IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A Novel Exponent Power Rayleigh (NovEPR) distribution Properties and its Applications

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Abstract:

In this paper, we introduce a new two parameter a Novel Exponent Power-Rayleigh (NovEPR) distribution. A new method has been proposed to introduce an extra parameter to a family of distributions for more flexibility .The proposed family may be named as a Novel Exponent Power-Rayleigh (NovEPR) distribution. The new class is called, a novel exponent power-Y (NovEP-Y) family of distributions. By implementing the NovEP-Y approach, a new model, namely, a novel exponent power-Rayleigh (NovEP- Rayleigh) distribution is introduced. To show the applicability of the NovEP- Rayleigh model, two data sets from the sports and health sciences are considered. The first data set represents the time-to-even data collected from different football matches during the period 1964–2018. Whereas, the second data set is taken from the health sector, representing the survival times of the COVID-19 infected patients. Based on some well-known statistical tests, it is observed that the NovEP- Rayleigh model is a very competitive distribution for modeling the data sets in the sports and health sectors.

Some distributional properties of the NovEP-Y family such as identifiability which include Cumulative Generating Function (CGF), Probability density function (Pdf), Distribution Function (DF), Quantile Function(QF), Moment Generating Function (MGF), Cumulative Generating, order statistics, quantile function, and moments are obtained. The recommended distribution reveals increasing, decreasing and bath-shaped probability Density, Distribution and Hazard rate function and Survival function.

Keywords: NovEPR, Quantile Function(QF), Moment Generating Function (MGF), Cumulative Generating, order statistics, quantile function, and moments.

1. Introduction

In statistical literature a good quantity of work has been dedicated to Rayleigh distribution. Several authors and the references cited therein have carried out extensive studies as relate to the estimation, prediction and several other inferences with respect to Rayleigh distribution. The reliability function of Rayleigh distribution decreases at a large amount higher rate than the reliability function of exponential distribution. To pick up the characteristics of these traditional distributions, researchers have been raising various extensions and modified forms of these distributions. However, in the recent literature, researchers have shown a deep interest in proposing new families of distributions. The literature is packed with such methods that are quite prosperous and still growing quickly.

This study presents a new two-parameter probability distribution called modified Rayleigh distribution. The one parameter modified Rayleigh distribution is used as a base line to construct the new mode Simulation is constructed to declare theoretical properties, to show the flexibility of the new model. For more brief information Ali et al. (2022) studied the A novel Frechet-type probability distribution: its properties and applications, Salahuddin et al. (2021) proposed On the properties of the new generalized Pareto distribution and its applications, Eloranta et al. (2021) proposed the), Cancer survival statistics for patients and healthcare professionals-a tutorial of real-world data analysis, . Laba Handique *et al* (2020) studied the New Extended Burr-III Distribution: Its Properties and Applications. Mi Zichun *et al* (2020) studied A New Extended X-family of distribution: Properties and Applications, Vijaya lakshmi and Anjaneyulu (2018) studied The Odd Generalized Exponential Type-I Generalized Half Logistic Distribution: Properties and Application. Vijaya Lakshmi and Anjaneyulu (2019) studied Quadratic Rank Transmuted Half Logistic Lomax Distribution: Properties and Application. Vijaya Lakshmi and Anjaneyulu (2019) studied Half Logistic Pareto Distribution: Properties and Application.

Major inspiration behind the proposed family is to find an expansion of the Rayleigh distribution with one additional parameters to bring in more flexibility with respect to skewness, kurtosis, tail weight and length.

This encompasses number known distributions as special and connected cases, also to ensure that it provides better alternative in the data modeling not only to its sub models including the Rayleigh distribution, but to other recent extensions.

The chapter is organized as follows. The new a Novel Exponent Power-Rayleigh (NovEPR) distribution and its probability density function, survival function and hazard function is developed in section 1.1,1.2, 1.3 and 1.4. A comprehensive account of statistical properties of the new distribution is provided in Section 2.1, 2.2, 2.3, 2.4, 2.5 such as IP (identifiability property), QF (quantile function) and Random number generation, r^{th} moment, Moment Generating Function(MGF) and Cumulative Generating Function(CGF) for a Novel Exponent Power-Rayleigh (NovEPR) distribution. In section 2.6 derive the order statistics for a Novel Exponent Power-Rayleigh (NovEPR) distribution.

