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Some Basic Properties of Near Plithogenic Neutrosophic Hypersoft Sets

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Abstract

The aim of the paper is to combine the plithogenic neutrosophic hypersoft set with the near set theory given by James. F. Peters and to introduce near plithogenic neutrosophic hypersoft set. Near plithogenic neutrosophic hypersoft sets are considered as mathematical tools for dealing with ambiguities. We first present the basic operations like union, intersection and some basic properties of the set. Also, some of the theoretical properties of the set are discussed.

Keywords: Near set; Hypersoft set; Plithogenic hypersoft set; Near plithogenic neutrosophic hypersoft set

1. Introduction

The rough sets were presented by Pawlak in order to solve the problem of uncertainty in 1982. In this set, the universal set is presented by the lower and upper approaches. This theory aimed to introduce some approaches into the sets. The theory of near sets was presented by James.F. Peters [1]. While Peters defines the nearness of objects, he is dependent on the nature of the objects, so he classifies the universal set according to the available information of the objects. Moreover, through entrenching the notions of rough sets, numerous applications of the near set theory have been enlarged and varied. Near sets and rough sets are like two sides of a coin, the only difference is the fact that what is focused on for rough sets is the approach of sets with nonempty boundaries.

The soft-set concept was developed by as a completely new math tool for solving difficulty in dealing with uncertainty. Molodtsov [2] defined a soft set that is sub-set as a parameterized family of the set of the universe. In the past few years, the fundamentals of soft set theory have been studied by different researchers.

Florentin Smarandache [3] generalized the soft set to hypersoft set by transforming the function F into a multi-argument function to deal with uncertainty, where one can have multiple parameters and so it can be used in several applications.

Florentin Smaradache[3,4] introduces the plithogenic set as a generalization of crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets. A plithogenic set is characterized by one or more parameters and each parameter may have several values.

2. Basic Definitions

Definition 2.1[1] Let U be the Global (Universal) set of objects, A, B⊆U and Þ be the set of all functions representing object features (probe functions), D⊆ Þ. Sets A and B are said to be near if a ϵA, b ϵB and α_i ϵ D, $1 \leq \alpha \leq n$ and $a \sim_{\{\alpha i\}} b$.

Table 1

Notations	Definition
$\sim_{Dq}$	$\sim_{Dq} = \{(a,b)/\ \alpha(a)= \alpha(b)$, for all $\alpha \in D_q\}$ (similarity relation)
$[a]_{Dq}$	$[a]_{Dq} = \{b \in U/a\sim_{Dq}b\}$ (equivalence class)
$U/\sim_{Dq}$	$U/\sim_{Dq}= \{[a]_{Dq}\ /a \in U\}$ (quotient set)
ρ_{Dq}	$\rho_{Dq}= U/\sim_{Dq}$ (partitions)
$\underline{\Gamma q(D)}(A)$	$\bigcup_{a:[a]Dq\subseteq A} [a]Dq$ (lower approximation)
$\overline{\Gamma q(D)}(A)$	$\bigcup_{a:[a]Dq\cap A\neq\emptyset} [a]Dq$ (upper approximation)
$B_{\Gamma q(D)}(A)$	$\overline{\Gamma q(D)}(A) - \underline{\Gamma q(D)}(A)$ (boundary)

Definition 2.2 [1] A nearness approximation space is a collection NAS= $(U,Þ, \sim_{Dq}, \Gamma_q, \zeta_{\Gamma q})$ where U represents the global set of objects, Þ denotes the probe functions, $\sim_{Dq}$ is the similarity relation on $D_q \subseteq D \subseteq$ Þ, Γ_q denotes the pile of partitions (collection of neighborhoods) and $\zeta_{\Gamma q}$ denotes the neighborhood overlap function.

The lower and upper near approximations of A with respect to NAS is given by,

$$\underline{\Gamma q(D)}(A) = \cup_{a:[a]Dq\subseteq A} [a]Dq \text{ and}$$

$$\overline{\Gamma q(D)}(A) = \cup_{a:[a]Dq\cap A\neq\emptyset} [a]Dq \text{ respectively}$$

The boundary of A with respect to NAS is given by, $B_{\Gamma q(D)}(A) = \overline{\Gamma q(D)}(A) - \underline{\Gamma q(D)}(A)$

If $B_{\Gamma q(D)}(A) \geq 0$, then A is a near set. [By Neighbourhoods Approximation Boundary Theorem].

