



Optimal Placement of Micro-PMUs in Reconfigurable Smart Distribution System

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Abstract: Monitoring of Distribution networks is the need of the day that requires high degree of precision owing to the dynamic nature of the networks. These networks are very complex in terms of number of nodes, their interconnections, closely distributed voltage levels and angles and also lack of proper documentation. A highly accurate monitoring system needs to be in place to assess the situation and act dynamically in times of emergencies. In order to achieve this, micro-phasor measurement unit (μ PMU) is being used in distribution network environment. These devices being complex and costly, optimal placement plays a vital role in the economics of monitoring system. In this work, Depth First Search algorithm has been used to determine preferred locations of placing μ PMUs and the remaining are optimally determined by Particle Swarm Optimization (PSO). The location of μ PMUs at buses with distributed generation and buses adjacent to radial buses makes it different from the algorithms used for PMU allocation in transmission systems. System Observability Redundancy Index (SORI) is used to determine the optimal solution out of all possible PMU placement sets. The sets having same number of μ PMUs with high SORI value is being chosen as the optimal set. The optimal μ PMU placement problem is implemented on IEEE 33 bus radial distribution feeder and on IEEE 85 bus radial distribution test feeders integrated with distributed energy resources. The optimal placement strategies of PMU are also examined for reconfigured distribution systems that aim at reducing the power loss in the system.

Index Terms - Micro-phasor measurement unit, Depth First Search, Particle Swarm Optimization, System Observability Redundancy Index.

I. INTRODUCTION

The distribution systems in recent times are undergoing fast changes with respect to their composition and operation as well. The large-scale integration of Distributed Energy Resources (DERs), Electric Vehicles (EVs) and Demand Response (DR) has resulted in complex configuration of the systems. Consequently, small disturbance would result in large uncertainties and also bidirectional power flows in the network. All these processes must be precisely monitored and controlled to ensure the integrity of the system. Presently, Supervisory Control and Data Acquisition (SCADA) system is being used for monitoring purposes. The SCADA system gathers information from Remote Terminal Units (RTUs) and has a provision to measure voltage, current, real power and reactive powers. But this SCADA systems suffers from a short coming that the resolution is low and is insufficient to capture the complex dynamics of the present distribution system.

The Phasor Measurement Unit (PMU) is a device developed for monitoring and controlling of power systems. It does so by providing time synchronized measurements of voltages, phase angles, currents, power factor, powers, etc with the help of Global Positioning System (GPS). There is very high resolution and accuracy, making it suitable for real-time monitoring. Usually, PMUs are employed in transmission system owing to their higher cost. Monitoring of Distribution networks is the need of the day that requires high degree of precision owing to the dynamic nature of the networks. These networks are very complex in terms of number of nodes, their interconnections, closely distributed voltage levels and angles and also lack of proper documentation. A highly accurate monitoring system needs to be in place to assess the situation and act dynamically in times of emergencies. In order to achieve this, micro-phasor measurement unit (μ PMU) or Distribution-level PMU (D-PMU) is being used in distribution network environment. The accuracy of μ PMU angle is close to $\pm 0.01^\circ$, precision is near $\pm 0.05\%$, the resolution of angle is about $\pm 0.002^\circ$, and that of magnitude is $\pm 0.0002\%$. The range of number of samples per second for a 60 Hz system is 10-120. Owing to all these features, μ PMU devices are preferred in distribution networks. These devices being complex and costly, optimal placement plays a vital role in the economics of monitoring system (Milosevic, 2003, Xu, 2004 and Dua, 2008).

II. LITERATURE REVIEW

The usage of μ PMUs for distribution networks has been reviewed in (Gou, 2008). Many aspects like uses of PMUs, monitoring and controlling functions etc are described here, as the PMUs help in improving reliability and security of the system. The optimal placement of μ PMUs becomes very crucial in economic view point, and hence needs advanced optimization algorithms to ensure that all the objectives are achieved. Literature presents various techniques used for this placement problem.

Genetic Algorithm (GA) is used for optimal placement in (Marin, 2003 and Aminifar, 2009). Graph Theory along with simple GA has been used by authors in (Milosevic, 2003) for optimal placement solutions during normal and contingent situations of the system. Desired depth of observability concept is proposed in (Borghetti, 2011) that makes use of spanning trees for the optimal placement problem. Simulated Annealing (SA) is the tool used for optimization purpose. Dual Bisecting Search algorithm (DBS) and SA methods were used in (Zhou, 2015) for the optimal placement problem. Tabu Search (TS) based method is used in (Peng, 2006) for the purpose. The minimum condition number of the normalized measurement matrix is used as basis in (Aminifar, 2010 and Pregararo, 2012) for optimal PMU placement. All the above works use meta-heuristic algorithms, but these took longer time for convergence, especially for large systems.