1.1 Novel Exponent Power-Rayleigh (NovEPR) distribution function

We further contribute to this area by proposing another new approach, namely, a novel exponent power-Y (NovEP-X) family. A random variable say X is said to follow a NovEP-Y family, if its DF (distribution function) K(x;σ^2,α) is given by

$$K(x;\sigma^2,\alpha) = 1-\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha} \quad \ldots (1.1)$$

Where$\alpha > 0$, x $\in [0,\infty)$, W(x; σ^2,α) and α> 0 is baseline distribution function.

1.2 Novel Exponent Power-Rayleigh (NovEPR) Probbility density function

A random variable say X is said to follow a NovEP-Y family, if its PDF (Probability distribution function) k(x;σ^2,α) is given by

$$k(x;\sigma^2,\alpha) = \frac{\lambda(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha-1} \quad \ldots(1.2)$$

Where x $\in [0,\infty)$, W(x; σ^2,α) and α > 0 is baseline distribution function.

1.3 Novel Exponent Power-Rayleigh (NovEPR) Survival Function

A random variable say X is said to follow a NovEP-Y family, if its SF (Survival Function) S(x;σ^2,α) is given by

$$S(x;\sigma^2,\alpha) = \left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right) \quad \ldots (1.3)$$

Where x $\in [0,\infty)$, W(x; σ^2,α) and α > 0 is baseline distribution function.

1.4 Novel Exponent Power-Rayleigh (NovEPR) Hazard Function

A random variable say X is said to follow a NovEP-Y family, if its HF (Hazard Function) H(x;σ^2,α) is given by

$$H(x;\sigma^2,\alpha) = \frac{\alpha(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{-1} \quad \ldots (1.4)$$

Where x $\in [0,\infty)$, W(x; σ^2, α) and $\alpha > 0$ is baseline distribution function.

2. Statistical Properties

2.1 The identifiability property Novel Exponent Power-Rayleigh (NovEPR) distribution

Let α_1 has the DF K(x;σ^2, α_1) and α_2 has the DF K(x;$\sigma^2,\ \alpha_2$) The parameter α is called identifiable, if $\alpha_1 = \alpha_2$

Proof:

Let assume

K(x;σ^2, α_1) = K(x;$\sigma^2,\ \alpha_2$) … (2.1)

Using (1.1) in (2.1), we obtained

$$1-\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha_1} = 1-\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha_2}$$

$$\alpha_1 \log\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right) = \alpha_2 \log\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)$$

Finally, we observe that

$\alpha_1 = \alpha_2$

2.2 Quantile Function (QF) and Random number generation Novel Exponent Power- Rayleigh (NovEPR) distribution

Quantile Function (QF)

The QF of the NovEP-R distributions is its inverse DF and derived as

A random variable X ~ NovEP-R (σ^2, α) has Quantile function and is in the form The p^{th} quantile x_p of NovEP-R distribution is the root of the equation

y = Q(u) = $K^{-1}(u) = W^{-1}(z)$,

where p = $e^{-x^2/2\sigma^2}$ is the solution of

y = Q(p) = 1- $\left(1-\frac{1-e^p}{e^{e^p}}\right)^{\alpha}$ … (2.2.1)

Random number generation

Let U~ U(0,1), then equation (2.2) can be used to simulate a random sample of size n from the NovEP-R distribution as follows

$x_i = 1- \left(1-\frac{1-e^{u_i}}{e^{e^{u_i}}}\right)^{\alpha}$ … (2.2.2)

2.3 The r^th moment of NovEP-R distribution

Theorem 1: The r^th moment about the origin of X ~ NovEP-R (σ^2, α) is given by

$$\mu_r^1 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\lambda\sigma^{r+1}}{l!}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\frac{\gamma(\frac{r+1}{2},((-(i+1)l+k+j)x)}{2((i+1)l-k-j)^{\frac{r+1}{2}}(-1)^{\frac{r-1}{2}}}$$

Proof: We have

$$\mu_r^1 = E(x^r)$$

$$= \int_{-\infty}^{\infty} x^r k(x;\sigma^2,\alpha)dx$$

$$= \int_{-\infty}^{\infty} x^r \frac{\lambda(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha-1} dx \quad \text{… (2.3.1)}$$

$$= \lambda\int_{-\infty}^{\infty} x^r \frac{(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\sum_{i=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\left(\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^i dx$$

… (2.3.2)

