IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Discussion of The Lost Sales Case of A Probabilistic Fixed Order Interval System and Solution By Intuitionistic Fuzzy Geometric Programming Technique

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Abstract: We apply Intuitionistic Fuzzy Optimization[IFO] technique as well as Intuitionistic Fuzzy Geometric Programming[IFGP] Technique to solve the problem in the lost sales case .This paper models a multi-objective Economic Order Interval [EOI] system with fuzzy cost components, where the demand during lead-time and order interval follows uniform probability distribution. Our objective is to establish the fact that IFGP method performs better than IFO method.

Index Terms: Fixed order interval, Intuitionistic Fuzzy Geometric Programming, fuzzy cost components, stochastic inventory.

1. Introduction

Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that, IFS can be used to simulate human decision-making process and any activitities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity (1986). Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov(1989) discussed an Open problems in intuitionistic fuzzy sets theory. An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov(1999). Atanassov and Kreinovich(1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are discussed also by Atanossov (1999). Intuitionistic fuzzy soft sets are considered by Maji, Biswas and Roy (2001). Nikolova, Nikolov, Cornelis and Deschrijver(2002) presented a Survey of the research on intuitionistic fuzzy sets. Rough intuitionistic fuzzy sets are analyzed by Rizvi, Naqvi and Nadeem(2002).

There are many references on application and the methods of GP in the survey papers (like Eckar (1980), Beightler et.al. (1979), Zener (1971). Hariri et. al. (1997) discussed the multi-item production lot-size inventory model with varying order cost under a restriction Jung and Klain (2001) developed single item inventory problems and solved by GP method. Ata Fragany and Wakeel (2003) considered some inventory problems solved by GP technique.

Zadeh (1965) first gave the concept of fuzzy set theory. Later on Bellman and Zadeh (1970) used the fuzzy set theory to the decision making problem Tanaka (1974) introduced the objective as fuzzy goal over the α-cut of a fuzzy constraint set and Zimmerman (1978) gave the concept to an inventory and production problem. Jeddi, Shultes and Haji (2004) considered a multi-product continuous review inventory system with stochastic demand, backorders and a budget constraint.

In this paper, a stochastic inventory model with fuzzy cost components is discussed here. In this Economic Order Interval System Intuitionistic fuzzy geometric programming technique performs better than Intuitionistic fuzzy Optimization technique.

2. Mathematical Model

The basic problem in this system is determining the order interval T and the desired maximum inventory level E. The economic order interval can be obtained by the minimization of the total annual cost. If stockouts are not permitted, Figure-1 and the following formula give the total annual inventory cost:

Total annual cost = (purchase cost) + (order cost) + (holding cost)

$$TC = RP + mC + \frac{RFP}{2m} = RP + \frac{C}{T} + \frac{RFPT}{2}$$

$m = \frac{1}{T}$ = Number of orders or reviews per year

$\frac{R}{2m} = \frac{RT}{2}$ = Average inventory in units

$T = \frac{1}{m}$ = Order interval in years

In retailing and wholesaling, a separate order is rarely placed for each item. All items from the same source are likely to be listed together on a single order. Frequently, a supplier provided numerous items and it is economical to have joint orders. In a joint order, many items are ordered from the same source or supplier. The quantity of each item to order depends on the same time interval between orders for the entire group. Thus for Economic Order Interval system of multiple items, the basic problem is determining the order interval T and the desired maximum inventory level E_i for each item.