The use of μ PMUs for distribution networks have been investigated as mentioned above using various conventional optimization methods as well as heuristic algorithms. However, the need for time efficient algorithm still persists specially for larger systems.

III. OBSERVABILITY RULES AND RECONFIGURABLE SYSTEMS

The PMU placement model comprises of buses, branches and injections. Bus refers to a substation where a μ PMU can be placed. A branch is the link between two connecting buses whose impedance is given. An injection is a varying load or source that supplies current to the bus it is connected to. This model is a simple version of the actual distribution system considering only topology of the system, that is adequate to determine its observability.

The following are the set of rules required to determine the observability of a system. The basis for these rules is that a μ PMU can take measurements from the bus it is placed at and also can assess the measurements of connected branches and buses using the Ohm's law (Srilatha, 2021).

3.1 Observability Rules

- 1. A μ PMU installed bus can measure its own voltage phasor and currents phasors of all the connecting branches.

A μ PMU is installed at Bus 1. The measurements that are possible with this are V_1 , I_{12} , I_{13} , and I_{41} can be measured directly by the PMU as shown in Figure 1.

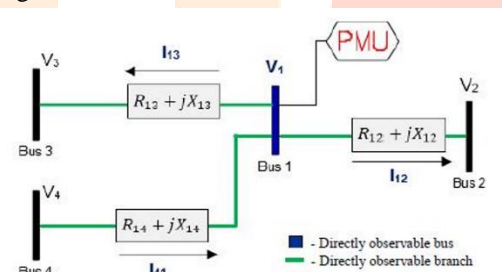


Figure 1. Description of observability rule 1

- 2. In case voltage at a bus and current in connecting branch is known, the voltage at the other end of this branch can be calculated.

From Figure 2, with the known values of I_{12} , I_{13} , and I_{41} , the voltages V_2 , V_3 and V_4 are calculated.

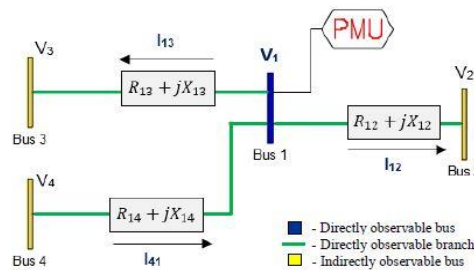


Figure 2. Description of observability rule 2

- 3. If the values of voltages at two interconnected buses are known, the connecting branch current can be calculated.

If the value of V_1 and V_2 are known, the interconnecting branch current I_{12} is calculated by using the Ohm's law.

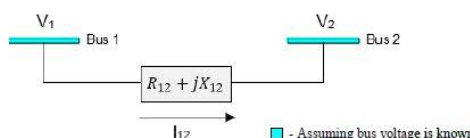


Figure 3. Description of observability rule 3

4. If power flow through a branch is known along with voltage at one of its end buses, the voltage of the bus at another end of the branch can be calculated.

In case the power flow measurements are placed in branches 1-3 and 2-3 and a PMU is placed at bus 4, the voltages at buses 1 and 2 can be calculated using the voltage at bus 3 as depicted in Figure 4.

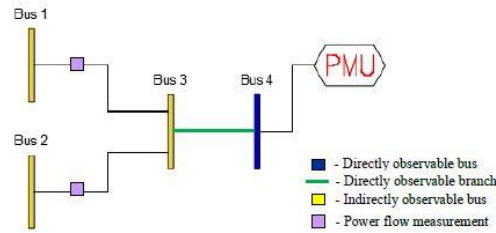


Figure 4. Description of observability rule 4

3.2 Reconfigurable Distribution Systems

Reconfiguration process aims to determine a configuration that improves the system voltage profile and minimizes the system power losses. The reconfiguration procedure is performed by modifying the feeder topological structure to efficiently manage the switching status of sectionalizing and tie-switches in the system. Sectionalizing switches are Normally closed switches and Tie switches are Normally Open switches.

IV. PROBLEM FORMULATION

μ PMU placement is of prime importance in power systems as the cost associated with it is very high and at the same time observability of the system needs to be ensured. Total observability of a system implies the voltages at all the buses and current through all the branches must be known. Also, the number of distribution network nodes is higher than transmission system, and thus requires a greater number of monitoring devices. This is also the aspect where the cost minimization needs to be considered. This task of determining the minimal set of μ PMUs required to ensure total observability of the system is termed as Optimal μ PMU Placement (OMP) task.