Based on Binomial expansion Eq. (2.3.1) becomes to (2.3.2)

$$=\lambda\int_{-\infty}^{\infty} x^r \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}e^{-x^2/2\sigma^2 l(i+1)}dx$$

$$=\lambda\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\int_0^{\infty} x^r e^{(x^2/\sigma^2)l(i+1)}e^{-(x^2/\sigma^2)(j+k)}dx$$

… (2.3.3)

Based on Maclaurin series Eq. (2.3.2) becomes to (2.3.3)

Let consider,

$$\int_{-\infty}^{\infty} x^r e^{(x^2/\sigma^2)l(i+1)}e^{-(x^2/\sigma^2)(j+k)}dx \quad \text{… (2.3.4)}$$

Let $x^2/\sigma^2 = z \Rightarrow x = \sigma\sqrt{z}$, $dx = \sigma\frac{1}{2\sqrt{z}}dz$ … (2.3.5)

Substitute (2.3.5) in (2.3.4) we get the following equation

$$=\frac{\sigma^{r+1}}{2}\int_0^{\infty} z^{r+1/2}e^{zl(i+1)}e^{-z(j+k)}dz$$

$$=\frac{\sigma^{r+1}\gamma(\frac{r+1}{2},((-(i+1)l+k+j)x)}{2((i+1)l-k-j)^{\frac{r+1}{2}}(-1)^{\frac{r-1}{2}}} \qquad \text{… (2.3.6)}$$

Substitute eq.(2.3.6) in (2.3.3) we get

$\mu_r^1 =$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\lambda\sigma^{r+1}}{l!}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\frac{\gamma(\frac{r+1}{2},((-(i+1)l+k+j)x)}{2((i+1)l-k-j)^{\frac{r+1}{2}}(-1)^{\frac{r-1}{2}}}$$

… (2.3.7)

Note that it assumed that $(i+1)l-k-j \neq 0$, –r- 4 ≠ 0 and –r- 2 ≠ 0

Hence proved.

2.4 Moment Generating Function (MGF)

The follows theorem gives the moment generating function of NovEP-R disribution

Theorem 2: The moment generating function of X ~ NovEP-R (σ^2, α) is given by

$$M_X(t) = M_X^t = \mathrm{E}(e^{tx})$$

$$= \int_{-\infty}^{\infty} e^{tx}\mathrm{k}(\mathrm{x};\sigma^2,\alpha)dx$$

$$= \int_0^{\infty} e^{tx}\frac{\lambda(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha-1}dx$$

… (2.4.1)

$$=\lambda\int_{-\infty}^{\infty} e^{tx}\frac{(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\sum_{i=0}^{\infty}(-1)^i\binom{\alpha-1}{i}\left(\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^i dx$$

…(2.4.2)

Based on Binomial expansion Eq. (2.4.1) becomes to (2.4.2)

$$\lambda\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{\sigma^{m+1}t^m}{m!}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\int_0^{\infty} x^m e^{(x^2/\sigma^2)l(i+1)}e^{-(x^2/\sigma^2)(j+k)}dx$$

… (2.4.3)

Based on Maclaurin series Eq. (2.4.2) becomes to (2.4.3)

Let ${x^2}/{\sigma^2} = z => x = \sigma\sqrt{z}$,dx = $\sigma \frac{1}{2\sqrt{z}}$ dz … (2.4.4)

Substitute (2.4.4) in (2.4.3) we get the following equation

$$= \lambda \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \frac{\sigma^{m+1}t^m}{m!}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\int_0^{\infty} z^{\frac{m-1}{2}} e^{zl(i+1)} e^{-z(j+k)} dz$$

$$= \lambda \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \frac{\sigma^{m+1}t^m}{m!}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\frac{\gamma(\frac{m+1}{2},((-(i+1)l+k+j)x)}{2((i+1)l-k-j)^{\frac{r+1}{2}}(-1)^{\frac{m-1}{2}}}$$

… (2.4.5)

2.5 Cumulative Generating Function (CGF)

A random variable X ~ NovEP-R (σ^2, α) has function and is in the form

$K_X(t) = \log(M_x(t))$ =

$$\log\left(\lambda \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \frac{\sigma^{m+1}t^m}{m!}(-1)^{i+j+k}\binom{\alpha-1}{i}\binom{i+1}{j}\binom{i+2}{k}\frac{\gamma(\frac{m+1}{2},((-(i+1)l+k+j)x)}{2((i+1)l-k-j)^{\frac{r+1}{2}}(-1)^{\frac{m-1}{2}}}\right)$$

… (2.4.6)

CONCLUSION

1. The NovEP-R Distribution failure rate function can have the following forms depending on its shape parameters: (i) decreasing
   (ii) upside down bathtub and
   (iii) reversed J-shaped shaped.
   Therefore, it can be used quite electively in analyzing lifetime data.
2. The NovEP-R new distribution gives Incomplete moments.