Definition 2.3 [3] Let U be the global set of objects, P(U) the power set of U. Let $n_1,n_2,\ldots n_m$, $m\geq 1$ be the parameters whose values belong to the sets $N_1,N_2,\ldots N_m$ respectively and $N_i \cap N_j = \emptyset$, $i \neq j$, $i,j \in \{1,2,3,\ldots n\}$. Then the set $(F, N_1xN_2x\ldots N_m)$ where $F: N_1xN_2x\ldots N_m \longrightarrow P(U)$ is the hypersoft set (H_s) over U.

Definition 2.4 [4] Let U be the global set of objects, A⊆U and let $n_1,n_2,\ldots n_m$, $m\geq 1$ be the parameters, R be the range of values of the parameter and among the range of parameter values, there is a dominant attribute value d which is the most essential value that one is interested in. Also, let d_a be the degree of appurtenance of each parameter value to the set A and d_c is the degree of contradiction between values of the parameter.

Then the tuple (A,n_m,R,d_a, d_c) is the plithogenic set.

3. Basic Properties of Near Plithogenic Neutrosophic Hypersoft Set

Definition 3.1 Let U be the global set of objects, A⊆U and P(U) the power set of U. Let $n_1, n_2, \ldots n_m$, $m \geq 1$ be the parameters whose values belong to the sets $N_1, N_2, \ldots N_m$ respectively and $N_i \cap N_j = \emptyset$, $i \neq j$, $i,j \in \{1,2,3,\ldots n\}$, R be the range of values of the parameter, d_a be the degree of appurtenance of each parameter value to the set A and d_c be the degree of contradiction between values of the parameter.. Then the set (F_p, $N_1 x N_2 x \ldots N_m$) where F_p: $N_1 x N_2 x \ldots N_m \longrightarrow P(U)$ is the plithogenic hypersoft set (PH_s) over U.

Definition 3.2 Let U be the global set of objects, A⊆U, **Ω** be a plithogenic neutrosophic hypersoft set over U and NAS= (U, Þ, $\sim_{Dq}$, Γ_q, $\zeta_{\Gamma q}$) be the nearness approximation space. The lower and upper near approximations of **Ω** with respect to NAS is given by,

$\underline{\Gamma q(D)}(\Omega) = \cup_{\alpha:[a]Dq \subseteq \Omega} [a]Dq$ and

$\overline{\Gamma q(D)}(\Omega) = \cup_{\alpha:[a]Dq \cap \Omega \neq \emptyset} [a]Dq$

respectively. The boundary of Ω with respect to NAS is given by, $B_{\Gamma q(D)}(\Omega) = \overline{\Gamma q(D)}(\Omega) - \underline{\Gamma q(D)}(\Omega)$

If $B_{\Gamma q(D)}(\Omega) \geq 0$, then Ω is a near plithogenic neutrosophic hypersoft set.

Definition 3.3 The union of two near plithogenic neutrosophic hypersoft sets $Þ_A$ and Q_B is given by,

$T(\alpha) = T_Þ(\alpha)$ if $\alpha \in A-B$

$= T_Q(\alpha)$ if $\alpha \in B-A$

$= \max(T_Þ(\alpha), T_Q(\alpha))$ if $\alpha \in A \cap B$

$I(\alpha) = I_Þ(\alpha)$ if $\alpha \in A-B$

$= I_Q(\alpha)$ if $\alpha \in B-A$

$= (I_Þ(\alpha) + I_Q(\alpha))/2$ if $\alpha \in A \cap B$

$F(\alpha) = F_Þ(\alpha)$ if $\alpha \in A-B$

$= F_Q(\alpha)$ if $\alpha \in B-A$

$= \min(F_Þ(\alpha), F_Q(\alpha))$ if $\alpha \in A \cap B$

Definition 3.4 The intersection of two near plithogenic neutrosophic hypersoft sets $Þ_A$ and Q_B is given by,