The cost and observability are contradicting features of any system. Higher is the observability of a system required, greater is the cost involved, but the cost needs to be minimized. In spite of this contradiction, μ PMUs have a feature that they can also have the measurements of the neighbouring branches and buses it is connected to along with the measurement of the bus it is placed at. This feature enables total observability with reduced number of μ PMUs. Also, multiple combinations of such placements may have an optimal solution so that the cost involved is minimum.

The objective of the Optimal μ PMU Placement (OMP) task is to determine the optimal set of μ PMUs to ensure total observability of the power system at minimum possible cost. Hence, the objective function for OMP task can be formulated as follows (Samantaray, 2017 and Chauhan, 2018).

For a N-bus system,

$$\text{Objective - Minimize } \sum_{i=1}^N c * \mu P_i \quad (1)$$

$$\text{subject to } A \cdot f(\mu P) \geq B \quad (2)$$

$$F = A * f(\mu P) \quad (3)$$

where μP – installed quantity of μ PMUs

c – cost coefficient of μ PMU (not considered as all μ PMUs are similar)

$[A]_{D \times D}$ – binary connectivity matrix of the network

$[\mu P]_{1 \times N}$ – binary μ PMU position array

B – observability matrix

F – vector whose nonzero entries indicate the observability of the corresponding buses whereas zero entries indicate un-observability

$$[A_{i,j}]_{N \times N} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if bus } i \text{ and } j \text{ are connected} \\ 0, & \text{connected} \end{cases} \quad (4)$$

$$[\mu P_i]_{1 \times N} = \begin{cases} 1, & \text{if a } \mu\text{PMU is placed at bus } i \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

For total observability of a system,

$$[B_i]_{N \times 1} \geq [1 \ 1 \ 1 \ 1 \ \dots \ 1 \ 1]^T \quad (6)$$

4.1 Measurement Redundancy

Usually, an optimization algorithm results in multiple answers. The OMP task also results in multiple μ PMU placement sets that consist of matching number of optimal μ PMUs. Among these multiple sets, the set that ensures maximum observability is considered as candidate solution. Maximum observability is illustrated by redundancy in measurements, that are defined using Bus Observability Index (BOI) and System Observability Redundancy Index (SORI) (Ahmedi, 2011).

BOI is defined as the number of times a bus is observed by the μ PMUs placement set; and SORI is the sum of BOI for all the buses, as illustrated by equations 7 and 8. The μ PMUs placement set with highest SORI is considered as the set with maximum redundancy.

$$BOI_i = A * X_i \quad (7)$$

$$SORI = \sum_{i=1}^N BOI_i \quad (8)$$

4.2 Micro-PMU Placement Strategy

The Optimal μ PMU Placement (OMP) task that is formulated in the previous section is usually used for determining the optimal placement set. However, in a practical distribution system, Distributed Energy Resources (DER) are connected at some of the buses. Moreover, there are few other buses in the system where loads are prominent in terms of quantity of load or type of load. The buses where such loads and DERs are present will be referred to as preferred locations for placing μ PMUs, and they can be straight away placed, even without resorting to optimize their placement.

Also, the buses that are adjacent to radial buses (buses at the end point of the laterals) are chosen as preferred locations for placing μ PMUs, as the μ PMU placed at a bus can also observe its neighbouring bus (observability rule 1). This strategy of placing the μ PMUs at preferred locations in the beginning of the placement task minimizes the computational load on the optimization algorithm. Only theremaining unobservable buses are made observable by placing additional required number of μ PMUs optimally. The above strategy is explained in the form of a flowchart in Figure 5. The process of identifying preferred buses for locating μ PMUs is implemented using Depth First Search (DFS) algorithm and optimization is implemented using Particle Swarm Optimization (PSO) and are explained in the further sections.

