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Cotangent Similarity Measures of Pythagorean Fuzzy Set

[1]M. Sowmiya, [2]A. Stanis Arul Mary

[1]PG Scholar, [2]Assistant Professor
[1]Department of Mathematics,
[1]Nirmala College For Women, Coimbatore, India

Abstract: In this paper, a new cotangent similarity measure between two Pythagorean fuzzy sets [PFS] was proposed and its properties were studied. Also, using the cotangent similarity measure and weighted cotangent similarity measures of Pythagorean fuzzy set we have given a solution to the Automobile problem.

***Key Words* – Pythagorean fuzzy set, Cotangent similarity measure, Weighted Cotangent similarity measure.**

I.INTRODUCTION

Similarity measure is an essential research topic in the current fuzzy, Pythagorean, neutrosophic and different hybrid environments .Fuzzy sets were introduced by L.A. Zadeh in 1965.Zadeh's idea of fuzzy set evolved as a new tool having the ability to deal with uncertainities in real life problems and discussed only membership function. After the extensions of fuzzy set theory Atanassov generalised this concept and introduced a new set called intuitionistic fuzzy set(IFS) in 1986 ,which can be describe the non-membership grade of an imprecise event along with its membership grade under a restriction that the sum of both membership and non-membership grades does not exceed 1. IFS has its greatest use in practical multiple attribute decision making problems. In some practical problems. In some practical problems, the sum of membership and non-membership degree to which an alternative satisfying attribute provided by decision maker (DM) may be bigger than 1.

Yager was decided to introduced the new concept known as Pythagorean fuzzy sets. Pythagorean fuzzy sets has limitation that their square sum is less than or equal to 1.

Recently , Ye presented the correlation coefficient of single-valued neutrosophic sets (SVNSs) and the cross-entropy measure of SVNSs and applied them to single-valued neutrosophic decision making problems. Then , Ye proposed similarity measures between interval neutrosophic sets and their applications in multicriteria decision making. Ye also proposed three vector similarity measures for SVNSs and instance of SVNSs and interval valued neutrosophic set, including the Jaccard , Dice , and cosine similarity and applied them to multi-criteria decision making problems with simplified neutrosophic information. Pramanik and Mondal proposed cotangent similarity measure of rough neutrosophic sets and its application to application to automobile problem. Pramanik and Mondal also proposed weighted fuzzy similarity measure based on tangent function and its application to automobile problem. Pramanik and Mondal proposed tangent similarity measures between intuitionistic fuzzy sets and studied some of its properties and applied it for automobile problem. Broumi and Smarandache defined Hausdorff distance measure between two neutrosophic sets . Broumi and Smarandache extended the concept of cosine similarity measure of SVNSs into INSs and applied it to pattern recognition.

In this paper we propose cotangent similarity measures for Pythagorean fuzzy sets [PFS] . We also proposed similarity measures for automobile problem.

II.PRELIMINARIES

Definition 2.1

Let E be a universe. An Intuitionistic fuzzy set A in E is defined as object of following form:

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in E\}$$

Where M_A: E→ [0,1], N_A: E→ [0,1] define the degree of membership and degree of non-membership of element x ∈ E respectively.

$$0 \leq M_A(x) + N_A(x) \leq 1 \text{ for any } x \in E$$

Here, $M_A(x)$ and $N_A(x)$ is the degree of membership and non-membership of the element of x respectively.

Definition 2.2

Let X be universe set. Then a Pythagorean fuzzy set A which is set of ordered pairs over X

$$A = \{\langle x, M_A(x), N_A(x)\rangle: x \in X\}$$

Where M_A: X→ [0,1], N_A: X→ [0,1] denote the degree of membership and degree of non-membership of element x ∈ X to the set A which is a subset of X and

$$0 \leq (M_A(x))^2 + (N_A(x))^2 \leq 1 \text{ for any } x \in X$$

$M_A(x)$ and $N_A(x)$ is the degree of membership and non-membership of the element of x respectively.

