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# CONTRA REGULAR ALPHA GENERALIZED CONTINUOUS MAPPINGS AND ALMOST REGULAR ALPHA GENERALIZED CONTINUOUS MAPPINGS IN BIPOLAR PYTHAGOREAN FUZZY TOPOLOGICAL SPACES

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***Abstract:*** In this paper, the concept of Contra and Almost Regular $\alpha$ Generalized Continuous Mappings are introduced and investigated some of their properties. Also, We have provided some characterization of Bipolar Pythagorean Fuzzy Contra Regular $\alpha$ generalized Continuous Mappings and Bipolar Pythagorean Fuzzy Almost Regular $\alpha$ Generalized Continuous Mappings.

***Keywords:*** **Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagorean Fuzzy Regular $\alpha$ Generalized Closed sets, Bipolar Pythagorean Fuzzy Regular closed sets, Bipolar Pythagorean Fuzzy Open sets.**

## I. INTRODUCTION

In 1965, Zadeh[11] introduced the concept of Fuzzy set which has a framework to encounter uncertainity, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the extensions of fuzzy set theory, a new concept called intuitionistic Fuzzy set[2] was introduced. In intuitionistic Fuzzy set with elements comprising membership and non membership degree. Yager[3] familarized the model of Pythagorean fuzzy sets. After the Pythagorean fuzzy sets, it was widely used in the field of decision making and was applied for the real life applications. Zhang [11] introduced the extension of fuzzy set with Bipolarity, called Bipolar value fuzzy sets. Chen et.al[10] develops extension of bipolar fuzzy sets.

In this paper, we introduced Bipolar Pythagorean Fuzzy Contra Regular $\alpha$ Generalized Continuous Mappings and Bipolar Pythagorean Fuzzy Almost Regular $\alpha$ Generalized Continuous Mappings.

## II PRELIMINARIES

**Definition 2.1:** Let X be the non empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0,1]$ is the membership of $x \in X$.

**Definition 2.2:** Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS)
A in X is given by A=$\{x, \mu_A(x), \nu_A(x) : x \in X\}$ where the functions $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$. The degree of indeterminacy $I_A = 1 - (\mu_A(x) + \nu_A(x))$ for each $x \in X$.

**Definition 2.3:** Let X be the non empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set P, respectively, and 0$\leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant$1 for each $x \in X$. The degree of indeterminacy $I_P = \sqrt{1 - (\mu_P^2(x) + \nu_P^2(x))}$ for each $x \in X$.

**Definition 2.4:** Let X be a non empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-) : x \in X\}$ where $\mu_A^+ : X \to [0,1], \nu_A^+ : X \to [0,1], \mu_A^- : X \to [-1,0], \nu_A^- : X \to [-1,0]$ are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant$1 and $0 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x)^2 \leqslant$1 where $\mu_A^+(x)$ denote the positive membership degree. $\nu_A^+(x)$ denote the positive non membership degree. $\mu_A^-(x)$ denote the negative membership degree. $\nu_A^-(x)$ denote the negative non membership degree.

**Definition 2.5:** Bipolar Pythagorean Fuzzy Topological Spaces: Let $X \neq \emptyset$ be a set and $\tau_p$ be a family of Bipolar Pythagorean fuzzy subsets of X. If

(i) $0_X, 1_X \in \tau_p$

(ii) For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.

(iii) $P_i \in \tau_p$ for arbitrary family $\{P_i$ s uch that $i \in J\} \subseteq \tau_p$.

Then $\tau_p$ is called Bipolar Pythagorean Fuzzy Topology on X and the pair $(X, \tau_p)$ is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of $\tau_p$ is called Bipolar Pythagorean fuzzy open set(BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set(BPFCS).

**Definition 2.6:** Let $(X, \tau_p)$ be a BPFTS and $P = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ be a BPFS over X. Then the Bipolar Pythagorean Fuzzy Interior, Bipolar Pythagorean Fuzzy Closure of P are defined by:

(i) BPFint(P) = $\cup \{G/G \;\; is \;\; a \;\; BPFOS \;\; in \;\; (X, \tau_p) \;\; and \;\; G \subseteq P\}$

(ii) BPFcl(P) = $\cap \{K \;\; / K \;\; is \;\; a \;\; BPFCS \;\; in \;\; (X, \tau_p) \;\; and \;\; P \subseteq K\}$

It is clear that

a. BPFint(P) is the biggest Bipolar Pythagorean Fuzzy Open set contained in P.

b. BPFcl(P) is the smallest Bipolar Pythagorean Fuzzy Closed set containing P.

**Definition 2.7:** If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS $(X, \tau_p)$ is said to be

(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $int(cl(A)) \subseteq A$

(b) Bipolar Pythagorean Fuzzy Semi open set (BPFSOS) if $A \subseteq cl(int(A))$

(c) Bipolar Pythagorean Fuzzy Pre-closed set(BPFPCS) if $cl(int(A)) \subseteq A$

(d) Bipolar Pythagorean Fuzzy Pre-open set(BPFPOS) if $A \subseteq int(cl(A))$

(e) Bipolar Pythagorean Fuzzy $\alpha$ closed set (BPFR$\alpha$CS) if $cl(int(cl(A)) \subseteq A$

(f) Bipolar Pythagorean Fuzzy $\alpha$ open set (BPF$\alpha$OS) if $A \subseteq int(cl(int(A))$

(g) Bipolar Pythagorean Fuzzy $\gamma$ closed set (BPF$\gamma$CS) if $A \subseteq int(cl(A)) \cup cl(int(A))$

(h) Bipolar Pythagorean Fuzzy $\gamma$ open set (BPF$\gamma$OS) if $cl(int(A)) \cup int(cl(A)) \subseteq A$

(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = cl(int(A))$

(j) Bipolar Pythagorean Fuzzy regular open set (BPFROS) if $A = int(cl(A))$

(k) If BPF set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Generalized closed set(BPFGCS), if $cl(A) \subseteq U$whenever $A \subseteq U$ and $U$ is BPFOS in X.

(l) If BPF set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Generalized open set(BPFGOS), if $A^c$ is a BPFGCS in X.

(m) If BPF set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Regular Generalized closed set(BPFRGCS), if $cl(A) \subseteq U$whenever $A \subseteq U$ and $U$ is BPFROS in X.

(n) If BPF set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Regular Generalized open set(BPFRGOS), if $A^c$ is a BPFRGCS in X.