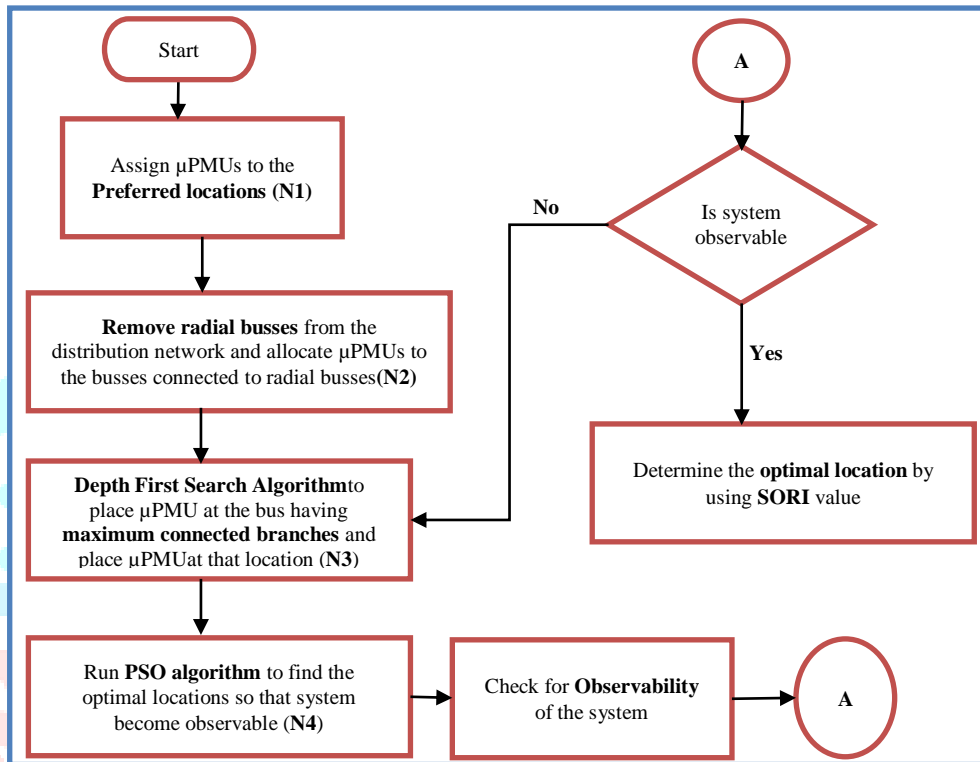


Figure 5. Flow chart implementing the micro-PMUs placement strategy

4.3 Depth First Search Algorithm

As discussed earlier, the strategy for Optimal μ PMU Placement involves placing of μ PMUs at preferred locations. While the buses with DERs are clearly given for any system, the other set of preferred buses are supposed to be the buses with maximum number of interconnections. Depth First Search (DFS) algorithm is used for this purpose.

Searching or tracking is one of the methods used for solving AI (Artificial Intelligence) problems. The process of exploring for a solution in a search space is referred to as Searching. It is implemented by going through each and every node in a graph, and searches for solution. An example of DFS algorithm is illustrated in Figure 6. The search process starts at the first node of the graph, and proceeds deeper and deeper until it finds the goal node or the node which has no children. The search algorithm then backtracks from the dead end towards the most recent node that is yet to be completely unexplored.

The applications of DFS Algorithm include finding the path, finding the strongly connected components of a graph and detecting cycles in a graph. DFS Algorithm is used to place μ PMU at the bus having maximum connections. Hence this list of buses can be considered as the next set of preferred locations for placing μ PMUs and also reduces the computational burden on PSO algorithm

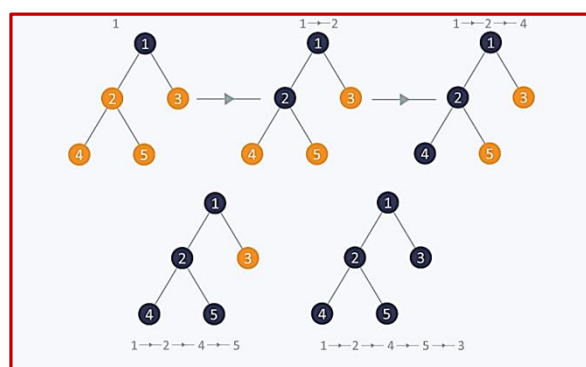


Figure 6. Example illustrating DFS algorithm

4.4 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is used as the method for optimally determining the DG capacities at the bus locations fixed by least VSI values. PSO is one of the meta-heuristic techniques that simulates the social behavioral intelligence of flocking of birds and movement of fish. Random values are initially assigned to the particles and are allowed to roam through the search space. During this course, the solution that is best for a particle is called Pbest and the best location for whole of the swarm is called Gbest. These particles are further allowed to move through the search space with corresponding velocities that aim the particle to move towards the best solution. These velocities and best positions are updated to obtain convergence as the iterations proceed, as represented by equations 9 to 12 (Srilatha, 2021).

$$x_p = (x_{p1}, x_{p2}, \dots, x_{pn}) \quad (9)$$

$$v_p = (v_{p1}, v_{p2}, \dots, v_{pn}) \quad (10)$$

$$v_p^{(iter)} = wv_p^{(iter-1)} + c_1\phi_1(x_{pbest}^{(iter-1)} - x_p^{(iter-1)}) + c_2\phi_2(x_{gbest}^{(iter-1)} - x_p^{(iter-1)}) \quad (11)$$

$$x_p^{(iter)} = x_p^{(iter-1)} + v_p^{(iter)} \quad (12)$$

Here, w is the inertia weight used for control the change in velocities of all particles and maintain the equilibrium between exploration and exploitation capabilities. Generally, a value between 0.4 and 0.9 is preferred for the values of w . c_1 and c_2 are the acceleration coefficients. ϕ_1 and ϕ_2 are random positive numbers.