Definition 2.3

Let A and B be Pythagorean fuzzy sets in a topological space X of the form

$A = \{\langle x, M_A(x), N_A(x)\rangle: x \in X\}$, $B = \{\langle x, M_B(x), N_B(x)\rangle: x \in X\}$

$$A \cup B = \{x, \max(M_A(x), M_B(x)), \min(N_A(x), N_B(x))| \, x \in X\}$$
$$A \cap B = \{x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x))| \, x \in X\}$$
$$A^C = \{(x, N_A(x), M_A(x))| \, x \in X\}$$

III.COTANGENT SIMILARITY MEASURES OF PYTHAGOREAN FUZZY SETS

Definition 3.1

Let P = {(x, $M_P(x), N_P(x)): x \in X\}$ and

Q = {(x, $M_Q(x), N_Q(x): x \in X\}$ be a two Pythagorean fuzzy sets.A cotangent similarity measures between pythagorean fuzzy sets P and Q is proposed as follows

$$COT_{PFS}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}[cot(\frac{\pi}{8}[2 + \left|M_P^2(x_i) - M_Q^2(x_i)\right| + \left|N_P^2(x_i) - N_Q^2(x_i)\right|])]$$

Theorem 3.2

Let P and Q be Pythagorean fuzzy sets then

1) $0 \leq COT_{PFS}(P,Q) \leq 1$;
2) $COT_{PFS}(P,Q) = 1$ iff P = Q;
3) $COT_{PFS}(P,Q) = COT_{PFS}(Q,P)$;
4) If O is a Pythagorean fuzzy set in X and $P \subseteq Q \subseteq O$ then $COT_{PFS}(P,O) \leq COT_{PFS}(P,Q)$ and $COT_{PFS}(P,O) \leq COT_{PFS}(Q,O)$.

Proof:

1) Since $\frac{\Pi}{4} \leq (\frac{\Pi}{8}(2+\left|M_P^2(x_i) - M_Q^2(x_i)\right| + \left|N_P^2(x_i) - N_Q^2(x_i)\right|)) \leq \frac{\Pi}{2}$
So that the cotangent function $COT_{PFS}(P,Q)$ are within 0 and 1.
Hence $0 \leq COT_{PFS}(P,Q) \leq 1$
2) For any two Pythagorean fuzzy sets P and Q if P = Q, this implies
$M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i)$
Hence $\left|M_P^2(x_i) - M_Q^2(x_i)\right| = 0, \left|N_P^2(x_i) - N_Q^2(x_i)\right| = 0,$
Thus $COT_{PFS}(P,Q) = 1$.
Conversely, if $COT_{PFS}(P,Q) = 1$,
To prove: P=Q .So that $M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i)$,
Hence P = Q.
3) Since $COT_{PFS}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}[cot(\frac{\pi}{8}[2 + \left|M_P^2(x_i) - M_Q^2(x_i)\right| + \left|N_P^2(x_i) - N_Q^2(x_i)\right|])]$
And $COT_{PFS}(Q,P) = \frac{1}{n}\sum_{i=1}^{n}[cot(\frac{\pi}{8}[2 + \left|M_Q^2(x_i) - M_P^2(x_i)\right| + \left|N_Q^2(x_i) - N_P^2(x_i)\right|])]$
Here values inside the modulus does not make any differences.
Hence $COT_{PFS}(P,Q) = COT_{PFS}$(Q,P)
4) Given:O is a Pythagorean fuzzy set in X and $P \subseteq Q \subseteq O$
To prove : $COT_{PFS}(P,O) \leq COT_{PFS}(P,Q)$ and $COT_{PFS}(P,O) \leq COT_{PFS}(Q,O)$.
If $P \subseteq Q \subseteq O$, then $M_P(x_i) \leq M_Q(x_i) \leq M_o(x_i), N_P(x_i) \geq N_Q(x_i) \geq N_0(x_i)$,

$$\left|M_P^2(x_i) - M_Q^2(x_i)\right| \leq |M_P^2(r_i) - M_O^2(x_i)|,$$
$$\left|M_Q^2(x_i) - M_O^2(x_i)\right| \leq |M_P^2(x_i) - M_O^2(x_i)|,$$
$$\left|N_P^2(x_i) - N_Q^2(x_i)\right| \leq |N_P^2(x_i) - N_o^2(x_i)|,$$
$$\left|N_Q^2(x_i) - N_O^2(x_i)\right| \leq |N_P^2(x_i) - N_o^2(x_i)|,$$

