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Accelerated Sequential Procedures For The Bounded Risk Point Estimation

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Abstract: we develop the classes of 'accelerated' sequential procedures of an absolute continuous population is developed for estimating the parameters for bounded risk point estimation problem under the set up of distributional relationship developed by Chaturvedi,A .,Pandey S,Gupta.M,(1991[2])

Index terms: Bounded risk, Asymptotic distribution

In order to construct fixed-range confidence interval for a normal mean, assuming the variance to be unknown ,Hall [3](1983) proposed an accelerated' sequential Procedure which combines the rates of two-stage and purely sequential procedures and also is more flexible in nature because the number of sampling stages can be reduced only by introducing finite number of observations .Several other experimenters have also developed and studied the same for other distributions also.

In the present Chapter, we develop the classes of 'accelerated' sequential procedures to construct fixed size confidence region for the parameter $\underline{\theta}$ for the bounded risk point estimation. The set up [2]of the problem is:

$\mathbf{X}_1, \ldots, \mathbf{X}_n$ be a random sample of size $n(\geqslant t+1)$,from a t variate continuous population ,with parameter θ of order $t \times 1$ of interest and Ψ a scalar unknown parameter , let $(\theta', \Psi)' \in R^t \times R^+$. The estimators of θ and Ψ are $\hat{\theta}_n = \hat{\theta}(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ and $\hat{\Psi}_n = \hat{\Psi}(\mathbf{X}_1, \ldots, \mathbf{X}_n)$. The following hypotheticals are made

(A,): A known positive definite matrix Q, of order t by t, a number $\delta \in (0,1]$ and a positive integer $r \geq 1$ exist ,s.t. $n[\psi^{-1}(\theta_n - \theta)' Q(\theta_n - \theta)]^{\delta} \sim \chi^2_{(r)}$

(A_2) : $\hat{\theta}_n$ and $\hat{\Psi}_n$ are independent for all values of n.

(A_3) : For integers $s(\geqslant 1)$,then for all n greater than or equal to s+1,

$r(n-s)\hat{\Psi}_n/\Psi = \sum_{j=1}^{n-s} Z_j^{(r)}$

where $Z_j^{(r)}$'s are iid rv's with $Z_j^{(r)} \sim \chi^2_{(r)}$.[2]

(A_4): $\hat{\Psi}_n$ is a consistent estimator of ψ.

Let $[y]^+$ denote the positive integral part of y. The class C_A^* of 'accelerated' sequential procedure is as follows: let $\eta \in (0,1)$ and $L \in (0,\infty)$ be specified.

Let the initial sample size taken to be $m \geq \max\{s+1, t+1\}$,where

$$m = o(A^{\delta/\alpha}) \text{ as } A \to \infty \text{ and } \lim_{A\to\infty}(m/D) < 1$$

Starting sequentially with the stopping time N_1 ,where

$N_1 = \text{lnf} \cdot [n_1 \geq m: n_1 \geq \eta\{K^*(\alpha,\delta,r)/N\}^{\delta/\alpha}\hat{\psi}_{n_1}]$……………….(1)

on the basis of N_1 observations, find estimate $\hat{\psi}_{N_1}$ and jump by taking N_2 observations. S.t.

$N_2 = [\{K^*(\alpha,\delta,r)/W\}^{\delta/\alpha}\hat{\psi}_{N_1} + L]^+ + 1$……………..(2)

$N = max \cdot (N_1, N_2)$ and estimate $\underline{\theta}$ by $\hat{\theta}_N$. Let us prove some lemmas which are used in proving the main theorem.

Lemma 1:$\lim_{A\to\infty}(N_1) = \lim_{A\to\infty}(N_2) = \infty$

From the definition of N_1 and N_2, respectively, Lemma follows.

Lemma 2: $\lim_{A\to\infty}(N/n_0) = 1$, a.s.