IV. RESULTS AND DISCUSSION

The case studies considered for demonstration of optimal micro-PMU placement are IEEE 33 bus and IEEE 85 bus radial distribution systems. These systems are modified with inclusion of distributed energy resources mentioned as follows. The OPM task will be analyzed for the base case as well as for the reconfigured system cases to understand the optimal placement task.

IEEE 33 bus radial distribution system is modified by including 4 diesel-based distributed energy resources and 3 wind turbine units (Chauhan, 2020). Figure 7 shows the topology of 33 bus test system with sectionalizing and tie switches indicated by solid lines and dashed lines respectively (Abdolahi, A., 2021). Figure 8 represents the topology of IEEE 85 bus radial distribution system with sectionalizing and tie switches, wherein the system is modified by including with three PV generators and seven WindDGs at random locations. The location of DGs installed buses for both the test systems are mentioned in Table 1. These buses are considered as preferred locations and μ PMUs are pre-assigned to them. These locations for μ PMUs are termed as N1 for respective systems.

Table 1: Locations of DGs installed (Preferred μ PMU locations-N1)

Test system	Preferred Locations at DGs(N1)
IEEE 33 bus	8, 13, 14, 16, 25, 32
IEEE 85 bus	3,7,10,13,21,35,45,53,58,61

Table 2 represents the radial buses and the buses that are directly connected to these buses. The task of the optimization algorithm is to exclude these radial buses from the list of buses to find optimal μ PMUs. Moreover, these adjacent buses are considered as preferred locations for placing μ PMUs, and are termed as N2. This step reduces the computational burden of the algorithm as distribution systems have many radial buses.

DFS algorithm is then used on the rest of the buses after removing radial buses, to determine the branches with maximum connections. These buses are also considered as preferred locations for placing PMUs as given in Table 3. Figure 9 and Figure 10 represent the output of DFS algorithm for both the test systems. The encircled nodes indicate the nodes with maximum number of connections.

PSO method is implemented on the rest of the buses to determine optimal locations of micro-PMUs. It is found that there are 15 number of μ PMUs which make system completely observable for IEEE 33 bus system and 35 number of μ PMUs which make system completely observable for IEEE 85 bus system. Optimal location set is obtained by using System Observability Redundancy Index (SORI). SORI is calculated for each placement set. For same number of optimal placement μ PMUs, the set having high SORI value is selected as best μ PMU Placement set. These results are presented in Table 4. The total μ PMU locations are also presented that is the sum of all preferred locations and optimal locations.

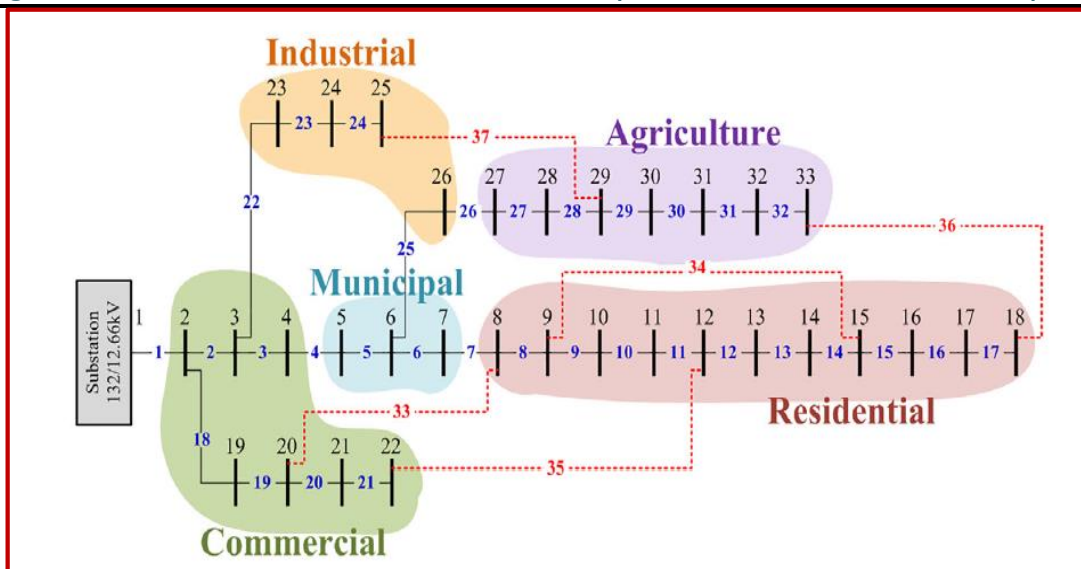


Figure 7. IEEE 33 bus distribution radial distribution system with sectionalizing and tie switches