**Definition 2.8:** A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space $(X, \tau_p)$ is called Bipolar Pythagorean Regular $\alpha$ Generalized closed set (BPFR$\alpha$GCS in short), if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is BPF regular open set in X.

**Definition 2.9:** A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space $(X, \tau_p)$ is called Bipolar Pythagorean Regular $\alpha$ Generalized open set (BPFR$\alpha$GOS in short), if $\alpha int(A) \supseteq U$ whenever $A \supseteq U$ and $U$ is BPF regular closed set in X.

**Definition 2.10:** A function $f: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is called BPFR$\alpha$G continuous mapping if the inverse image of every BPF closed set in Y is BPFR$\alpha$G closed set in X.

**Definition 2.11:** A mapping $f: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is said to be

(i) BPF semi continuous mapping if $f^{-1}(A) \in BPFSO(X)$ for every $A \in (Y, \sigma_p)$.

(ii) BPF$\alpha$ continuous mapping if $f^{-1}(A) \in BPF\alpha O(X)$ for every $A \in (Y, \sigma_p)$.

(iii) BPF Pre continuous mapping if $f^{-1}(A) \in BPFPO(X)$ for every $A \in (Y, \sigma_p)$.

(iv) BPF$\gamma$ continuous mapping if $f^{-1}(A) \in BPF\gamma O(X)$ for every $A \in (Y, \sigma_p)$.

**Definition 2.12:** A mapping $f: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is said to be BPF Generalized Continuous mapping (BPFG continuous mapping) if $f^{-1}(A) \in BPFGC(X)$ for every BPFCS A in $(Y, \sigma_p)$.

**Definition 2.13:** A mapping $f: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is said to be BPF$\alpha$ Generalized Continuous mapping (BPF$\alpha$G continuous mapping) if $f^{-1}(A) \in BPF\alpha C(X)$ for every BPFCS A in $(Y, \sigma_p)$.

**Definition 2.14:** A BPFTS $(X, \tau_p)$ is said to be a $BPFR\alpha_c T_{1/2}$ space (Bipolar Pythagorean Fuzzy Regular $\alpha_c T_{1/2}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPFCS in $(X, \tau_p)$.

**Definition 2.15:** A BPFTS $(X, \tau_p)$ is said to be a BPFR$\alpha_g T_{1/2}$ space (Bipolar Pythagorean Fuzzy Regular $\alpha_g T_{1/2}$ space) if every BPFR$\alpha$GCS in $(X, \tau_p)$ is a BPFGCS in $(X, \tau_p)$.

**Definition 2.16:** A BPFTS $(X, \tau_p)$ is said to be a BPFR$\alpha_\alpha T_{1/2}$ space (Bipolar Pythagorean Fuzzy Regular $\alpha_\alpha T_{1/2}$ space) if every BPFR$\alpha$GCS in $(X, \tau_p)$ is a BPF$\alpha$CS in $(X, \tau_p)$.

**Definition 2.17:** A mapping $f: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy irresolute mapping (BPF irresolute mapping) if $f^{-1}(A) \in BPFCS(X)$ for every BPFCS A in $(Y, \sigma_p)$.

**Definition 2.18:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Generalized irresolute mapping (BPFG irresolute mapping) if $f^{-1}(A) \in BPFGCS(X)$ for every BPFGCS A in $(Y,\sigma_p)$.

**Definition 2.19:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra continuous mapping (BPF contra continuous mapping) if $f^{-1}(A) \in$ BPFOS(X) for every BPFCS A in $(Y,\sigma_p)$.

**Definition 2.20:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra $\alpha$ continuous mapping (BPF contra $\alpha$ continuous mapping) if $f^{-1}(A) \in BPF\alpha OS(X)$ for every BPFCS A in $(Y,\sigma_p)$.

**Definition 2.21:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra Pre continuous mapping (BPF contra Pre continuous mapping) if $f^{-1}(A) \in BPFPOS(X)$ for every BPFCS A in $(Y,\sigma_p)$.

**Definition 2.22:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra semi continuous mapping (BPF contra semi continuous mapping) if $f^{-1}(A) \in BPFSOS(X)$ for every BPFCS A in $(Y,\sigma_p)$.

**Definition 2.23:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra Generalized continuous mapping (BPF contra G continuous mapping) if $f^{-1}(A) \in BPFGOS(X)$ for every BPFCS A in $(Y,\sigma_p)$.

**Definition 2.24:** A mapping $f:(X,\tau_p) \to (Y,\sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra$\alpha$G continuous mapping (BPF contra$\alpha$G continuous mapping) if $f^{-1}(A) \in BPFP\alpha GOS(X)$ for every BPFCS A in $(Y,\sigma_p)$.

## III BIPOLAR PYTHAGOREAN FUZZY CONTRA REGULAR $\alpha$ GENERALIZED CONTINUOUS MAPPINGS

In this section we introduced Bipolar Pythagorean Fuzzy Contra Regular $\alpha$ Generalized continuous mappings and studied some of its properties.

**Definition 3.1:** A mapping $\phi : (X,\tau_p) \to (Y,\sigma_p)$ is called Bipolar phythagorean fuzzy contra regular $\alpha$ generalized continuous (BPFCR$\alpha$G continuous in short) mapping if $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X for every BPFOS $\omega$ of Y.

**Example 3.2:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.7, 0.6), (0.5, 0.3), (-0.6, -0.7), (-0.4, -0.3)), $T_2$ = (x, (0.3, 0.2), (0.7, 0.6), (-0.2, -0.1), (-0.8, -0.7)) and $T_3$ = (y, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)). Define a mapping $\phi:(X,\tau_p) \to (Y,\sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3$ = (y, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)) is BPFOS in Y and $T_1, T_2$ are BPFROS in X. Now $\phi^{-1}(T_3)$ = (x, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)) is a BPFR$\alpha$GCS in X, as $\alpha cl(\phi^{-1}(T_3)) = T_1^c \subseteq T_1$ whenever $\phi^{-1}(T_3) \subseteq T_1$ and $T_1$ is BPFROS in X. Therefore, $\phi$ is BPFCR$\alpha$G continuous mapping in X.

