{}^q(\triangle^p{}_s)$ of non-absolute type

In this section, for $s \geq 0$, we define the Riesz sequence space $r^q_{\mathcal{F}}(\triangle^p_s)$, and prove that the space $r^q_{\mathcal{F}}(\triangle^p_s)$ is a complete paranormed linear space and show it is linearly isomorphic to the space $l(p)$.

Let Λ be a real or complex linear space, define the function $\tau : \Lambda \to \mathrm{R}$ with R as set of real numbers. Then, the paranormed space is a pair $(\Lambda;\tau)$ and τ is a paranorm for Λ, if the following axioms are satisfied for all $\zeta, \eta \in \Lambda$ and for all scalars β:

(i) $\tau(\theta) = 0$,

(ii) $\tau(-\zeta) = \tau(\zeta)$,

(iii) $\tau(\zeta + \eta) \leq \tau(\zeta) + \tau(\eta)$, *and*

(iv) scalar multiplication is continuous, that is,

$|\beta_n - \beta| \to 0$ and $h(\zeta_n - \zeta) \to 0$ imply $\tau(\beta_n\zeta_n - \beta\zeta) \to 0$ for all $\beta's$ in R and $\zeta's$ in Λ, where θ is a zero vector in the linear space Λ. Assume here and after that (p_k) be a bounded sequence of strictly positive real numbers with $\sup_k p_k = H$ and $M = max\{1,H\}$. Then, the linear space

$\ell_\infty(p)$ was defined by Maddox [30] as follows :

$$\ell_\infty(p) = \{\zeta = (\zeta_k) : \sup_k |\zeta_k|^{p_k} < \infty\}$$

which is complete space paranormed by

$$\tau_1(\zeta) = \left[\sup_k |\zeta_k|^{p_k}\right]^{1/M}.$$

We shall assume throughout that $p_k^{-1}+\{p_k'\}^{-1}$ provided 1 < $\inf p_k \leq H < \infty$, and we denote the collection of all finite subsets of N by F, where N={0, 1, 2, ... }.

Following Altay et al. [1]-[2], Ba¸sarir et al. [5], Choudhary et al. [6], Ganie et al. [9, 18], Ruckle [36], Sengo¨nu¨l [37], Mursaleen [33], Sheikh et al. [38]-[41], we define the difference sequence space $r_{\mathcal{F}}^{q}(\triangle_s^p)$ as follows:

$$r_{\mathcal{F}}^{q}(\triangle_s^p)=\left\{\zeta=(\zeta_k)\in\Lambda:\sum_k k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k}q_j\triangle\zeta_j\right)\right|^{p_k}<\infty\right\}$$

where, $0 < p_k \leq H < \infty$ and $s \geq 0$.

By (1), it can be redefined as

$$r_{\mathcal{F}}^{q}(\triangle_s^p)=\{l(p)\}_{R_{\mathcal{F}}^{q}(\triangle_g)}.$$

Define the sequence $\xi = (\xi_k)$, which will be used, by the $R_F^q \triangle_g$ transform of a sequence $\zeta = (\zeta_k)$, i.e.,

$$\xi_k=f\frac{k^{-s}}{Q_k}\sum_{j=0}^{k}q_j\triangle\zeta_j \quad (2)$$

Now, we begin with the following theorem which is essential in the text.

Theorem 2.1. *$r_F^q(\triangle_s^p)$ is a complete linear metric space paranormed by $h_\triangle$, defined as*

$$h_\triangle(\zeta)=\left[\sum_k k^{-s}\left|\mathcal{F}\left[\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q_k}\zeta_k\right]\right|^{p_k}\right]^{\frac{1}{M}}$$

with $0 < p_k \leq H < \infty$.