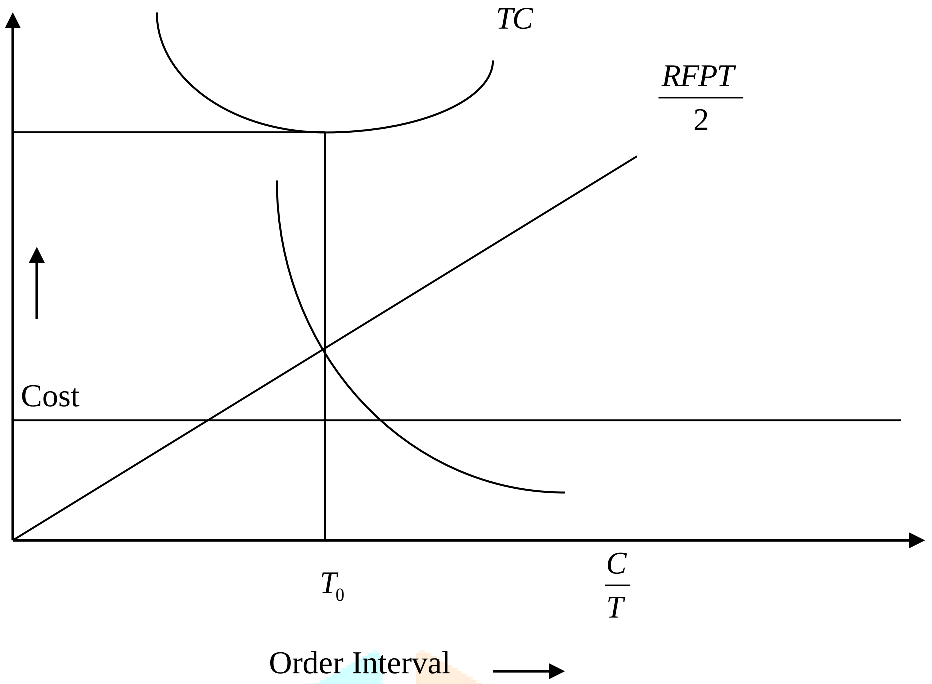

Figure – 1: Annual inventory cost

The economic order interval can be obtained by minimizing the total annual cost. Neglecting stockout cost, the formulation is as follows:

Total annual cost = (purchase cost) + (order cost) + (holding cost)

$$TC = \sum_{i=1}^{n} R_i P_i + \frac{(C+nc)}{T} + \frac{1}{2} TF \sum_{i=1}^{n} R_i P_i$$

R_i = Annual requirement for item i

P_i = Purchase cost of item i

n = Total number of joint order item

C = Order cost for the joint order

c = Order cost associated with each individual item

T = Order interval in years

F = Holding cost as a fraction of purchase cost

The expected annual cost of operating the fixed order interval for a single item with constant lead-time is as follows:

Expected annual cost=(purchase cost)+(order cost)+(holding cost)+(stockout cost)

$$TC(E,T) = RP + \frac{C}{T} + H(E - (\frac{RT}{2} + RL)) + \frac{S}{T} \int_{E}^{\infty} (M-E) f(M) dM$$

$$0 < T \le 1$$

where,

TC = expected annual cost

R = average annual demand in units

P = unit price

C = ordering and review cost per occurrences

H = holding cost per unit per year

E = maximum inventory level in units

T = order interval in years

L = lead-time in years

S = stockout cost per unit

M = demand during order interval and lead-time

$\overline{M}$ = RT+RL=average demand during order interval and lead-time

f(M) = probability density function of demand during order interval and lead-time

$$E(M > E) = \int_{E}^{\infty} (M - E) f(M) dM$$

= Expected stockout quantity during order interval.

2.1 Lost Sales Case

This case is slightly different than backorder case. Only difference is the calculation of safety stock. In case of backordering the safety stock is assumed to be E-$\overline{M}$, but in the lost sales case safety stock also includes the expected number of stockouts. Thus, the safety stock is slightly higher in this case than backorder case. The safety stock is determined by the following expression:

$$(E-\overline{M}) + \int_{E}^{\infty} (M - E) f(M) dM$$

The appropriate mathematical notation for the lost sales case with stockout cost per unit is as follows:

Annual cost of safety stock = holding cost + stockout cost

$$TC(E,T) = H(E - \overline{M}) + (H + \frac{S}{T}) \int_{E}^{\infty} (M - E) f(M) dM$$

$$0 < T \le 1$$

2.2 Stochastic model

Demand follows Uniform distribution

We assume that demand during order interval and lead-time for the period is a random variable which follows uniform distribution and if the decision maker feels that demand values below a or above b are highly unlikely and values between a and b are equally likely, then the probability density function of demand during order interval and lead-time $f(M)$ is given by:

$$f(M)=\begin{cases}\dfrac{1}{b-a} & \text{if a } \leq M \leq b \\ 0 & \text{otherwise}\end{cases}$$

So, $$\int_E^{\infty}(M-E)f(M)dM=\frac{(b-E)^2}{2(b-a)} \qquad ...(1)$$

2.3 Multi-Objective Stochastic Inventory Model [MOSIM] with Fuzzy Cost Coefficients

$$MinTC_i(E_i,T_i)=\tilde{H}_i(E_i-\overline{M}_i)+(\tilde{H}_i+\frac{\tilde{S}_i}{T_i})\int_{E_i}^{\infty}(M-E_i)f_i(M)dM) \qquad ...(2)$$

$E_i,T_i>0,\forall i=1,2,.....,n$

$T_i \leq 1$

Here, $\tilde{H}_i=(H_i^-,H_i^0,H_i^+)$ and $\tilde{S}_i=(S_i^-,S_i^0,S_i^+)$ are two triangular fuzzy numbers.

3. Geometric Programming Problem

Geometric Programming (GP) can be considered to be an innovative modus operandi to solve a nonlinear problem in comparison with other nonlinear techniques. It was originally developed to design engineering problems. It has become a very popular technique since its inception in solving nonlinear problems. The advantages of this method is that, this technique provides us with a systematic approach for solving a class of nonlinear optimization problems by finding the optimal value of the objective function and then the optimal values of the design variables are derived, also. This method often reduces a complex nonlinear optimization problem to a set of simultaneous equations and this approach is more amenable to the digital computers.

GP is an optimization problem of the form:

$Min\ g_0(t)$ …(3)

subject to

$g_j(t) \leq 1,$

j = 1, 2, ………, m.

$h_k(t)=1,$ k=1, 2, ……….., p

$t_i > 0,$ i = 1, 2, ………., n

where, $g_j(t)$ (j = 1, 2, ………, m) are posynomial or signomial functions and $h_k(t)$ (k=1, 2, ……….., p) are monomials t_i (i = 1, 2, ………., n) are decision variable vector of n components.

The problem (3) can be written as:

$Min\ g_0(t)$

subject to

$g'_j(t) \le 1$, j = 1, 2, ………, m.

t > 0, [since $g_j(t) \le 1,\ h_k(t) = 1 \Rightarrow g'_j(t) \le 1$ where $g'_j(t)(= g_j(t)/h_k(t))$ be a posynomial (j=1, 2, ………, m ; k=1, 2, ………, p)].

3.1 Fuzzy Geometric Programming Problem

Multi-objective geometric programming (MOGP) is a special type of a class of MONLP problems. Biswal (1992) and Verma (1990) developed a fuzzy geometric programming technique to solve a MOGP problem. Here, we have discussed a fuzzy geometric programming technique based on max-min and max-convex combination operators to solve a MOGP problem.

To solve the MOGP problem we use the Zimmerman's technique. The procedure consists of the following steps.

Step 1. Solve the MOGP problem as a single GP problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions. Repeat the process k times for k different objectives. Let $x^1, x^2, \ldots\ldots, x^k$ be the ideal solutions for the respective objective functions, where $x^r = (x_1^r, x_2^r, \ldots\ldots, x_n^r)$

Step 2. From the ideal solutions of Step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each solution, the pay-off matrix of size (k x k) can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \cdots \\ x^k \end{array}
\begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \cdots\cdots & f_k(x) \end{array} \\
\begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \cdots & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \cdots & f_k(x^2) \\ \cdots & \cdots & \cdots & \cdots \\ f_1(x^k) & f_2(x^k) & \cdots & f_k^*(x^k) \end{bmatrix} \end{array}$$