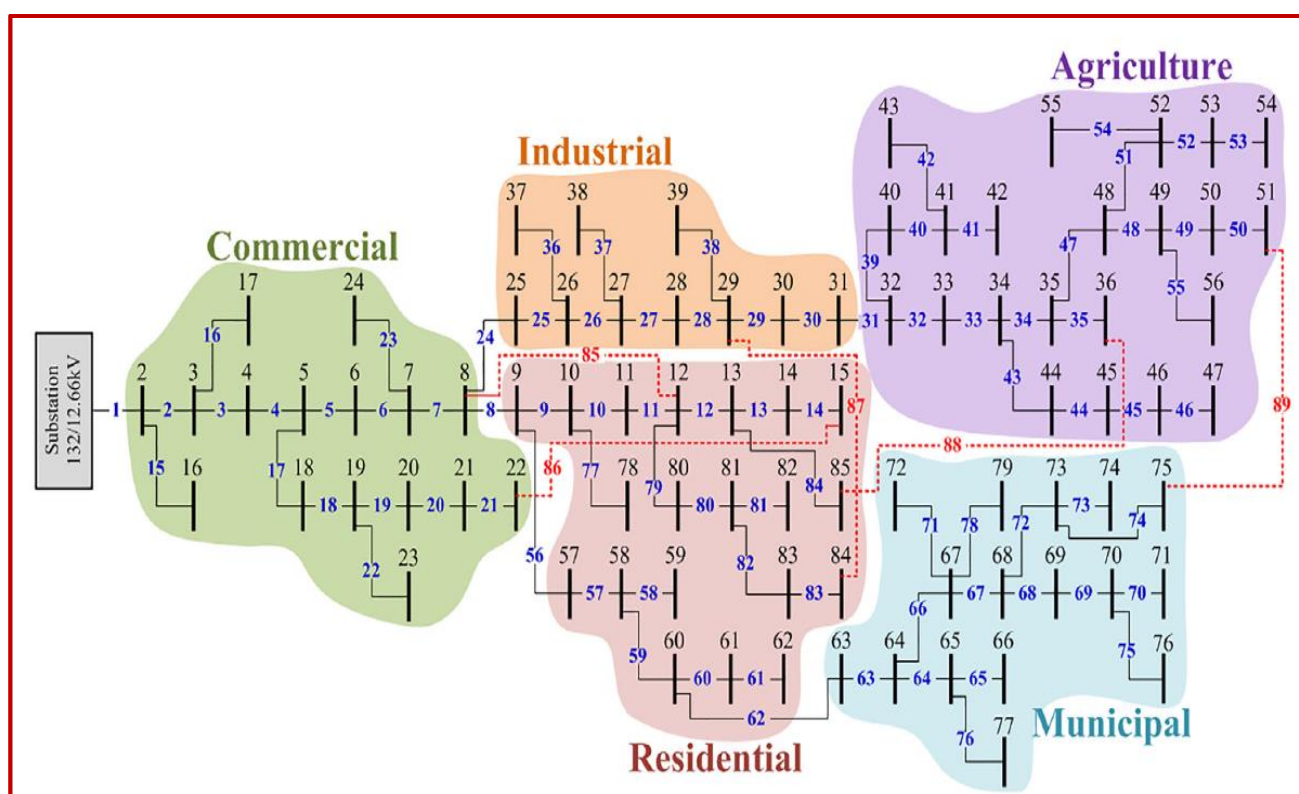


Figure 8. IEEE 85 bus distribution radial distribution system with sectionalizing and tie switches

Table 2: Buses connected to radial buses (Preferred μ PMU locations-N2)

Test System	Radial Buses	Buses connected to radial buses (N2)
IEEE 33 bus	1, 22, 25, 33, 18	2, 21, 24, 32, 17
IEEE 85 bus	116152337383940 4244464756515562 6677727971767475 8284	2141926272941464950 52656770738183

Table 3: Preferred locations using DFS algorithm (Preferred μ PMU locations-N3)

Test system	Preferred Locations using DFS (N3)
IEEE 33 bus	3, 6
IEEE 85 bus	5, 7, 8, 10, 13, 35, 58 and 67