Proof : From (1) the inequality

$\eta\{k^*(\alpha,\delta,r)/N\}^{\delta/\alpha}\hat{\psi}_{N_1} \leq N_1 \leq \{k^*(\alpha,\delta,r)/\omega\}^{\delta/\alpha}\hat{\psi}_{N_1-1} + 1$…………..(3)

or,

$$(\hat{\psi}'_{N_1}/\psi) \leq \left(\frac{N_1}{n_0}\right) \leq (\hat{\psi}_{N_1-1}/\psi) + (\eta_{n_0})^{-1} \text{……………………(4)}$$

Applying Kolmogorov's strong law of large numbers and (A_3) ,gives the result that $\hat{\psi}_{N_1} \to \psi$ as $n \to \infty$.Using this result and Lemma 1 and equation (4),the result follows.

Lemma 3:As A→ ∞,$(\eta n_0)^{-\frac{1}{2}}(N_1 - \eta n_0) \to N(0, 2q^{-1})$

Proof: Using (A_3), the rule (3)can be rewritten as

$N_1 = \text{lnf} \cdot \left[n_1 \geq m: \sum_{j=1}^{n_1-s} q^{-1}z_j^{(q)} \leq (n_1 - s)(n_1/\eta n_0)\right]$…………(5)

Defining a new stopping variable N_1^* as

$N_1^* = \text{lnf} \cdot [n_1 \geq m - s: \sum_{j=1}^{n_l} q^{-1}z_j^{(q)} \leq n_1^2(1 + sn_1^{-1})/\eta n_0]$

With the help of Lemma 1 of Swanepoel and Vanwyk [4](1982), it can be proved that the stopping variables N_1 and N_1^* have same probability distribution

On Comparing (6) with equation (1.1) of Woodroofe [5](1977), , $\alpha = 2, A = 1, \mu = 1$ and $\tau^2 = 2q^{-1}$. Using result of Bhattacharya and Malik (1973)[1] the lemma follows .

$$(\eta n_0)^{-\frac{1}{2}}(N_1 - \eta n_0) \xrightarrow{L} N(O, \beta^2\tau^2\mu^{-2})$$

Lemma 4 : For all $m \geq s + 2q^{-1}$, as $A \to \infty$

$$\mathbb{E}(N_1) = \eta n_0 + \nu - (s + 2q^{-1}) + o(1)$$

where ν is specified.

Proof: In the notations of Woodroofe[5] (1977), $a = q/2$, $\lambda = \eta n_0 \cdot L(n_1) = 1 + sn_1^{-1}$ and $L_0 = s$. The lemma now follows from Theorem 2.4 of Woodroofe [5] (1977) that, as $A \to \infty$,

$$\begin{aligned} E(N_1) &= \lambda + \beta\mu^{-1}\nu - \beta L_0 - \frac{1}{2}\alpha\beta^2\tau^2\mu^{-2} + o(1) \\ &= \eta n_0 + \nu - (s + 2q^{-1}) + o(1) \end{aligned}$$

Lemma5: For all $m > s + 2q^{-1}$, as $A \to \infty$

$E(N) = n_0 + L - \eta^{-1}(s + 2q^{-1}) + o(1)$............(7)

$$\text{var}\,\langle N\rangle = 2\eta^{-1}n_0 + o\left(\lambda^{\delta/\alpha}\right).\ \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

and, for $\gamma(> 0)$.

$$E(|N - E(N)|^\gamma) = 0\left(\lambda^{\frac{\gamma\delta}{2\alpha}}\right) \ldots\ldots\ldots\ldots\ldots\ldots.(9)$$

Proof: Rewriting the stopping rule (5) as

$$N_1 = \text{lnf}\left[n_1 \geq m: q(n_1 - s)\left(\hat{\psi}_{n_1}/\psi\right) < qn_1(n_1 - s)/\{w/K^*(\alpha,\delta,r)\}^{\delta/\alpha}\right].$$

Let us consider the difference

$$D_A = \{qN_1(N_1 - \delta)/\eta\psi\}\{w/K^*(\alpha,\delta,r)\}^{\delta/\alpha} - q(N_1 - s)\left(\hat{\psi}_{N_1}/\psi\right).......(10)$$