Proof: The linearity of $r_{\mathcal{F}}^{q}(\triangle_s^p)$ with respect to the co-ordinatewise addition and scalar multiplication follows from from the inequalities which are satisfied for $\zeta,\xi\in r_{\mathcal{F}}^{q}(\triangle_s^p)$(see [14], p.30])

$$\left[\sum_k k^{-s}\left|\mathcal{F}\left[\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})(\zeta_j+\eta_j)+\frac{q_k}{Q_k}(\zeta_k+\eta_k)\right]\right|^{p_k}\right]^{\frac{1}{M}}$$
$$\leq\left[\sum_k k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q_k}\zeta_k\right)\right|^{p_k}\right]^{\frac{1}{M}}$$
$$+\left[\sum_k k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\eta_j+\frac{q_k}{Q_k}\eta_k\right)\right|^{p_k}\right]^{\frac{1}{M}} \quad (3)$$

and for any $\alpha \in \mathbf{R}$ (see, [13])

$$|\alpha|^{p_k}\leq max(1,|\alpha|^M). \quad (4)$$

It is clear that, $h_\triangle(\theta)$=0 and $h_\triangle(\zeta) = h_\triangle(-\zeta)$ for all $\zeta \in r_F^q(\triangle_s^p)$. Again the inequality (3) and (4), yield the subadditivity of $h_\triangle$ and

$h_\triangle(\alpha\zeta) \leq max(1,|\alpha|)h_\triangle(\zeta)$.

Let $\{\zeta^n\}$ be any sequence of points of the space $r^q_{\mathcal{F}}(\triangle^p_s)$ such that $h_\triangle(\zeta^n - \zeta) \to 0$ and (α_n) is a sequence of scalars such that $\alpha_n \to \alpha$.
Then, since the inequality,

$h_\triangle(x^n) \leq h_\triangle(x) + h_\triangle(x^n - x)$

holds by subadditivity of $h_\triangle$, $\{h_\triangle(\zeta^n)\}$ is bounded and we thus have

$$h_\triangle(\alpha_n\zeta^n - \alpha\zeta) = \left[\sum_k k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k}(q_j - q_{j+1})(\alpha_n\zeta^n_j - \alpha\zeta_j)\right)\right|^{p_k}\right]^{\frac{1}{M}}$$

$\leq |\alpha_n - \alpha|^{\frac{1}{M}} h_\triangle(\zeta^n) + |\alpha|^{\frac{1}{M}} h_\triangle(\zeta^n - \zeta)$

which tends to zero as $n \to \infty$, which shows that the scalar multiplication is continuous. Hence, $h_\triangle$ is paranorm on the space $r^q_{\mathcal{F}}(\triangle^p_u)$.

It remains to prove the completeness of the space $r^q_{\mathcal{F}}(\triangle^p_s)$. Let $\{\zeta^i\}$ be any Cauchy sequence in the space $r^q_{\mathcal{F}}(\triangle^p_s)$, where $\zeta^i = \{\zeta^i_0, \zeta^i_1, \dots\}$. Then,
for a given $\epsilon > 0$, there exists a positive integer $n_0(\epsilon)$ such that

$$h_\triangle(\zeta^i - \zeta^j) < \epsilon \tag{5}$$

for all $i,j \geq n_0(\epsilon)$. Using definition of $h_\triangle$ and for each fixed $k \in \mathbf{N}$, we have

$$\left|(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k - (R^q_s\triangle\zeta^j)_k\right| \leq \left[\sum_k\left|(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k - (R^q_{\mathcal{F}}\triangle_s\zeta^j)_k\right|^{p_k}\right]^{\frac{1}{M}} < \epsilon$$

for $i,j \geq n_0(\epsilon)$, which leads us to the fact that $\{(R_F{}^q \triangle_s\zeta^0)_k, (R_F{}^q \triangle_s\zeta^1)_k, \dots\}$ is a Cauchy sequence of real numbers for every fixed $k \in \mathbf{N}$. Since R is complete, it converges, say, $(R_F{}^q \triangle_s\zeta^i)_k \to ((R_F{}^q \triangle_s\zeta)_k$ as $i \to \infty$. Using these infinitely many limits $(R^q_{\mathcal{F}}\triangle_s\zeta)_0, (R^q_s\triangle\zeta)_1, \cdots$, we define the sequence $\{(R_F{}^q \triangle_s\zeta)_0, (R_F{}^q \triangle_s\zeta)_1, \dots\}$. From (5) for each $m \in \mathbf{N}$ and $i,j \geq n_0(\epsilon)$,