OBSERVATION FOR THE SIMULATION

1. When sample size increases variance decreases. So we conclude that fitted new NovEP-R distribution is good.
2. It is observe that, Mean and Median are approximately equal in NovEP-R distribution.

3. Application

Real life data set I

Data analysis in sport: Time-to-event data

This subsection illustrates the NovEP-Rayleigh distribution by taking the time-to-event data from different seventy-eight football matches. These matches were played during the period 1964– 2018. The observations of this data set represent the time waiting till the first goal is scored. The summary measures (SMs) of the time-to-event data are: minimum =3.20, 1st quartile =5.60, median = 7.32, mean =6.58, 3rd quartile =9.00, maximum =13.00, variance = 8.52 and range = 9.80. In link to the time-to-event data of football matches. Furthermore, the box plot is also presented in Fig. 3.1. The plot in Fig. 3.2, show that the data set, QQ-Plot.

Fig. 3.1. Box Plot for football matches

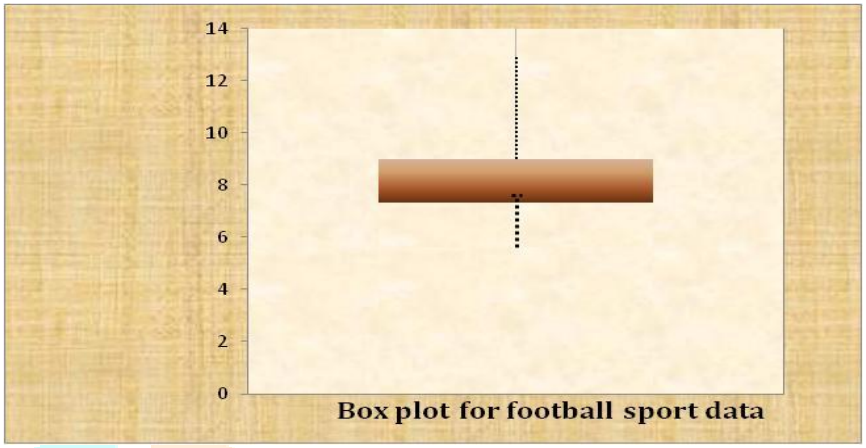
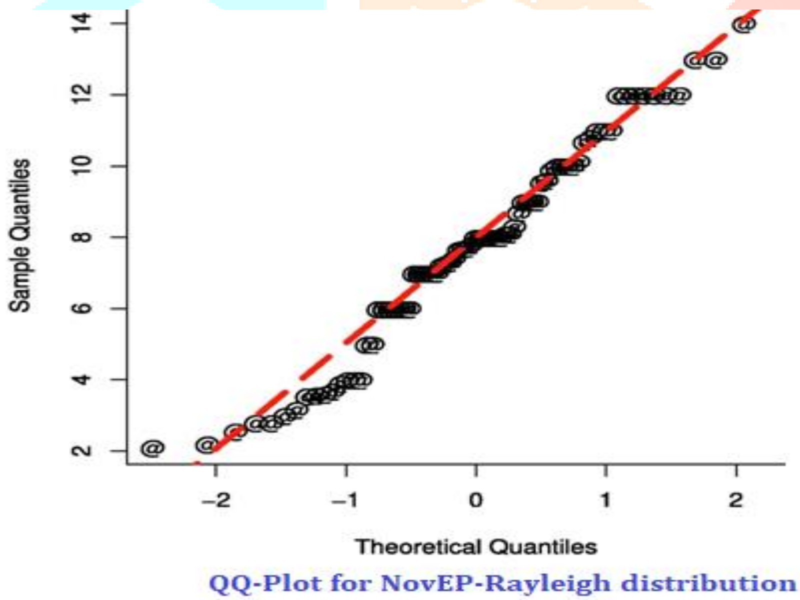


Fig. 3.2. QQ-Plot for NovEP-Rayleigh distribution for foot ball data



Real data set 2:

This subsection offers a second illustration of the NovEP-Rayleigh distribution by taking a data set from the health sector. The data set is taken from https://www.worldometers.info/coronavirus, and represents the survival times of the patients affected by the COVID-19 pandemic. The summary measures (SMs) of the survival times data are: minimum = 0.510, 1st quartile = 3.125, median =3.981, mean =4.237, 3rd quartile =5.623, maximum =7.632, variance =2.983 and range = 6.122.