The mean of the asymptotic (as $A \to \infty$) distribution 0 D_A is ν. Let us define

$$D_A^* = \eta\{q(N_1 - s)\}^{-1}\left\{\frac{K^*(a,\delta,r)}{w}\right\}^{\frac{\delta}{\alpha}} \cdot D_A$$

$$\text{since } z_j(q) \text{ 's are positive, } q(N_1 - s)\left(\frac{\hat{\psi}_{N_1}}{\psi}\right) \geq q\left(N_1 - s - 1\right)\left(\frac{\hat{\psi}_{N_{1-1}}}{\psi}\right) \quad (11)$$

using this resuit and basic inequality (3), we obtain from (.10) and 11) that

$$D_A^* = \eta\{q(N_1 - s)\}^{-1}\left\{\frac{K^*(a,\delta,r)}{w}\right\}^{\frac{\delta}{\alpha}}\left[\frac{q(N_1(N_1 - s)}{\eta\psi'} \cdot \left\{\frac{w}{K^*(\alpha,\delta,r)}\right\}^{\delta/\alpha} - q(N_1 - s)^{\hat{\psi}}N_1\right]$$

$$\leq \eta\{q(N_1 - s)\}^{-1}\left\{\frac{K^*(a,\delta,r)}{w}\right\}^{\frac{\delta}{\alpha}}\psi \cdot \left[\frac{qN_1(N_1 - s)}{\eta\psi}\left\{\frac{w}{K^*(\alpha,\delta,r)}\right\}^{\delta/\alpha} - q(N_1 - s - 1)\,\hat{\psi}_{N_1-1}/\psi\right]$$

$$\leq \eta\{q(N_1 - s)\}^{-1}\left\{\frac{K^*(\alpha,\delta,r)}{w}\right\}^{\delta/\alpha} \psi\left[\frac{q_1(N,-s)}{\eta\psi}\left\{\frac{K^*(\alpha,\delta,r)}{w}\right\}^{\delta/\alpha} - \left\{\frac{q(N_1 - s - 1)}{\eta\psi}\right\}\left\{\frac{K^*(\alpha,\delta,r)}{w}\right\}^{\delta/\alpha}(N_1 - 1)\right]$$

$$\leq \eta\{q(N_1 - s)\}^{-1}\left\{\frac{k^*(a,\delta,r)}{W}\right\}^{\delta/\alpha} \psi\left[\frac{qN_1(N,-s)}{\eta\psi}\cdot\left\{\frac{w}{K^*(a,\delta,r)}\right\}^{\delta/\alpha}(N_1 - 1)\right]$$

.

$$\leq N_1 - (N_1 - s - 1)$$
$$= s + 1.$$

Moreover, again using (.3), it follows from (.10) and (.11) that.

$$D_A^* \geq \eta \quad \{q(N_1 - s)\}^{-1}\left\{\frac{K^*(\alpha,\delta,r)}{w}\right\}^{\delta/\alpha} \psi\left[\frac{qN_1(N_1 - s)}{\eta\psi} \cdot \left\{\frac{W}{K^*(\alpha,\delta,r)}\right\}^{\delta/\alpha} - \frac{q(N_1 - s)}{\psi}\eta^{-1}\cdot\left\{\frac{w}{K^*(\alpha,\delta,r)}\right\}^{\delta/\alpha} N_1\right]$$

$$= \quad 0$$

Thus: $0 \leq D_A^* \leq s + 1$ and from dominated convergence, as A tends to infinity $E(D_A^*)$=v. Using this result and Lemma 4, gives,

for all m> $s + 2q^{-1}$

$$\nu \quad = E(D_A^*)$$
$$= E\left[N_1 - \eta\{K^*(\alpha,\delta,r)/w\}^{\delta/\alpha}\hat{\psi}N_1\right]^{\delta/\alpha}$$