**Proposition 3.3:** Every BPFC continuous mapping is a BPFCR$\alpha$G continuous mapping but not conversely.

**Proof:** Let $\phi:(X,\tau_p) \to (Y,\sigma_p)$ be a BPFC continuous mapping. Let $\omega$ be a BPFOS in Y. By hypothesis, $\phi^{-1}(\omega)$ is a BPFCS in X. Since every BPFCS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Example 3.4:** From Example 3.2, $\phi$ is BPFCR$\alpha$G continuous mapping but not BPFC continuous mapping, as $cl(\phi^{-1}(T_3)) = T_1^c \neq \phi^{-1}(T_3)$.

**Proposition 3.5:** Every BPFC$\alpha$ continuous mapping is a BPFCR$\alpha$G continuous mapping but not conversely.

**Proof:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFC continuous mapping. Let $\omega$ be a BPFOS in Y. By hypothesis, $\phi^{-1}(\omega)$ is a BPF$\alpha$CS in X. Since every BPF$\alpha$CS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Example 3.6:** From Example 3.2, $\phi$ is BPFCR$\alpha$G continuous mapping but not BPF$\alpha$C continuous mapping, as $\alpha cl(\phi^{-1}(T_3)) = cl(int(cl(\phi^{-1}(T_3))) = T_1^c \subseteq \phi^{-1}(T_3)$.

**Proposition:3.7:** Every BPFCR continuous mapping is a BPFCR$\alpha$G continuous mapping but not conversely.

**Proof :** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFC continuous mapping. Let $\omega$ be a BPFOS in Y. By hypothesis, $\phi^{-1}(\omega)$ is a BPFCS in X. Since every BPFCS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Example: 3.8:** From Example 3.2, $\phi$ is BPFCR$\alpha$G continuous mapping but not BPFCR continuous mapping, as $\alpha cl(\phi^{-1}(T_3)) = cl(int(cl(\phi^{-1}(T_3))) = T_1^c \subseteq \phi^{-1}(T_3)$.

**Proposition:3.9:** Every BPFCG continuous mapping is a BPFCR$\alpha$G continuous mapping but not conversely.

**Proof:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFCG continuous mapping. Let $\omega$ be a BPFOS in Y. By hypothesis, $\phi^{-1}(\omega)$ is a BPFGCS in X. Since every BPFGCS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Example: 3.10:** From Example 3.2, $\phi$ is BPFCR$\alpha$G continuous mapping but not BPFCG continuous mapping, as $cl(\phi^{-1}(T_3)) = T_2^c \nsubseteq U$.

**Proposition:3.11:** Every BPFC$\alpha$G continuous mapping is a BPFCR$\alpha$G continuous mapping but not conversely.

**Proof:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFC$\alpha$G continuous mapping. Let $\omega$ be a BPFOS in Y. By hypothesis, $\phi^{-1}(\omega)$ is a BPF$\alpha$GCS in X. Since every BPF$\alpha$GCS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Example: 3.12:** From Example 3.2, $\phi$ is BPFCR$\alpha$G continuous mapping but not BPFC$\alpha$G continuous mapping, as $\alpha cl(\phi^{-1}(T_3)) = cl(int(cl(\phi^{-1}(T_3))) = T_2^c \nsubseteq U$.

**Remark 3.13:** Every BPFCP continuous mapping and BPFCR$\alpha$G continuous mapping are independent of each other.

**Example 3.14:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.7, 0.6), (0.5, 0.3), (-0.6, -0.7), (-0.4, -0.3)), $T_2$ = (x, (0.3, 0.2), (0.7, 0.6), (-0.2, -0.1), (-0.8, -0.7)) and $T_3$ = (y, (0.4, 0.2), (0.5, 0.6), (-0.5, -0.3), (-0.8, -0.7)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3$ = (y, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)) is BPFOS in Y and $T_1, T_2$ are BPFROS in X. Then $\phi^{-1}(T_3)$ = (x, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)) is a BPFR$\alpha$GCS in X but $(\phi^{-1}(T_3))$ is not BPFPCS, as $cl(int(\phi^{-1}(T_3))) = T_1^c \nsubseteq \phi^{-1}(T_3)$. Therefore, $\phi$ is not a BPFCP continuous mapping in X.

**Example 3.15:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.7, 0.6), (0.5, 0.3), (-0.6, -0.5), (-0.4, -0.3)), $T_2$ = (x, (0.3, 0.2), (0.7, 0.6), (-0.3, -0.1), (-0.7, -0.6)) and $T_3$ = (y, (0.1, 0.2), (0.8, 0.8), (-0.1, -0.2), (-0.6, -0.6)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3$ = (y, (0.1, 0.2), (0.8, 0.8), (-0.1, -0.2), (-0.6, -0.6)) is BPFOS in Y and $T_1, T_2$ are BPFROS in X. Then $\phi^{-1}(T_3)$ = (x, (0.1, 0.2), (0.8, 0.8), (-0.1, -0.2), (-0.6, -0.6)) is a BPFPCS in X but $\phi^{-1}(T_3)$ is not BPFR$\alpha$GCS, as $\alpha cl(\phi^{-1}(T_3)) = T_1^c \subseteq T_1 \nsubseteq T_2$. Therefore, $\phi$ is not a BPFCR$\alpha$G continuous mapping in X.

**Remark 3.16:** Every BPFCS continuous mapping and BPFCR$\alpha$G continuous mapping are independent of each other.

**Example 3.17:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.7, 0.6), (0.5, 0.3), (-0.6, -0.7), (-0.4, -0.3)), $T_2$ = (x, (0.3, 0.2), (0.7, 0.6), (-0.2, -0.1) (-0.8, -0.7)) and $T_3$ = (y, (0.4, 0.2), (0.5, 0.6), (-0.5, -0.3), (-0.8, -0.7)). Define a mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3$ = (y, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)) is BPFOS in Y and $T_1, T_2$ are BPFROS in X. Then $\phi^{-1}(T_3)$ = (x, (0.4, 0.2), (0.8, 0.7), (-0.5, -0.3), (-0.8, -0.7)) is a BPFR$\alpha$GCS in X but $(\phi^{-1}(T_3))$ is not BPFSCS, as $int(cl((\phi^{-1}(T_3))) = T_1^c \nsubseteq \phi^{-1}(T_3)$. Therefore, $\phi$ is not a BPFCS continuous mapping in X.