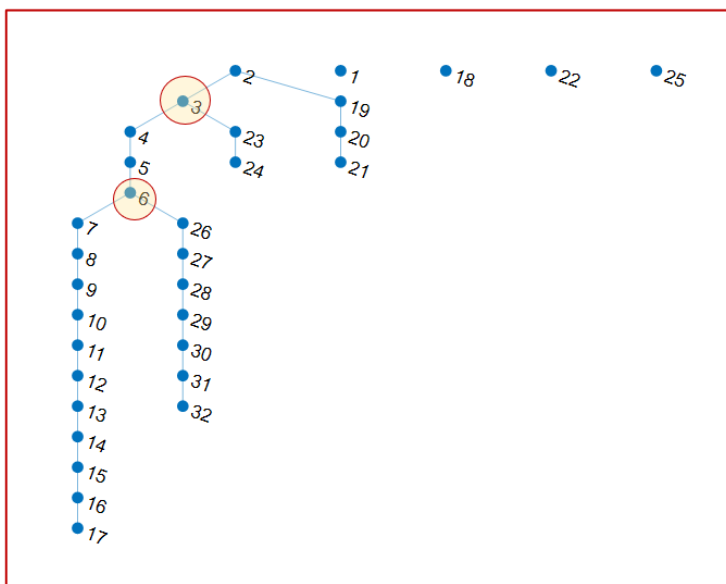


Figure 9. Output of DFS algorithm for IEEE 33 bus system

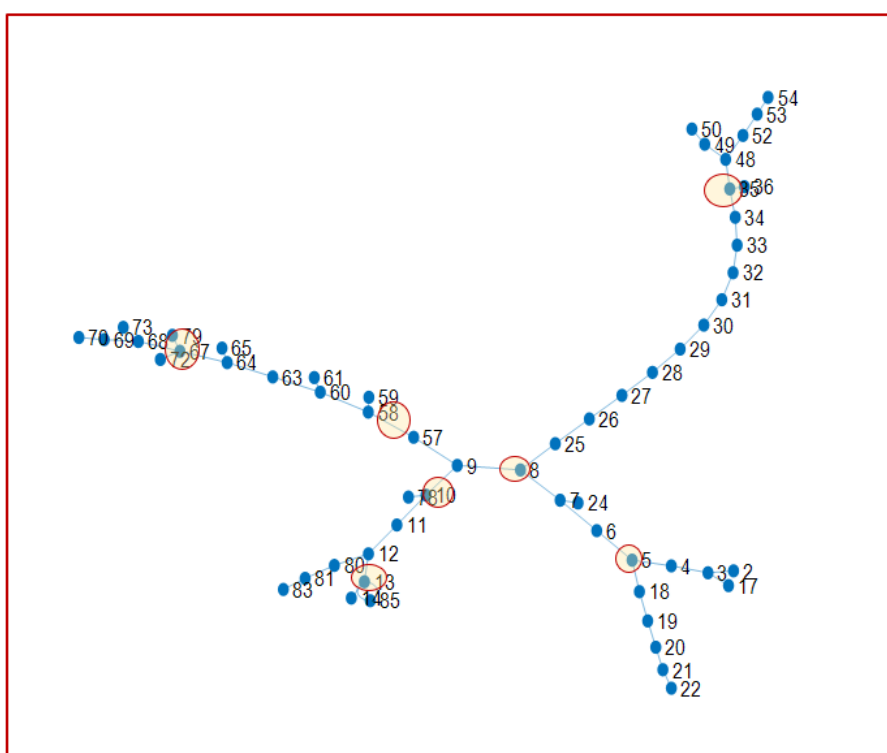


Figure 10. Output of DFS algorithm for IEEE 85 bus system

Table 4: Optimal locations using PSO algorithm (optimal μ PMU locations-N4)

Test System	Using PSO (N4)	Total No of μ PMUs (N1+N2+N3+N4)	Final μ PMU Locations	SORI
IEEE 33 bus	10, 24, 27, 30	15	2 3 6 8 11 13 14 16 17 21 24 25 27 29 32	47
IEEE 85 bus	11, 18, 28, 30, 33, 48, 63	35	2, 3, 7, 10, 11, 13, 14, 18, 19, 21, 26, 27, 28, 29, 30, 33, 35, 41, 45, 46, 48, 49, 50, 52, 53, 58, 61, 63, 65, 67, 70, 73, 81, 83, 84	125

The following reconfiguring strategies have been considered with an objective of reduced power losses in the system [17]. These configurations as displayed as case I to case V for IEEE 33-bus and cases VI to case X for IEEE 85 bus systems are considered for placement of PMU using the procedure explained in chapter 3. The results corresponding to these cases for optimally placing the PMUs are recorded in Tables 5, 6 7 and 8 respectively.