The cotangent is decreasing function within the interval [[$\frac{\pi}{4}, \frac{\pi}{4}$]

$$COT_{PFS}(P,O) \leq COT_{PFS}(P,Q) \text{ and } COT_{PFS}(P,O) \leq COT_{PFS}(Q,O)$$

Definition 3.3

If we consider the weights of each element x_i ,weighted cotangent similarity measure between Pythagorean fuzzy sets Pand Q can be defined as follows

$$COT_{PFS}(P,Q) = \sum_{i=1}^{n} w_i[cot(\frac{\pi}{8}[2 + \left|M_P^2(x_i) - M_Q^2(x_i)\right| + \left|N_P^2(x_i) - N_Q^2(x_i)\right|])]$$

Where $w_i \in [0,1], i = 0,1,2 \ldots n$ are the weights and $\sum_{i=1}^{n} w_i = 1$.

If we take w_i=1/n,i=1,2,3…then $COT_{WPFS}(P,O) = COT_{PFS}(P,Q)$

Theorem 3.4

The weighted cotangent similarity measure $COT_{WPFS}(P,Q)$ between Pythagorean fuzzy set P and Q satisfies the following properties

1) $0 \leq COT_{WPFS}(P,Q) \leq 1$;
2) $COT_{WPFS}(P,Q) = 1$ iff P = Q;
3) $COT_{WPFS}(P,Q) = COT_{WPFS}(Q,P)$;
4) If O is a Pythagorean fuzzy set in X and $P \subseteq Q \subseteq O$ then $COT_{WPFS}(P,O) \leq COT_{WPFS}(P,Q)$ and $COT_{WPFS}(P,O) \leq COT_{WPFS}(Q,O)$.

Proof

1) Since $\frac{\Pi}{4} \leq (\frac{\Pi}{8}(2+|M_P^2(x_i) - M_Q^2(x_i)|+|N_P^2(x_i) - N_Q^2(x_i)|)) \leq \frac{\Pi}{2}$
$\sum_{i=1}^{n} w_i = 1$. So that the cotangent function $COT_{WPFS}(P,Q)$ are within 0 and 1.
Hence $0 \leq COT_{WPFS}(P,Q) \leq 1$.
2)Given P = Q,
To prove : $COT_{WPFS}(P,Q) = 1$
When P=Q , $\sum_{i=1}^{n} w_i = 1$.
Then $COT_{WPFS}(P,Q)$=1
If $COT_{WPFS}(P,Q) = 1$ $then$ $M_P(x_i) = M_Q(x_i), N_P(x_i) = N_Q(x_i)$,
Hence P = Q.
3)Since $COT_{WPFS}(P,Q) = \sum_{i=1}^{n} w_i[cot(\frac{\pi}{8}[2 + |M_P^2(x_i) - M_Q^2(x_i)| + |N_P^2(x_i) - N_Q^2(x_i)|])]$

$$COT_{WPFS}(Q,P) = \sum_{i=1}^{n} w_i[cot(\frac{\pi}{8}[2 + |M_Q^2(x_i) - M_P^2(x_i)| + |N_Q^2(x_i) - N_P^2(x_i)|])]$$

Here the values inside the modulus does not make any differences.
Hence $COT_{WPFS}(P,Q) = COT_{WPFS}(Q,P)$
4) If O is a Pythagorean fuzzy set in X and $P \subseteq Q \subseteq O$
To prove:
$COT_{WPFS}(P,O) \leq COT_{WPFS}(P,Q)$ and $COT_{WPFS}(P,O) \leq COT_{WPFS}(Q,O)$
If $P \subseteq Q \subseteq O$ then $M_P(x_i) \leq M_Q(x_i) \leq M_O(x_i), N_P(x_i) \geq N_Q(x_i) \geq N_O(x_i)$,

$$|M_P^2(x_i) - M_Q^2(x_i)| \leq |M_P^2(x_i) - M_O^2(x_i)|,$$
$$|M_Q^2(x_i) - M_O^2(x_i)| \leq |M_P^2(x_i) - M_O^2(x_i)|,$$
$$|N_P^2(x_i) - N_Q^2(x_i)| \leq |N_P^2(x_i) - N_O^2(x_i)|,$$
$$|N_Q^2(x_i) - N_O^2(x_i)| \leq |N_P^2(x_i) - N_O^2(x_i)|,$$