**Example 3.18:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.5, 0.3), (0.6, 0.7), (-0.4, -0.2), (-0.5, -0.6)), $T_2$ = (x, (0.2, 0.2), (0.7, 0.7), (-0.2, -0.1), (-0.5, -0.6)) and $T_3$ = (y, (0.5, 0.3), (0.6, 0.7), (-0.4, -0.2), (-0.5, -0.6)). Define a mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3$ = (y, (0.1, 0.2), (0.8, 0.8), (-0.1, -0.2), (-0.6, -0.6)) is BPFOS in Y and $T_1$ is BPFROS in X. Then $\phi^{-1}(T_3)$ = (x, (0.1, 0.2), (0.8, 0.8), (-0.1, -0.2), (-0.6, -0.6)) is a BPFSCS in X but $(\phi^{-1}(T_3))$ is not BPFR$\alpha$GCS, as $\alpha cl(\phi^{-1}(T_3))) = T_1^c \nsubseteq T_1$). Therefore, $\phi$ is not a BPFCR$\alpha$G continuous mapping in X.

**Theorem 3.19:** A mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ is BPFCR$\alpha$G continuous mapping if and only if the inverse image of each BPFCS in Y is a BPFR$\alpha$GOS in X.

**Proof: (Necessity:)** Let $\omega$ be BPFCS in Y. This implies $\omega^c$ is BPFOS in Y. Since $\phi$ is BPFCR$\alpha$G continuous mapping, $\phi^{-1}(\omega^c)$ is BPFR$\alpha$GCS in X. Since $\phi^{-1}(\omega^c) = (\phi^{-1}(\omega))^c$. Thus, $\phi^{-1}(\omega)$ is BPFR$\alpha$GOS in X.

**(Sufficiency:)** Suppose that $\omega$ is BPFOS in Y. This implies $\omega^c$ is BPFCS in Y. By hypothesis, $\phi^{-1}(\omega^c)$ is BPFR$\alpha$GOS in X. Since $\phi^{-1}(\omega^c) = (\phi^{-1}(\omega))^c$, where $(\phi^{-1}(\omega))^c$ is BPFR$\alpha$GOS in X, $\phi^{-1}(\omega)$ is BPFR$\alpha$GCS in X. Hence $\phi$ is BPFCR$\alpha$G continuous mapping.

**Theorem 3.20:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping and let $\phi^{-1}(\omega)$ be a BPFROS in X for every BPFCS $\omega$ in Y. Then $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Proof:** Let $\omega$ be a BPFCS in Y. By hypothesis, $\phi^{-1}(\omega)$ is BPFROS in X. Since every BPFROS is BPFR$\alpha$GOS, $\phi^{-1}(\omega)$ is BPFR$\alpha$GOS in X. Thus, $\phi$ is BPFCR$\alpha$G continuous mapping, by Theorem 3.12.

**Theorem 3.21** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be BPFCR$\alpha$G continuous mapping and $\psi: (Y, \sigma_p) \to (Z, \gamma_p)$ be BPF continuous mapping, then $(\psi \circ \phi): (X, \tau_p) \to (Z, \gamma_p)$ is BPFCR$\alpha$G continuous mapping.

**Proof:** Let $\omega$ be BPFOS in Z. Then $\psi^{-1}(\omega)$ is BPFOS in Y, by hypothesis. Since $\phi$ is BPFCR$\alpha$G continuous mapping, $\phi^{-1}(\psi^{-1}(\omega))$ is BPFR$\alpha$GCS in X. Hence $(\psi \circ \phi)$ is BPFCR$\alpha$G continuous mapping.

**Theorem 3.22:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFCR$\alpha$G continuous mapping and $\psi: (Y, \sigma_p) \to (Z, \gamma_p)$ be a BPFG continuous mapping and Y is a $BPFT_{1/2}$ space, then $\psi \circ \phi: (X, \tau_p) \to (Z, \gamma_p)$ is a BPFCR$\alpha$G continuous mapping.

**Proof:** Let $\omega$ be a BPFOS in Z. Then $\psi^{-1}(\omega)$ is a BPFOS in Y, by hypothesis. Since Y is a BPF$T_{1/2}$ space, $\psi^{-1}(\omega)$ is a BPFOS in Y. Therefore, $\phi^{-1}(\psi^{-1}(\omega))$ is a BPFR$\alpha$GCS in X, by hypothesis. Hence $\psi \circ \phi$ is a BFCR$\alpha$G continuous mapping.

**Theorem 3.23:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. Suppose that one of the following properties hold:

(i) $\phi(\alpha cl(\omega)) \subseteq int(\phi(\omega))$ for each BPFS $\omega$ in X.

(ii) $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(int(\delta))$ for each BPFS $\delta$ in Y.

(iii) $\phi^{-1}(cl(\delta)) \subseteq \alpha int(\phi^{-1}(\delta))$ for each BPFS $\delta$ in Y.

Then $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Proof:** (i) → (ii) Suppose that $\delta$ is a BPFS in Y. Then, $\phi^{-1}(\delta)$ is a BPFS in X. By hypothesis, $\phi(\alpha cl(\phi^{-1}(\delta))) \subseteq int(\phi(\phi^{-1}(\delta))) \subseteq int(\delta)$. Now $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\phi(\alpha cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(int(\delta))$.

(ii) → (iii) is obvious by taking complement in (ii).

Suppose (iii) holds: Let $\omega$ be a BPFCS in Y. Then, $cl(\omega) = \omega$ and $\phi^{-1}(\omega)$ is a BPFS in X. Now $\phi^{-1}(\omega) = \phi^{-1}(cl(\omega)) \subseteq \alpha int(\phi^{-1}(\omega)) \subseteq \phi^{-1}(\omega)$, by hypothesis. This implies, $\phi^{-1}(\omega)$ is a BPF$\alpha$OS in X and hence $\phi^{-1}(\omega)$ is a BPFR$\alpha$GOS in X. Thus $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Theorem 3.24:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a bijective mapping. Then $\phi$ is a BPFCR$\alpha$G continuous mapping if $cl(\phi(\omega)) \subseteq \phi(\alpha int(\omega))$ for every BPFS $\omega$ in X.