Table 5: Reconfiguration strategies for 33 bus test system

Switch Status	Measurement Redundancy	Power Loss (kW)	Analysis Case
S7, S9, S14, S32, S37	37	139.55	Case-I
S7, S9, S14, S36, S37	40	142.16	Case-II
S11, S28, S33, S34, S36	37	146.03	Case-III
S11, S32, S33, S34, S37	39	152.96	Case-IV
S7, S17, S34, S35, S37	37	159.35	Case-V

Table 6: Reconfiguration strategies for 85 bus test system

Switch Status	Measurement Redundancy	Power Loss (kW)	Analysis Case
S11, S21, S35, S50, S83	112	231.35	Case-VI
S9, S86, S87, S88, S89	118	247.87	Case-VII
S85, S86, S87, S88, S89	114	249.90	Case-VIII
S11, S18, S50, S87, S88	110	284.17	Case-IX
S17, S85, S87, S88, S89	110	308.13	Case-X

Table 7: Optimal Placement Results for 33 bus test system

Case	No of μ PMUs	Optimal Locations	SORI
Base Case	15	2, 3, 6, 8, 10, 13, 14, 16, 17, 21, 24, 25, 27, 30, 32	47
Case-I	15	2, 3, 6, 8, 11, 13, 14, 16, 17, 21, 24, 25, 28, 29, 32	47
Case-II	16	2, 3, 6, 8, 11, 13, 14, 16, 17, 21, 24, 25, 26, 28, 31, 32	50
Case-III	16	2, 3, 6, 8, 11, 13, 14, 16, 17, 21, 24, 25, 26, 28, 31, 32	49
Case-IV	17	2, 3, 6, 8, 11, 13, 14, 16, 17, 21, 24, 25, 26, 28, 29, 31, 32	50
Case-V	15	2, 3, 6, 8, 11, 13, 14, 16, 17, 21, 24, 25, 28, 29, 32	50

Table 8: Optimal Placement Results for 85 bus test system

Case	No of μ PMUs	Optimal Locations	SORI
Base Case	35	2, 3, 7, 10, 11, 13, 14, 18, 19, 21, 26, 27, 28, 29, 30, 33, 35, 41, 45, 46, 48, 49, 50, 52, 53, 58, 61, 63, 65, 67, 70, 73, 81, 83, 84	125
Case-VI	33	2, 3, 7, 10, 13, 14, 18, 19, 21, 24, 26, 27, 29, 32, 35, 41, 45, 46, 49, 50, 52, 53, 58, 61, 64, 65, 67, 68, 70, 73, 80, 81, 83	121
Case-VII	33	2, 3, 6, 7, 9, 10, 13, 14, 19, 21, 25, 26, 27, 29, 32, 35, 41, 45, 46, 49, 50, 52, 53, 58, 61, 63, 65, 67, 68, 70, 73, 81, 83	122
Case-VIII	34	2, 3, 4, 7, 10, 11, 13, 14, 19, 21, 26, 27, 28, 29, 31, 33, 35, 41, 45, 46, 49, 50, 52, 53, 58, 61, 63, 65, 67, 69, 70, 73, 81, 83	122
Case-IX	33	2, 3, 6, 7, 10, 13, 14, 19, 21, 26, 27, 29, 31, 33, 35, 41, 45, 46, 49, 50, 52, 53, 58, 59, 61, 63, 65, 67, 70, 73, 78, 81, 83	117
Case-X	33	2, 3, 6, 7, 10, 13, 14, 17, 19, 20, 21, 26, 27, 29, 32, 35, 41, 45, 46, 49, 50, 52, 53, 58, 60, 61, 65, 67, 68, 70, 73, 81, 83	121

Thus, allocation of μ PMUs using (PSO) Particle Swarm Optimization algorithm has been implemented in radial distribution systems for base case and reconfigured systems. System Observability Redundancy Index (SORI) is used to determine the optimal solution out of all similar numbered placement sets. It can be observed that the optimal placement strategies for reconfigured systems also are performing similar to the base case.

V. CONCLUSION

Allocation of μ PMUs using (PSO) Particle Swarm Optimization algorithm has been implemented. System of Redundancy Index (SORI) is used to determine the optimal solution out of all existing possible placement sets. Placement sets having same number of μ PMUs with high SORI value is being chosen as the optimal set. The optimal μ PMU placement problem is implemented on IEEE 33 bus radial distribution feeder and on IEEE 85 bus radial distributions test feeder integrated with distributed energy resources. Pre-allocation of certain micro-PMUs is done in distribution systems, unlike in transmission systems. This reduces the burden on the optimization algorithm used to optimize PMU placements for unobserved buses.

Pre-allocation at preferred buses are the location of Distributed energy resources and the buses adjacent to radial buses. The optimal placement strategies for reconfigured systems also are performing similarly to the basecase.

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