Fig. 3.3. Box Plot for COVID-19 data

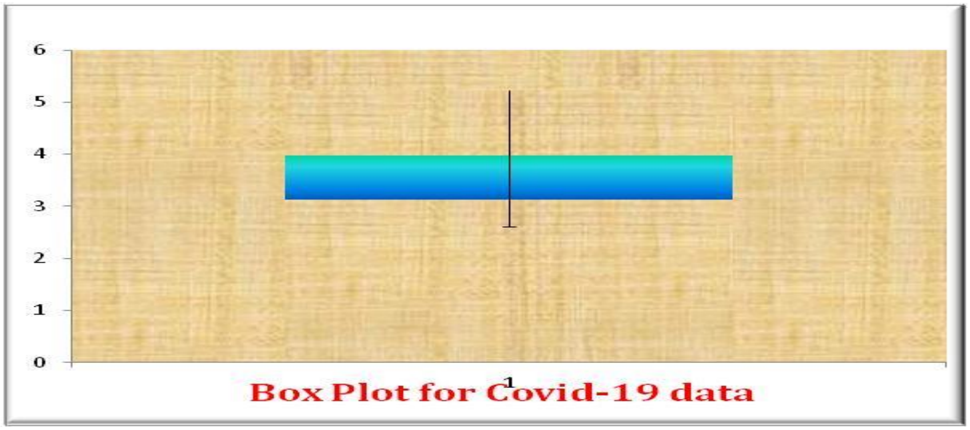
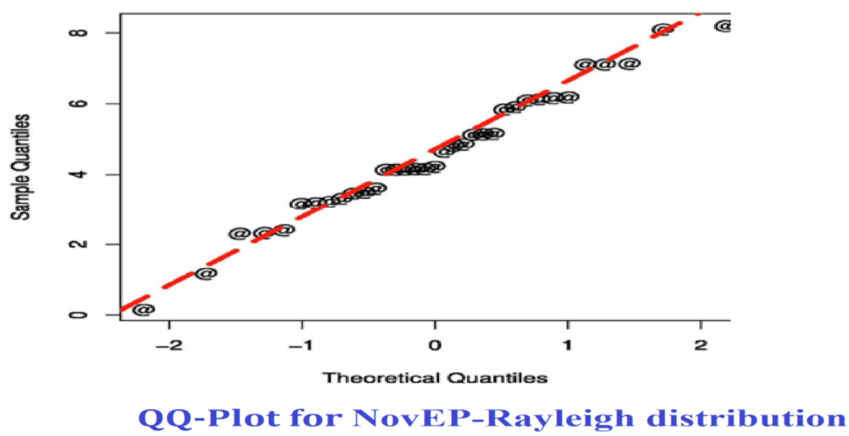


Fig. 3.4 QQ-Plot for NovEP-Rayleigh distribution COVID-19 data



The quantile function in (2.2.1) and the r^{th} raw moments in (2.3.7) of the NovEP-Rayleigh distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=20. These values are displayed in Table 3.1

Table 3.1

Mean, Median, Variance, Skewness and Kurtosis

For NovEP-Rayleigh (σ^2, α) distribution

σ^2	α	Mean	Median	Variance	Skewness	Kurtosis
1.5	0.5	0.1987	0.1915	0.1887	75.4976	22.6405
	1.5	0.9345	0.8769	0.7542	43.7836	21.8765
	2.5	1.766	0.9573	0.8641	58.2293	18.3747
	3.5	2.0652	1.8754	1.4578	48.6171	17.4297
	4.5	3.1421	2.1031	1.9863	61.2836	21.8856
2	0.5	0.1782	0.1017	0.1785	75.5184	22.6337
	1.5	0.912	0.8566	0.7983	43.7968	21.8682
	2.5	1.684	0.8521	0.7152	58.4116	18.3907
	3.5	1.9863	0.8522	0.7055	49.1485	17.4616
	4.5	3.0008	1.9985	1.5454	61.3934	21.8775
2.5	0.5	0.1584	0.1006	0.1682	75.6869	22.6167
	1.5	0.8568	0.8152	0.7112	43.8112	21.8637
	2.5	1.532	0.7987	0.6547	58.4276	18.38
	3.5	1.8524	0.6982	0.5132	49.5363	17.4299
	4.5	2.9863	1.3547	1.165	61.4155	21.9236
3	0.5	0.1356	0.0986	0.1545	75.8824	22.6803
	1.5	0.5478	0.4988	0.3211	43.8651	21.8322
	2.5	1.2651	0.6574	0.5227	58.6251	18.3672
	3.5	1.6862	0.6551	0.5142	49.6321	17.4352
	4.5	2.5413	1.2133	1.1682	61.5773	21.8864