**Proof:** Let $\omega$ be a BPFCS in Y. Then, $cl(\omega) = \omega$ and $\phi^{-1}(\omega)$ is a BPFS in X. By hypothesis, $cl(\phi(\phi^{-1}(\omega))) \subseteq \phi(\alpha int(\phi^{-1}(\omega))$. Since $\phi$ is an onto, $\phi(\phi^{-1}(\omega)) = \omega$.Therefore$\omega = cl(\omega) = cl(\phi(\phi^{-1}(\omega))) \subseteq \phi(\alpha int(\phi^{-1}(\omega)))$. Now $\phi^{-1}(\omega) \subseteq \phi^{-1}(\phi(\alpha int(\phi^{-1}(\omega)))) = \alpha int(\phi^{-1}(\omega)) \subseteq \phi^{-1}(\omega)$Hence $\phi^{-1}(\omega)$ is a BPF$\alpha$OS in X and hence a BPFR$\alpha$GOS in X. Thus $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Theorem 3.25:** If $\phi: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFCR$\alpha$G continuous mapping, where X is a BPFR$\alpha T_{1/2}$ space, then the following conditions hold:

(i) $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(int(\alpha cl(\delta)))$ for every BPFOS $\delta$ in Y.

(ii) $\phi^{-1}(cl(\alpha int(\delta))) \subseteq \alpha int(\phi^{-1}(\delta))$ for every BPFCS $\delta$ in Y.

**Proof:** (i) Let $\delta$ be a BPFOS in Y. By hypothesis $\phi^{-1}(\delta)$ is a BPFR$\alpha$GCS in X. Since X is a BPFR$\alpha T_{1/2}$ space, $\phi^{-1}(\delta)$ is a BPF$\alpha$CS in X. This implies, $\alpha cl(\phi^{-1}(\delta)) = \phi^{-1}(\delta) = \phi^{-1}(int(\delta)) \subseteq \phi^{-1}(int(\alpha cl(\delta)))$.(ii) can be proved easily by taking the complement of (i).

**Theorem 3.26:** A mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFCR$\alpha$G continuous mapping if $\phi^{-1}(\alpha cl(\delta)) \subseteq int(\phi^{-1}(\delta))$ for every BPFS $\delta$ in Y.

**Proof:** Let $\delta \subseteq Y$ be a BPFCS. Then $cl(\delta) = \delta$. Since every BPFCS is a BPF$\alpha$CS, $\alpha cl(\delta) = \delta$. Now by hypothesis, $\phi^{-1}(\delta) = \phi^{-1}(\alpha cl(\delta)) \subseteq int(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\delta)$. This implies, $\phi^{-1}(\delta) = int(\phi^{-1}(\delta))$. Therefore, $\phi^{-1}(\delta)$ is a BPFOS in X. Hence $\delta$ is a BPFC continuous mapping. Then by Remark 3.3, $\phi$ is a BPFCR$\alpha$G continuous mapping.

**Theorem 3.27:** A mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFCR$\alpha$G continuous mapping, where X is a BPFR$\alpha T_{1/2}$ space if and only if $\phi^{-1}(\alpha cl(\delta)) \subseteq \alpha int(\phi^{-1}(cl(\delta)))$ for every BPFS $\delta$ in Y.

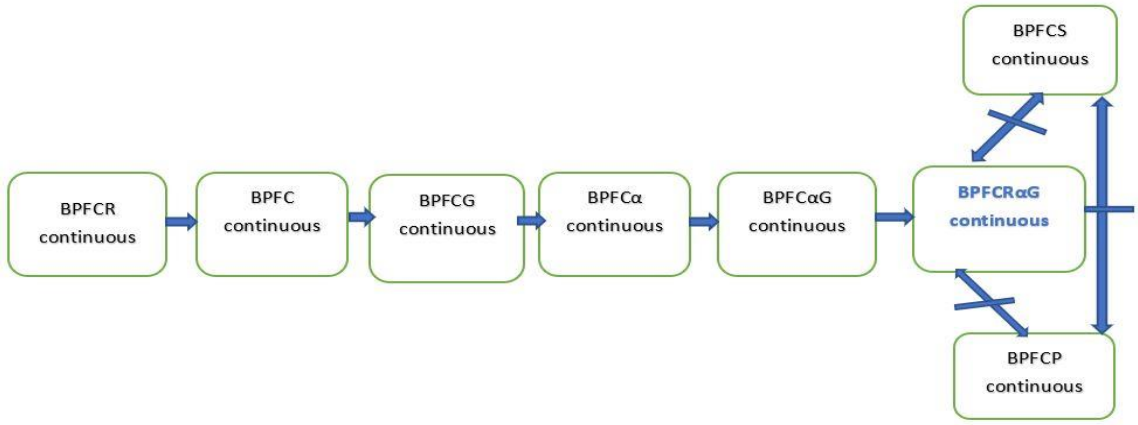
**Proof:** Necessity: Let $\delta \subseteq Y$ be a BPFS. Then $cl(\delta)$ is a BPFCS in Y. By hypothesis $\phi^{-1}(cl(\delta))$ is a BPFR$\alpha$GOS in X. Since X is a BPFR$\alpha T_{1/2}$ space, $\phi^{-1}(cl(\delta))$ is a BPF$\alpha$OS in X. This implies, $\phi^{-1}(cl(\delta)) = \alpha int(\phi^{-1}(cl(\delta)))$. Therefore, $\phi^{-1}(\alpha cl(\delta)) \subseteq \phi^{-1}(cl(\delta)) = \alpha int(\phi^{-1}(cl(\delta)))$.

**Sufficiency:** Let $\delta \subseteq Y$ be a BPFS. Then $cl(\delta)$ is a BPFCS in Y. By hypothesis, $\phi^{-1}(\alpha cl(\delta)) \subseteq \alpha int(\phi^{-1}(cl(\delta))) = \alpha int(\phi^{-1}(\delta))$. But $\alpha cl(\delta) = \delta$. Therefore, $\phi^{-1}(\delta) = \phi^{-1}(\alpha cl(\delta)) \subseteq \alpha int(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\delta)$. This implies, $\phi^{-1}(\delta)$ is a BPF$\alpha$OS in X and hence $\phi^{-1}(\delta)$ is a BPFR$\alpha$GOS in X. Hence $\delta$ is a BPFCR$\alpha$G continuous mapping.

**Theorem 3.28:** A BPF continuous mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFCR$\alpha$G continuous mapping, if BPFR$\alpha$GO(X) = BPFR$\alpha$GC(X).

**Proof:** Let $\omega \subseteq$ Y be a BPFOS. By hypothesis, $\phi^{-1}(\omega)$ is a BPFOS in X and hence $\phi^{-1}(\omega)$ is a BPFR$\alpha$GOS in X. Thus $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X, as BPFR$\alpha$GO(X) = BPFR$\alpha$GC(X). Therefore, $\phi$ is a BPFCR$\alpha$G continuous mapping.