$$\sum_{k=0}^{m}\left|(R^q_{\mathcal{F}}\triangle_s\zeta^i)_k - (R^q_{\mathcal{F}}\triangle_s\zeta^j)_k\right|^{p_k} \leq h_\triangle(\zeta^i - \zeta^j)^M < \epsilon^M. \tag{6}$$

Take any $i,j \geq n_0(\epsilon)$. First, let $j \to \infty$ in (6) and then $m \to \infty$, we obtain

$h_\triangle(\zeta^i - \zeta) \leq \epsilon$.

Finally, taking $\epsilon = 1$ in (6) and by letting $i \geq n_0(1)$, we have by Minkowski's inequality for each $m \in N$ that

$$\left[\sum_{k=0}^{m}|(R_{\mathcal{F}}^{q}\triangle_{g}\zeta)_k|^{p_k}\right]^{\frac{1}{M}} \leq h_{\triangle}(\zeta^i-\zeta)+h_{\triangle}(\zeta^i)\leq 1+h_{\triangle}(\zeta^i)$$

which implies that $\zeta \in r_{\mathcal{F}}^{q}(\triangle_s^p)$. Since $h_\triangle(\zeta - \zeta^i) \leq \epsilon$ for all $i \geq n_0(\epsilon)$, it follows that $\zeta^i \to \zeta$ as $i \to \infty$, hence we have shown that $r_{\mathcal{F}}^{q}(\triangle_s^p)$ is complete, hence the proof of the theorem follows.

Note that one can easily see the absolute property does not hold on the spaces $r_{\mathcal{F}}^{q}(\triangle_s^p)$, that is, $h_\triangle(\zeta) \neq h_\triangle(|\zeta|)$ for atleast one sequence in the space $r_{\mathcal{F}}^{q}(\triangle_s^p)$ and this says that $r_{\mathcal{F}}^{q}(\triangle_s^p)$ is a sequence space of nonabsolute type.

3. Inclusion relations

In this section, we investigate some of its inclusions properties .

Theorem 3.1. *For $s \geq 0$, if p_k and t_k are bounded sequences of positive real numbers with $0 < p_k \leq t_k < \infty$ for each $k \in \mathbf{N}$, then for any modulus function $\mathcal{F}$, $r_{\mathcal{F}}^{q}(\triangle_s^p) \subseteq r_{\mathcal{F}}^{q}(\triangle_s^t)$.*

Proof: For $\zeta \in r_{\mathcal{F}}^{q}(\triangle_s^p)$ it is obvious that

$$\sum_{k} k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q_k}\zeta_k\right)\right|^{p_k}<\infty.$$

Consequently, for sufficiently large values of k say $k \geq k_0$ for some fixed $k_0 \in \mathbf{N}$.

$$k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q_k}\zeta_k\right)\right|<\infty.$$

But F being increasing and $p_k \leq t_k$, we have

$$\begin{aligned}&\sum_{k\geq k_0} k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q_k}\zeta_k\right)\right|^{t_k}\\ &\leq \sum_{k\geq k_0} k^{-s}\left|\mathcal{F}\left(\frac{1}{Q_k}\sum_{j=0}^{k-1}(q_j-q_{j+1})\zeta_j+\frac{q_k}{Q_k}\zeta_k\right)\right|^{p_k}<\infty.\end{aligned}$$

From this, it is clear that $\zeta \in r_{\mathcal{F}}^{q}(\triangle_s^t)$ and the result follows.⋄

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