$T(\alpha) = T_Þ(\alpha)$ if $\alpha \in A-B$

$= T_Q(\alpha)$ if $\alpha \in B-A$

$= \min(T_Þ(\alpha), T_Q(\alpha))$ if $\alpha \in A \cap B$

$I(\alpha) = I_Þ(\alpha)$ if $\alpha \in A-B$

$= I_Q(\alpha)$ if $\alpha \in B-A$

$= (I_Þ(\alpha) + I_Q(\alpha))/2$ if $\alpha \in A \cap B$

$F(\alpha) = F_Þ(\alpha)$ if $\alpha \in A\text{-}B$

$= F_Q(\alpha)$ if $\alpha \in B\text{-}A$

$= \max(F_Þ(\alpha), F_Q(\alpha))$ if $\alpha \in A \cap B$

Definition 3.5 Let $Þ_D$ be a near plithogenic neutrosophic hypersoft set in U. Then $Þ_D$ is said to be

i. a null near plithogenic neutrosophic hypersoft set, if $Þ(\alpha) = \emptyset, \forall \alpha \in D$.
ii. a whole near plithogenic neutrosophic hypersoft set, if $Þ(\alpha) = U, \forall \alpha \in D$.

Definition 3.6 Let $Þ_D$ be a near plithogenic neutrosophic hypersoft set in U. Then $Þ_D{}^C$ is said to be the complement of $Þ_D$, where $Þ^c(\alpha) = U - Þ(\neg\alpha)\ \forall \alpha \in \neg D$ (i.e., $\alpha \in \neg N_1xN_2xN_3x\ldots xN_m$).

Definition 3.7 Let $Þ_D$ be a near plithogenic neutrosophic hypersoft set in U. Then $Þ^C{}_D$ is said to be the relative complement of $Þ_D$, where $Þ^c(\alpha) = U - Þ(\neg\alpha)\ \forall \neg\alpha \in D$ (i.e., $\neg\alpha \in N_1xN_2xN_3x\ldots xN_m$).

Definition 3.8 Let $Þ_D$ and Q_D be two near plithogenic neutrosophic hypersoft sets in U. If $Þ_D \subseteq Q_D$, then $Þ_D$ is a near plithogenic neutrosophic hypersoft subset of Q_D, if $\underline{\Gamma q(Þ(\alpha))} \subseteq \underline{\Gamma q(Q(\alpha))}\ \ \forall \alpha \in D$.

If $Þ_D \supseteq Q_D$, i.e., $Þ_D$ is called a near plithogenic neutrosophic hypersoft superset of Q_D, if Q_D is a near plithogenic neutrosophic hypersoft subset of $Þ_D$.

Definition 3.9 Let $Þ_D$ and Q_D be two near plithogenic neutrosophic hypersoft sets in U. If $Þ_D$ and Q_D are near plithogenic neutrosophic hypersoft subsets of each other, then they are equal, (i.e.,) $Þ_D = Q_D$.

Definition 3.10 Let $Þ_D$ and Q_D be two near plithogenic neutrosophic hypersoft sets. Then

1. $(Þ_D \cup Q_D)^C = Þ^C{}_D \cap Q^C{}_D$.

2. $(Þ_D \cap Q_D)^C = Þ^C{}_D \cup Q^C{}_D$.

Proposition 3.11

Let $Þ_D, Q_D, R_D, S_D$ be near plithogenic neutrosophic hypersoft sets of U. Then the following holds

1. $Þ_D \cap \emptyset = \emptyset$.

2. $Þ_D \cap U = Þ_D$.

3. $Þ_D \cup \emptyset = Þ_D$.

4. $Þ_D \cup U = U$.

5. $Þ_D \subseteq Q_D$ iff $Þ_D \cap Q_D = Þ_D, \forall \alpha \in D$.

6. $Þ_D \subseteq Q_D$ iff $Þ_D \cup Q_D = Q_D$, $\forall \alpha \in D$.

7. If $Þ_D \cap Q_D = \emptyset$, then $Þ_D \subseteq Q^c{}_D$.

8. $Þ_D \cup Þ^c{}_D = U$.

9. If $Þ_D \subseteq Q_D$ and $Q_D \subseteq R_D$, then $Þ_D \subseteq R_D$.

10. If $Þ_D \subseteq Q_D$ and $R_D \subseteq S_D$, then $Þ_D \cap R_D \subseteq Q_D \cap S_D$.

11. $Þ_D \subseteq Q_D$ iff $Q^c{}_D \subseteq Þ^c{}_D$.

Proof

Proofs of 1-4 and 8-10 are straight forward

5) Consider $Þ_D \subseteq Q_D$

$\Rightarrow \underline{\Gamma(\text{Þ}(\alpha))} \subseteq \underline{\Gamma(Q(\alpha))}\ \forall\ \alpha \in D$.