Figure 1: The relation between various types of BPFCRG continuous are given in the following diagram




## IV BIPOLAR PYTHAGOREAN FUZZY ALMOST REGULAR $\alpha$ GENERALIZED CONTINUOUS MAPPINGS

In this section we introduced Bipolar Pythagorean Fuzzy Almost Regular $\alpha$ Generalized continuous mappings and investigated some of its properties.

**Definition 4.1:** A mapping $\phi: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is said to be a Bipolar Pythagorean Fuzzy Almost Regular $\alpha$ Generalized Continuous (BPFaR$\alpha$G continuous in short) mapping if $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X for every BPFRCS $\omega$ in Y.

**Example 4.2:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.6, 0.5), (0.2, 0.2), (-0.6, -0.7), (-0.2, -0.1)), $T_2$ = (x, (0.3, 0.2), (0.7, 0.5), (-0.3, -0.2), (-0.7, -0.5)), $T_3$ = (y, (0.4, 0.4), (0.2, 0.2), (0.4, -0.4), (-0.2, -0.1)) and $T_4$ = (y, (0.3, 0.2), (0.6, 0.6), (-0.3, -0.2), (-0.6, -0.6)). Define a mapping $\phi: (X, \tau_p) \rightarrow (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_4^c$ = (y, (0.6, 0.6), (0.3, 0.2), (-0.6, -0.6), (-0.3, -0.2)) is BPFRCS in Y and $\phi^{-1}(T_4^c)$ = (x, (0.6, 0.6), (0.3, 0.2), (-0.6, -0.6), (-0.3, -0.2)) is BPFS in X. Then $\alpha cl(\phi^{-1}(T_4^c)) = T_2^c \subseteq 1_p$ as $\phi^{-1}(T_4^c) \subseteq 1_p$. Therefore, $\phi^{-1}(T_4^c)$ is BPFR$\alpha$GCS in X. Thus $\phi$ is BPFaR$\alpha$G continuous mapping in X.

**Proposition 4.3:** Every BPF continuous mapping is a BPFaR$\alpha$G continuous mapping but not conversely.

**Proof:** Suppose that $\phi: (X, \tau_p) \rightarrow (Y, \sigma_p)$ is a BPF continuous mapping. Let $\delta$ be a BPFRCS in Y. Since every BPFRCS is a BPFCS, $\delta$ is a BPFCS in Y. Then $\phi^{-1}(\delta)$ is a BPFCS in X, by hypothesis. Since every BPFCS is a BPFR$\alpha$GCS, $\phi^{-1}(\delta)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Example 4.4:** From Example 4.2, $\phi$ is BPFaR$\alpha$G continuous mapping but not BPF continuous mapping, as $cl(\phi^{-1}(T_4^c)) = T_2^c \neq \phi^{-1}(T_4^c)$.

**Proposition 4.5:** Every BPF$\alpha$ continuous mapping is a BPFaR$\alpha$G continuous mapping but not conversely.

**Proof:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$\alpha$ continuous mapping. Let $\omega$ be a BPFRCS in Y. Since every BPFRCS is a BPFCS, $\omega$ is a BPFCS in Y. Then $\phi^{-1}(\omega)$ is a BPF$\alpha$CS in X, by hypothesis. Since every BPF$\alpha$CS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Example 4.6:** From Example 4.2, $\phi$ is BPFaR$\alpha$G continuous mapping but not BPF continuous mapping, as $cl(int(cl(\phi^{-1}(T_4^c))) = T_2^c \nsubseteq \phi^{-1}(T_4^c)$.

**Proposition 4.7:** Every BPFR continuous mapping is a BPFaR$\alpha$G continuous mapping but not conversely.

**Proof :** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF continuous mapping. Let $\omega$ be a BPFRCS in Y. Since every BPFRCS is a BPFR$\alpha$GCS, $\omega$ is a BPFRCS in Y. Then, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X, by hypothesis. Hence $\phi$ is a BPFaR$\alpha$G Continuous mapping.

**Example 4.8:** From Example 4.2, $\phi$ is BPFaR$\alpha$G continuous mapping but not BPFR continuous mapping, as $int(cl(\phi^{-1}(T_4^c))) = T_2^c \neq \phi^{-1}(T_4^c)$.

**Proposition 4.9:** Every BPFG continuous mapping is a BPFaR$\alpha$G continuous mapping but not conversely.

**Proof :** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFG continuous mapping. Let $\omega$ be a BPFRCS in Y. Since every BPFRCS is a BPFGCS, $\omega$ is a BPFGCS in Y. $\phi^{-1}(\omega)$is a BPFGCS in X, by hypothesis. Since every BPFGCS is a BPFR$\alpha$GCS, $\phi^{-1}(\omega)$is a BPFRGCS in X. Hence $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Example 4.10:** From Example 4.2, $\phi$ is BPFaR$\alpha$G continuous mapping but not BPFG continuous mapping, as $cl(\phi^{-1}(T_4^c)) = T_2^c \nsubseteq U$.

**Proposition 4.11:** Every BPF$\alpha$G continuous mapping is a BPFaR$\alpha$G continuous mapping but not conversely.

**Proof:** Let$\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$\alpha$G continuous mapping. Let $\omega$ be a BPFRCS in Y. Since every BPFRCS is a BPF$\alpha$GCS, $\omega$ is a BPF$\alpha$GCS in Y. $\phi^{-1}(\omega)$ is a BPF$\alpha$GCS in X, by hypothesis. Since every BPF$\alpha$GCS is a BPFRGCS, $\phi^{-1}(\omega)$ is a BPFRGCS in X. Hence $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Example 4.12:** From Example 4.2, $\phi$ is BPFaR$\alpha$G continuous mapping but not BPF$\alpha$G continuous mapping, as $cl(int(cl(\phi^{-1}(T_4^c))) = T_2^c \nsubseteq U$.