The quantile function in (2.2.1) and the r^{th} raw moments in (2.3.7) of the NovEP-Rayleigh distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=60. These values are displayed in Table 3.2

Table 3.2

Mean, Median, Variance, Skewness and Kurtosis For NovEP-Rayleigh (σ^2, α) distribution

σ^2	α	Mean	Median	Variance	Skewness	Kurtosis
5	0.5	0.1996	0.1967	0.1854	90.1456	24.2361
	1.5	0.9465	0.8855	0.7432	54.6353	23.5114
	2.5	1.1854	0.9787	0.8405	60.5340	18.6847
	3.5	2.1587	1.9771	1.4378	50.5015	19.854
	4.5	0.176	0.1863	0.1742	86.2525	23.6668
6.5	0.5	0.1854	0.1842	0.1701	90.5687	24.1254
	1.5	0.9354	0.8548	0.7325	54.7894	23.4781
	2.5	1.1765	0.9417	0.7101	60.5748	18.5622
	3.5	2.0147	1.9025	1.3254	50.8114	19.7548
	4.5	0.1658	0.1647	0.1542	86.3521	23.5469
8	0.5	0.1752	0.1724	0.1652	90.8547	24.0054
	1.5	0.9122	0.8321	0.7224	74.8654	23.3476
	2.5	1.1045	0.9154	0.7009	60.8471	18.3242
	3.5	2.0047	1.8957	1.3012	50.9875	19.6354
	4.5	0.1524	0.1548	0.1500	86.5873	23.4747
10	0.5	0.1743	0.1688	0.1543	90.9863	23.9889
	1.5	0.9054	0.8211	0.7149	74.9021	23.1547
	2.5	1.0570	0.9054	0.6985	60.9011	18.1005
	3.5	1.9873	1.8765	1.2968	50.9963	19.5644
	4.5	0.2668	0.2741	0.1463	86.7957	23.3142

The quantile function in (2.2.1) and the r^{th} raw moments in (2.3.7) of the NovEP-Rayleigh distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=100. These values are displayed in Table 3.3

Table 3.3

Mean, Median, Variance, Skewness and Kurtosis For NovEP-Rayleigh (σ^2, α) distribution

σ^2	α	Mean	Median	Variance	Skewness	Kurtosis
5	0.5	0.1874	0.1857	0.1845	90.1456	24.2361
	1.5	0.9358	0.8752	0.7302	54.6353	23.5114
	2.5	1.1841	0.9658	0.8395	60.5340	18.6847
	3.5	2.1452	1.9652	1.4258	50.5015	19.854
	4.5	0.1724	0.1795	0.1668	86.2525	23.6668
6.5	0.5	0.1852	0.1846	0.1798	90.6589	24.5874
	1.5	0.9255	0.8628	0.7258	54.6787	23.6587
	2.5	1.1786	0.9567	0.8241	60.6648	23.6832
	3.5	2.125	1.9542	1.3658	50.9867	18.9763
	4.5	0.1659	0.1698	0.1587	86.9879	20.0148
8	0.5	0.1816	0.1785	0.1706	91.2583	24.6745
	1.5	0.8937	0.8698	0.6985	54.8796	23.8798
	2.5	1.1634	0.9305	0.8057	60.8792	24.2145
	3.5	2.0698	1.9364	1.2549	51.0259	19.1423
	4.5	0.1584	0.1617	0.1344	87.3651	20.1987
10	0.5	0.1798	0.1772	0.1698	91.5684	24.8887
	1.5	0.8725	0.8544	0.6784	54.9868	23.9983
	2.5	1.1659	0.9154	0.7986	60.9896	24.5736
	3.5	1.9864	1.8474	1.2364	51.6532	19.8724
	4.5	0.1576	0.1587	0.1247	87.5838	20.2542

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