or

$$E\left[\{K^*(a_0\delta,r)/\}^{\delta/\alpha}\hat{\psi}_{N_1}\right] \quad = \eta^{-1}\{E(N_1) - \nu\}$$
$$= n_0 - \eta^{-1}(s + 2q^{-1}) + 0(1).$$

from the definition of N ,it follows that

$$E(N) = n_0 + \mathrm{L} - \eta^{-1}(s + 2q^{-1}) \quad + 0(1)$$

and (.7) holds

From the definition of N, $\mathrm{Var}(N) = \eta^{-2}\mathrm{Var}(N_1)$

Let $h(N_1) = (\eta_0)^{-1/2}(N_1 - \eta_n)$. It fol1ows from Theorem 2. [5] of woodroofe (1977) that $\mathrm{h}^2(\mathrm{N}_1)$ 1s unifomby integrable for al $m > s + 2q^{-1}$. Hence. using Lemma 3, we get for all $m > s + 2q^{-1}$, as $A \to \infty$ $\mathrm{Var}\cdot(N) = \eta^{-2}[2\eta n_0\{1 + o(1)\}]$

$$= 2\eta^{-1}n_0 + o(\lambda^{\delta/\alpha}).$$

And (8) follows. The proof of (9) follows from Hall [3 . The following theorem gives the main result

.

Theorem 1: For all m> $\max\{t, s+2q^{-1}\}$ *and* sufficiently large A,

say $A \geq A_0, E\left[L\left(\underline{\theta}, \hat{\theta}_N\right)\right] \leq W$,if

$$L \geq \eta^{-1}\left(s+2q^{-1}+\frac{\alpha}{\delta}+1\right).$$

Proof: The risk associated with the sampling scheme $(1)-(2)$ is

$$E\left[L\left(\underline{\theta}, \underline{\hat{\theta}}_{\mathrm{N}}\right)\right] = WE\left\{(n_O/\mathrm{N})^{\alpha/\delta}\right\}$$

Using Taylor's expansion, we obtain

$$E\left[L\left(\underline{\theta}, \hat{\theta}_N\right)\right] = W{n_0}^{\alpha/\delta}\,[{n_0}^{-\alpha/}\ \ \delta - \frac{\alpha}{\delta} {n_0}^{-(\frac{\alpha}{\delta}+1)}$$

$$\text{-}\ E(N-n_0) + \frac{\alpha}{2\delta}\left(\frac{\alpha}{\delta}+1\right) n_0^{-\left(\frac{\alpha}{\delta}+2\right)} E(N-n_0)^2] + \xi_A.$$

- where the remainder term $\xi_A = O\left(A^{-3\delta/\alpha}\, E(N-E(N))^3\right)$.

Thus, applying Lemma 5. we obtain for all $m > s+2q^{-1}$,

$$E\left[L\left(\underline{\theta}, \hat{\theta}_N\right)\right] = w\Big[1 - \frac{\alpha}{\delta n_0}\left(L-\eta^{-1}(s+2q^{-1}\right) + o(1)$$
$$+\frac{1}{2n_0^2}\cdot\frac{\alpha}{\delta}\left(\frac{\alpha}{\delta}+1\right)\left\{(2\eta^{-1}n_0 + o\left(A^{\delta/\alpha}\right)\right) + (L-\eta^{-1}(s+2q^{-1}) + o(1))^2\Big\} + o\left(A^{-3\delta/2\alpha}\right)$$

$$= w\left[1 - \frac{\alpha}{\delta n_0}\{L - \eta^{-1\{}(s+2q^{-1}) + \left(\frac{\alpha}{\delta}+1\right)\}\}\right] + o\left(A^{-\delta/\alpha}\right) + o\left(\lambda^{-3\delta/2\alpha}\right)$$

$$= w\left[1 - \frac{\alpha}{\delta n_0}\left\{L - \eta^{-1}\left(s+2q^{-1}+\frac{\alpha}{\delta}+1\right)\right\}\right] + o\left(A^{-\delta/\alpha}\right) + o\left(A^{-2\delta/\alpha}\right) + o\left(A^{-3\delta/2\alpha}\right)$$

And the theorem follows.

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