**Remark: 4.13:** Every BPFP continuous mapping and BPFaR$\alpha$G continuous mapping are independent to each other.

**Example 4.14:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.6, 0.5), (0.6, 0.4), (-0.6, -0.4), (-0.6, -0.3)), $T_2$ = (x, (0.2, 0.3), (0.7, 0.6), (-0.4, -0.3), (-0.6, -0.6)), $T_3$ = (y, (0.6, 0.6), (0.3, 0.3), (-0.6, -0.5), (-0.3, -0.2)) and $T_4$ = (y, (0.1, 0.1), (0.9, 0.9), (-0.1, -0.1), (-0.9, -0.9)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3^c$ = (y, (0.3, 0.3), (0.6, 0.6), (-0.3, -0.2), (-0.6, -0.5)) is BPFRCS in Y and $T_1, T_2$ are BPFROS in X. Now, $\phi^{-1}(T_3^c)$ = (x, (0.3, 0.3), (0.6, 0.6), (-0.3, -0.2), (-0.6, -0.5)) is a BPFPCS in X but $\phi^{-1}(T_3^c)$ is not a BPFR$\alpha$GCS, as $\alpha cl(\phi^{-1}(T_3^c)) = T_1^c \nsubseteq T_1$. Therefore, $\phi$ is BPFP continuous mapping but not a BPFaR$\alpha$G continuous mapping in X.

**Example 4.15:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.7, 0.5), (0.1, 0.1), (-0.7, -0.4), (-0.1, -0.1)), $T_2$ = (x, (0.1, 0.2), (0.5, 0.5), (-0.1, -0.2), (-0.4, -0.4)), $T_3$ = (y, (0.6, 0.7), (0.2, 0.1), (-0.6, -0.5), (-0.1, -0.2)) and $T_4$ = (y, (0.1, 0.1), (0.8, 0.8), (-0.1, -0.1), (-0.8, -0.7)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_4^c$ = (y, (0.8, 0.8), (0.1, 0.1), (-0.8, -0.7), (-0.1, -0.1)) is BPFRCS in Y and $T_2$ is BPFROS in X. Now, $\phi^{-1}(T_4^c)$ = (x, (0.8, 0.8), (0.1, 0.1), (-0.8, -0.7), (-0.1, -0.1)) is a BPFR$\alpha$GCS in X but $\phi^{-1}(T_4^c)$ is not a BPFPCS, as $cl(int((\phi^{-1}(T_4^c))) = 1_p \nsubseteq \phi^{-1}(T_4^c)$. Therefore, $\phi$ is a BPFaR$\alpha$G continuous mapping but not a BPFP continuous mapping in X.

**Remark 4.16:** Every BPFS continuous mapping and BPFaR$\alpha$G continuous mapping are independent to each other.

**Example 4.17:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.5, 0.5), (0.3, 0.2), (-0.5,-0.4), (-0.3, -0.1)), $T_2$ = (x, (0.1, 0.2), (0.7, 0.6), (-0.1, -0.2), (-0.6, -0.6)), $T_3$ = (y, (0.5, 0.5), (0.3, 0.1), (-0.5, -0.4), (-0.3, -0.1)) and $T_4$ = (y, (0.3, 0.3), (0.8, 0.8), (-0.3, -0.2), (-0.5, -0.6)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_4^c$ = (y, (0.8, 0.8), (0.3, 0.3), (-0.5, -0.6), (-0.3, -0.2)) is BPFRCS in Y and $T_1$,$T_2$ are BPFROS in X. Now $\phi^{-1}(T_4^c)$ = (x, (0.8, 0.8), (0.3, 0.3), (-0.5, -0.6), (-0.3, -0.2)) is a BPFR$\alpha$GCS in X but $\phi^{-1}(T_4^c)$ is not BPFSCS, as $int(cl((\phi^{-1}(T_4^c))) = 1_p \nsubseteq \phi^{-1}(T_4^c)$. Therefore, $\phi$ is BPFaR$\alpha$G continuous mapping but not a BPFS continuous mapping in X.

**Example 4.18:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where$T_1$= (x, (0.6, 0.7), (0.2, 0.2), (-0.6, -0.5), (-0.3, -0.2)),$T_2$ = (x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.6, -0.5)), $T_3$ = (y, (0.2, 0.2), (0.6, 0.7), (-0.4, -0.3), (-0.6, -0.5)) and $T_4$ = (y, (0.5, 0.5), (0.4, 0.4), (-0.5, -0.5), (-0.5, -0.4)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3^c$ = (y, (0.6, 0.7), (0.2, 0.2), (-0.6, -0.5), (-0.4, -0.3)) is BPFRCS in Y and $T_1, T_2$ are BPFROS in X. Now $\phi^{-1}(T_3^c)$ = (x, (0.6, 0.7), (0.2, 0.2), (-0.6, -0.5), (-0.4, -0.3)) is a BPFSCS in X but $\phi^{-1}(T_3^c)$ is not a BPFR$\alpha$GCS, since $\alpha cl(\phi^{-1}(T_3^c)) = T_2^c \nsubseteq T_1$ as $\phi^{-1}(T_3^c) \subseteq T_1$ . Therefore, $\phi$ is a BPFS continuous mapping but not a BPFaR$\alpha$G continuous mapping in X.

**Theorem 4.19:** A mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFaR$\alpha$G continuous mapping if and only if the inverse image of each BPFROS in Y is a BPFR$\alpha$GOS in X.

**Proof: Necessity:** Let $\omega$ be a BPFROS in Y. This implies $\omega^c$ is a BPFRCS in Y. Since $\phi$ is a BPFaR$\alpha$G continuous mapping, $\phi^{-1}(\omega^c)$ is a BPFR$\alpha$GCS in X. Since $\phi^{-1}(\omega^c) = (\phi^{-1}(\omega))^c$, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GOS in X.

**Sufficiency:** Let $\omega$ be a BPFRCS in Y. This implies $\omega^c$ is a BPFROS in Y. By hypothesis, $\phi^{-1}(\omega^c)$ is a BPFR$\alpha$GOS in X. Since $\phi^{-1}(\omega^c) = (\phi^{-1}(\omega))^c$, $\phi^{-1}(\omega)$ is a BPFR$\alpha$GCS in X. Thus $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Theorem 4.20:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where $\phi^{-1}(\delta)$ is a BPFRCS in X for every BPFCS in Y. Then $\phi$ is a BPFaR$\alpha$G continuous mapping but not conversely.