Step 3. From the Step 2, find the desired goal L_r and worst tolerable value U_r of $f_r(x)$, r = 1, 2, …….., k as follows:

$L_r \le f_r \le U_r$, r = 1, 2, …….., k

Where, $U_r = \max\{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

$L_r = \min\{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

Step 4. Define a fuzzy linear or non-linear membership function $\mu_r[f_r(x)]$ for the r-th objective function $f_r(x)$, r = 1, 2, …….., k

$$\mu_r[f_r(x)] = \begin{cases} 0 \text{ or } \to 0 & \text{if } f_r(x) \ge U_r \\ d_r(x) & \text{if } L_r \le f_r(x) \le U_r \ (r = 1, 2, \ldots\ldots, k) \\ 1 \text{ or } \to 1 & \text{if } f_r(x) \le L_r \end{cases}$$

Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

Step 5. At this stage, either a max-min operator or a max-convex combination operator can be used to formulate the corresponding single objective optimization problem.

3.2 Through a Max-Min operator

According to Zimmerman (1978) the problem can be solved as:

$$\mu_D(x^*) = Max(Min(\mu_1(f_1(x)), \mu_2(f_2(x)),........, \mu_k(f_k(x))))$$

subject to

$g_j(x) \leq b_j$, j=1, 2, ….., m, $x > 0$

which is equivalent to the following problem as:

Max α …(4)

Subject to

$\alpha \leq \mu_r[f_r(x)]$, for r = 1, 2, …….., k

$g_j(x) \leq b_j$, j=1, 2, ….., m, $x > 0$

4. Mathematical Analysis

4.1 Formulation of Intuitionistic Fuzzy Optimization [IFO]

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and constraints and to minimize the degree of rejection of IF objectives and constraints, we can write:

$$\max\ \mu_i(\overline{X}), \overline{X}\varepsilon R, i = 1,2,......, K+n$$
$$\min\ \upsilon_i(\overline{X}), \overline{X}\varepsilon R, i = 1,2,......, K+n$$

Subject to

$$\upsilon_i(\overline{X}) \geq 0,$$
$$\mu_i(\overline{X}) \geq \upsilon_i(\overline{X})$$
$$\mu_i(\overline{X}) + \upsilon_i(\overline{X}) < 1$$
$$\overline{X} \geq 0$$

Where $\mu_i(\overline{X})$ denotes the degree of membership function of $(\overline{X})$ to the i^{th} IF sets and $\nu_i(\overline{X})$ denotes the degree of non-membership (rejection) of $(\overline{X})$ from the i^{th} IF sets.

4.2 An Intuitionistic Fuzzy Approach for Solving MOIP with Linear Membership and Non-Membership Functions

To define the membership function of MOIM problem, let L_k^{acc} and U_k^{acc} be the lower and upper bounds of the k^{th} objective function. These values are determined as follows: Calculate the individual minimum value of each objective function as a single objective IP subject to the given set of constraints. Let $\overline{X}_1^*, \overline{X}_2^*, ...\overline{X}_k^*$ be the respective optimal solution for the k different objective and evaluate each objective function at all these k optimal solution. It is assumed here that at least two of these solutions are different for which the k^{th} objective function has different bounded values. For each objective, find lower bound (minimum value) L_k^{acc} and the upper bound (maximum value) U_k^{acc}. But in intuitionistic fuzzy optimization (IFO), the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To define membership function of MOIM problem, let L_k^{rej} and U_k^{rej} be the lower and upper bound of the objective function $Z_k(\bar{X})$ where $L_k^{acc} \le L_k^{rej} \le U_k^{rej} \le U_k^{acc}$. These values are defined as follows:

The linear membership function for the objective $Z_k(\bar{X})$ is defined as:

$$\mu_k(Z_k(\bar{X})) = \begin{cases} 1 & \text{if } Z_k(\bar{X}) \le L_k^{acc} \\ \dfrac{U_k^{acc} - Z_k(\bar{X})}{U_k^{acc} - L_k^{acc}} & \text{if } L_k^{acc} \le Z_k(\bar{X}) \le U_k^{acc} \\ 0 & \text{if } Z_k(\bar{X}) \ge U_k^{acc} \end{cases} \quad \text{.... (5)}$$

$$\nu_k(Z_k(\bar{X})) = \begin{cases} 1 & \text{if } Z_k(\bar{X}) \ge U_k^{rej} \\ \dfrac{Z_k(\bar{X}) - L_k^{rej}}{U_k^{rej} - L_k^{rej}} & \text{if } L_k^{rej} \le Z_k(\bar{X}) \le U_k^{rej} \\ 0 & \text{if } Z_k(\bar{X}) \le L_k^{rej} \end{cases} \quad \text{.... (6)}$$

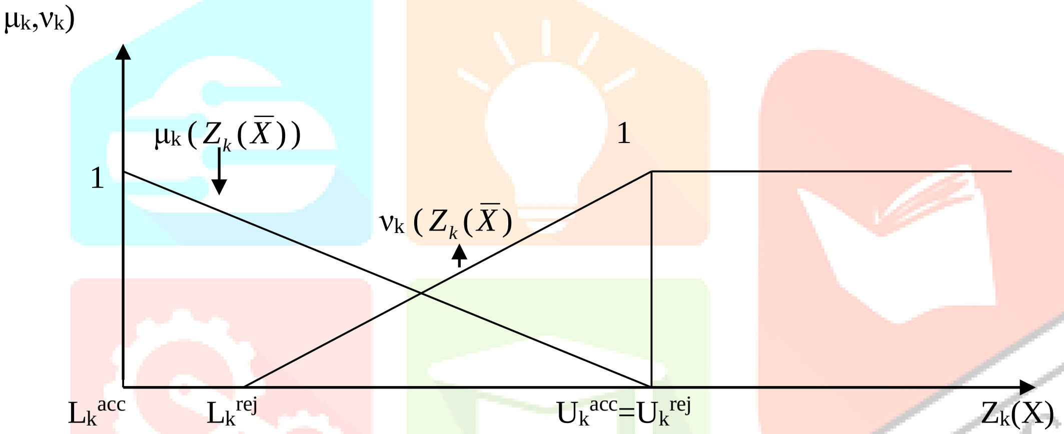

Figure- 2: Membership and non-membership functions of the objective goal

Lemma: In case of minimization problem, the lower bound for non-membership function (rejection)) is always greater than that of the membership function (acceptance).

Now, we take new lower and upper bound for the non-membership function as follows:

$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc})$ where $0 < t < 1$

$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc})$ for $t = 0$

Following the fuzzy decision of Bellman-Zadeh (1970) together with linear membership function and non-membership functions of (5) and (6), an intuitionistic fuzzy optimization model of MOIM problem can be written as:

$$\begin{aligned} &\max\ \mu_k(\bar{X}),\ \bar{X} \varepsilon R,\ k = 1,2,......,K \\ &\min\ \upsilon_k(\bar{X}),\ \bar{X} \varepsilon R,\ k = 1,2,......,K \end{aligned} \quad \text{....(7)}$$

Subject to

$\upsilon_k(\bar{X}) \geq 0,$

$\mu_k(\bar{X}) \geq \upsilon_k(\bar{X})$

$\mu_k(\bar{X}) + \upsilon_k(\bar{X}) < 1$

$\bar{X} \geq 0$

The problem of equation (7) can be reduced following Angelov (1997) to the following form:

$$Max\ \alpha - \beta \qquad \text{....(8)}$$

Subject to

$Z_k(\bar{X}) \leq U_k^{acc} - \alpha(U_k^{acc} - L_k^{acc})$

$Z_k(\bar{X}) \leq L_k^{rej} + \beta(U_k^{rej} - L_k^{rej})$

$\beta \geq 0$

$\alpha \geq \beta$

$\alpha + \beta < 1$

$\bar{X} \geq 0$

Then the solution of the MOIM problem is summarized in the following steps:

Step 1. Pick the first objective function and solve it as a single objective IP subject to the constraint, continue the process K-times for K different objective functions. If all the solutions (i.e. $\bar{X}_1^* = \bar{X}_2^* = ... = \bar{X}_k^*$) (k = 1,2,.....,K) same, then one of them is the optimal compromise solution and go to step 6. Otherwise go to step 2. However, this rarely happens due to the conflicting objective functions.

Then the intuitionistic fuzzy goals take the form

$Z_k(\bar{X}) \tilde{\leq} L_k(\bar{X})^*{}_k\ k = 1,2,........,K.,$

Step 2. To build membership function, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step 1, we find the values of all the objective functions at each ideal solution and construct pay off matrix as follows:

$$\begin{bmatrix} Z_1(\bar{X}_1^*) & Z_2(\bar{X}_1^*) & .... & .... & .... & Z_k(\bar{X}_1^*) \\ Z_1(\bar{X}_2^*) & Z_2(\bar{X}_2^*) & .... & .... & .... & Z_k(\bar{X}_2^*) \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ Z_1(\bar{X}_k^*) & Z_2(\bar{X}_k^*) & .... & .... & .... & Z_k(\bar{X}_k^*) \end{bmatrix}$$

Step 3. From Step 2, we find the upper and lower bounds of each objective for the degree of acceptance and rejection corresponding to the set of solutions as follows:

$$U_k^{acc} = \max_{1 \leq r \leq k}(Z_k(\bar{X}_r^*)) \quad \text{and} \quad L_k^{acc} = \min_{1 \leq r \leq k}(Z_k(\bar{X}_r^*))$$

For linear membership functions,

$$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ where } 0 < t < 1$$

$$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ for } t = 0$$

Step 4. Construct the fuzzy programming problem of equation (7) and find its equivalent NLP problem of equation (8).

Step 5. Solve equation (8) by using appropriate mathematical programming algorithm to get an optimal solution and evaluate the K objective functions at these optimal compromise solutions

Step 6. STOP.

5. Numericals

To solve the model (2) by Intuitionistic Fuzzy Optimization Technique[IFO] and Intuitionistic Fuzzy Geometric Programming Technique[IFGP] when probability density function of demand during order interval and lead-time follows uniform distribution, we consider the following data:

$\tilde{H}_1$=\$(10,12,14); a_1=10; b_1=30; $\tilde{S}_1$=\$(13,15,17);

$\tilde{H}_2$=\$(12,15,18); a_2=15; b_2=40; $\tilde{S}_2$=\$(17,20,23).

Now, the following results are obtained in Table-1.

Intuitionistic Fuzzy Optimization [IFO] Technique and Intuitionistic Fuzzy Geometric Programming Technique to solve the model ()

METHODS	E_1^*	E_2^*	T_1^*	T_2^*	TC_1^*(\$)	TC_2^*(\$)	ASPIRATION LEVEL
IFO	125	115	0.50	0.70	7819	4527	α=0.71
IFGP	120	111	0.49	0.71	7712	4486	α=0.74

Table-1

Analyzing the above Tables and Figures the following observations can be made:

From the Table-1 we conclude that, Intuitionistic Fuzzy Geometric Programming Technique [IFGP] obtained more minimized values of TC_1 and TC_2, in comparison to Intuitionistic Fuzzy Optimization Technique [IFO].

6. Conclusion

In this paper, our objective is to establish the better performance of the Intuitionistic Fuzzy Geometric Programming Technique i.e. to prove that Intuitionistic Fuzzy Geometric Programming Technique optimizes the objective function more than the usual Intuitionistic Fuzzy Optimization Technique. This stochastic model can also be analyzed in case of normal distribution, exponential distribution etc..

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