Let $\text{Þ}_D \cap Q_D = R_D$

Since $R(\alpha) = \text{Þ}(\alpha) \cap Q(\alpha)$

$= \text{Þ}(\alpha)\ \forall\ \alpha \in D$

$\Rightarrow \text{Þ}_D \cap Q_D = \text{Þ}_D$

Conversely, Consider $\text{Þ}_D \cap Q_D = \text{Þ}_D$

Let $\text{Þ}_D \cap Q_D = R_D$

Since $R(\alpha) = \text{Þ}(\alpha) \cap Q(\alpha)$

$= \text{Þ}(\alpha)\ \forall\ \alpha \in D$

We know that, $\underline{\Gamma(\text{Þ}(\alpha))} \subseteq \underline{\Gamma(Q(\alpha))}\ \forall\ \alpha \in D$

Thus $\text{Þ}_D \subseteq Q_D$.

6) Proof of (6) is similar to (5)

7) Consider $\text{Þ}_D \cap Q_D = \emptyset$

$\Rightarrow \text{Þ}(\alpha) \cap Q(\alpha) = \emptyset$

$\underline{\Gamma(\text{Þ}(\alpha))} \subseteq \underline{\Gamma(\mathbf{U} - Q(\alpha))}$

$\subseteq \Gamma(Q^{c}(\alpha))$

$\Rightarrow \text{Þ}(\alpha) \subseteq Q^{c}(\alpha)$

$\Rightarrow \text{Þ}_D \subseteq Q^{c}_D$

11) $\text{Þ}_D \subseteq Q^{c}_D \Leftrightarrow \underline{\Gamma(\text{Þ}(\alpha))} \subseteq \underline{\Gamma(Q(\alpha))}$

$\Leftrightarrow Q(\alpha)^{c} \subseteq \text{Þ}(\alpha)^{c}\ \forall\ \alpha \in D$.

$\Leftrightarrow Q^{c}(\alpha) \subseteq \text{Þ}^{c}(\alpha)\ \forall\ \alpha \in D$.

$\Leftrightarrow Q^{c}_D \subseteq \text{Þ}^{c}_D\ \forall\ \alpha \in D$.

Theorem 3.12

Let I be the index set and Þ_{Di}, $\forall\ i \in I$ be a near plithogenic neutrosophic hypersoft set. Then,

I. $[\bigcup_{i\in I} \text{Þ}_{D_i}]^c = \bigcap_{i\in I} \text{Þ}_{D_i}{}^c$
II. $[\bigcap_{i\in I} \text{Þ}_{D_i}]^c = \bigcup_{i\in I} \text{Þ}_{D_i}{}^c$

Proof

I. Consider $\left[\bigcup_{i\in I} \text{Þ}_{D_i}\right]^c$

$[\bigcup_{i\in I} \text{Þ}_{D_i}]^c = U - \bigcup_{i\in I} \text{Þ}_i(\alpha)$

$= \bigcap_{i\in I}[U - \text{Þ}_i(\alpha)],\ \forall\ \alpha \in D$ …….(1)

Consider $\bigcap_{i\in I} \text{Þ}_{D_i}{}^c$

$\bigcap_{i\in I} \text{Þ}_{D_i}{}^c = \bigcap_{i\in I}[U - \text{Þ}_i(\alpha)],\ \forall\ \alpha \in D$ …….(2)

From (1) and (2), $[\bigcup_{i\in I} Þ_{D_i}]^c = \bigcap_{i\in I} Þ_{D_i}{}^c$

II. Consider $[\bigcap_{i\in I} Þ_{D_i}]^c$

$[\bigcap_{i\in I} Þ_{D_i}]^c$ = U $- \bigcap_{i\in I} Þ_i(\alpha)$

$= \bigcup_{i\in I}[U - Þ_i(\alpha)]$, $\forall$ $\alpha \in$ D …….(3)

Consider $\bigcup_{i\in I} Þ_{D_i}{}^c$

$\bigcup_{i\in I} Þ_{D_i}{}^c = \bigcup_{i\in I}[U - Þ_i(\alpha)]$, $\forall$ $\alpha \in$ D …….(4)

From (3) and (4), $[\bigcap_{i\in I} Þ_{D_i}]^c = \bigcup_{i\in I} Þ_{D_i}{}^c$.

4. Conclusion

Thus, in this paper some basic properties of the near plithogenic neutrosophic hypersoft set was studied. Many theoretical properties and results of the set and the topology are to be defined in future.

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