**Proof:** Let $\delta$ be a BPFRCS in Y. Since every BPFRCS is a BPFCS, $\delta$ is a BPFCS in Y. Then $\phi^{-1}(\delta)$ is a BPFRCS in X. Since every BPFRCS is a BPFR$\alpha$GCS, $\phi^{-1}(\delta)$ is a BPFR$\alpha$GCS in X. Hence $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Example 4.21:** Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where $T_1$= (x, (0.7, 0.7), (0.3, 0.2), (-0.7, -0.5), (-0.3, -0.2)),$T_2$ = (x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.7, -0.6)), $T_3$ = (y, (0.7, 0.7), (0.3, 0.2), (-0.7, -0.6), (-0.2, -0.2)) and $T_4$ = (y, (0.2, 0.2), (0.9, 0.9), (-0.1, -0.1), (-0.8, -0.7)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here $T_3^c$ = (y, (0.3, 0.2), (0.7, 0.7), (-0.2, -0.1), (-0.7, -0.6)) is BPFRCS in Y and $\phi^{-1}(T_3^c)$ = (x, (0.3, 0.2), (0.7, 0.7), (-0.2, -0.1), (-0.7, -0.6)) is a BPFS in X. Now $\phi^{-1}(T_3^c) \subseteq T_1$ where $T_1$ is BPFROS in X and $\alpha cl(\phi^{-1}(T_3^c)) = T_1^c \subseteq T_1$. Therefore $\phi^{-1}(T_3^c)$ is a BPFR$\alpha$GCS in X but not a BPFRCS in X, since $T_3^c$ is BPFCS in Y but $cl(int(\phi^{-1}(T_3^c))) = T_1^c$. Thus $\phi$ is a BPFaR$\alpha$G continuous mapping but not the mapping in Theorem 4.14.

**Theorem 4.22:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. If $\phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta))$ for every BPFS $\delta$ in Y, then $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Proof:** Let $\delta$ be a BPFROS in Y. By hypothesis, $\phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta))$. Since $\delta$ is a BPFROS, it is a BPF$\alpha$OS in Y. Therefore $\alpha\ int(\delta) = \delta$. Hence $\phi^{-1}(\delta) = \phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\delta)$. Therefore $\phi^{-1}(\delta) = \alpha int(\phi^{-1}(\delta))$. This implies $\phi^{-1}(\delta)$ is a BPF$\alpha$OS in X and hence $\phi^{-1}(\delta)$ is a BPFR$\alpha$GOS in X. Thus $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Remark 4.23:** The converse of the above theorem 4.16 is true if $\delta$ is a BPFROS in Y and X is a BPFR$\alpha T_{1/2}$ space.

**Proof:** Let $\phi$ be a BPFaR$\alpha$G continuous mapping. Let $\delta$ be a BPFROS in Y. Then $\phi^{-1}(\delta)$ is a BPFR$\alpha$GOS in X. Since X is a BPFR$\alpha T_{1/2}$ space, $\phi^{-1}(\delta)$ is a BPF$\alpha$OS in X. This implies $\phi^{-1}(\delta) = \alpha int(\phi^{-1}(\delta))$.Now $\phi^{-1}(\alpha int(\delta)) \subseteq \phi^{-1}(\delta) = \alpha int(\phi^{-1}(\delta))$.Therefore $\phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta))$.

**Theorem 4.24:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. If $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\alpha cl(\delta))$ for every BPFS $\delta$ in Y, then $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Proof:** Let $\delta$ be a BPFRCS in Y. By hypothesis, $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\alpha cl(\delta))$. Since $\delta$ is a BPFRCS, it is a BPF$\alpha$CS in Y. Therefore $\alpha\ cl(\delta) = \delta$. Hence $\phi^{-1}(\delta) = \phi^{-1}(\alpha cl(\delta)) \supseteq \alpha cl(\phi^{-1}(\delta)) \supseteq \phi^{-1}(\delta)$. Therefore $\phi^{-1}(\delta) = \alpha cl(\phi^{-1}(\delta))$. This implies $\phi^{-1}(\delta)$ is a BPF$\alpha$CS in X and hence $\phi^{-1}(\delta)$ is a BPFR$\alpha$GCS in X. Thus $\phi$ is a BPFaR$\alpha$G continuous mapping.

**Remark 4.25:** The converse of the above theorem 4.18 is true if $\delta$ is a BPFRCS in Y and X is a BPFR$\alpha T_{1/2}$ space.

**Proof:** Let $\phi$ be a BPFaR$\alpha$G continuous mapping. Let $\delta$ be a BPFRCS in Y. Then $\phi^{-1}(\delta)$ is a BPFR$\alpha$GCS in X. Since X is a BPFR$\alpha T_{1/2}$ space, $\phi^{-1}(\delta)$ is a BPF$\alpha$CS in X. This implies $\alpha cl(\phi^{-1}(\delta)) = \phi^{-1}(\delta)$. Now $\phi^{-1}(\alpha cl(\delta)) \supseteq \phi^{-1}(\delta) = \alpha cl(\phi^{-1}(\delta))$.Therefore $\phi^{-1}(\alpha cl(\delta)) \supseteq \alpha cl(\phi^{-1}(\delta))$.

**Theorem 4.26:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where X is a BPFR$\alpha T_{1/2}$ space. If $\phi$ is a BPFaR$\alpha$G continuous mapping, then $cl(int(cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(\alpha cl(\delta))$ for every BPFRCS $\delta$ in Y.

**Proof:** Let $\delta$ be a BPFRCS in Y. By hypothesis, $\phi^{-1}(\delta)$ is a BPFR$\alpha$GCS in X. Since X is a BPFR$\alpha T_{1/2}$ space, $\phi^{-1}(\delta)$ is a BPF$\alpha$CS in X. This implies $\alpha cl(\phi^{-1}(\delta)) = \phi^{-1}(\delta)$. Now $cl(int(cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(\delta) \cup cl(int(cl(\phi^{-1}(\delta)))) \subseteq \alpha cl(\phi^{-1}(\delta)) = \phi^{-1}(\delta) = \phi^{-1}(\alpha cl(\delta))$, as every BPFRCS is a BPF$\alpha$CS. Hence $cl(int(cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(\alpha cl(\delta))$.

**Theorem 4.27:** Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where X is a BPFR$\alpha T_{1/2}$ space. If $\phi$ is a BPFaR$\alpha$G continuous mapping, then $\phi^{-1}(\alpha int(\delta)) \subseteq int(cl(int(\phi^{-1}(\delta))))$ for every BPFROS B in Y.

**Proof:** This theorem can be easily proved by taking complement in Theorem 4.26.

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