Since cotangent function is decreasing function within the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$

$$\sum_{i=1}^{n} w_i = 1.$$

Therefore, $COT_{WPFS}(P,O) \leq COT_{WPFS}(P,Q)$ and $COT_{WPFS}(P,O) \leq COT_{WPFS}(Q,O)$

IV. DECISION MAKING BASED ON COTANGENT SIMILARITY MEASURES

Let $A_1, A_2, \ldots A_m$ be a discrete set of candidates, $C_1, C_2, \ldots C_n$ be the set of criteria for each candidate and $B_1, B_2, \ldots, B_k$ are the alternatives of each candidate. The decision -maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performance of candidates $A_i(i = 1,2, \ldots m)$ against the criteria $C_j(j = 1,2 \ldots, n)$.The values associated with the alternatives for MADM problem can be presented in the following decision matrix(see Tab 1 and Tab 2). The relation between candidates and attributes are given in Tab 1. The relation between attributes and alternatives are given in the Tab 2.

Tab 1 : The relation between candidates and attributes

R_1	C_1	C_2	…	C_n
A_1	d_{11}	d_{12}	…	d_{1n}
A_2	d_{21}	d_{13}	…	d_{2n}
…	…	…	…	…
A_m	d_{m1}	d_{m2}	…	d_{mn}

Tab 2 : The relation between attributes and alternatives

R_2	B_1	B_2	…	B_k
C_1	δ_{11}	δ_{12}	…	δ_{1k}
C_2	δ_{21}	δ_{22}	…	δ_{2k}
…	…	…	…	…
C_n	δ_{n1}	δ_{n2}	…	δ_{nk}

Here d_{ij} and δ_{ij} are all Pythagorean Fuzzy numbers.
The steps corresponding to Pythagorean number based on tangent and cotangent functions are presented following steps.

Step 1: Determination of the relation between candidates and attributes

The relation between candidate $A_i(i = 1,2,\ldots m)$ and the attribute $C_j(j = 1,2\ldots n)$ is presented in Tab 3.

Tab 3 : The relation between candidates and attributes in terms of Pythagorean fuzzy numbers

R_3	C_1	C_2	...	C_n
A_1	(a_{11},b_{11})	(a_{12},b_{12})	...	(a_{1n},b_{1n})
A_2	(a_{21},b_{21})	(a_{22},b_{22})	...	(a_{2n},b_{2n})
...	...	...	...	...
A_m	(a_{m1},b_{m1})	(a_{m2},b_{m2})	...	(a_{mn},b_{mn})

Step 2: Determination of the relation between attributes and alternatives

The relation between attributes $C_i(i = 1,2,\ldots n)$ and the alternatives $B_t(t = 1,2\ldots k)$ is presented in Tab 4.

Tab 4 : The relation between attributes and alternatives in terms of Pythagorean fuzzy sets

R_2	B_1	B_2	...	B_n
C_1	(c_{11},d_{11})	(c_{12},d_{12})	...	(c_{1k},d_{1k})
C_2	(c_{21},d_{21})	(c_{22},d_{22})	...	(c_{2k},c_{2k})
...	...	...	...	...
C_n	(c_{n1},d_{n1})	(c_{n2},c_{n2})	...	(c_{nk},d_{nk})

Step 3: Determination of the Similarity measures

Determine the similarity measure between the Tab 3 and Tab 4 using $T_{PFS}(P,Q)$ $T_{PFS}(P,Q)$, $COT_{PFS}(P,Q)$ and $COT_{PFS}(P,Q)$.

Step 4:Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of the similarity measures. Highest value reflects the best alternative.

Step 5: End

Example 4.1

In day to day life new upcoming models are arriving in the automobile field which leads to confusion to conclude the best one .

For example

R={R_1, R_2, R_3 , R_4} be a set of Respondents

B={Cost,Mileage,Colour} be a set of benefits

A={Yamaha, Hero,Bajaj,TVS} be a set of automobiles

The solution strategy to examine respondent which will provide membership and non-membership for each respondent regarding relation between respondent and different benefits (Table i)

The correlation measure between R_1 and R_2 (Table iii)

Table (i)

Relation between Respondent and benefits

S_1	Cost	Mileage	Colour
R_1	(0.5,0.4)	(0.3,0.6)	(0.7,0.3)
R_2	(0.8,0.3)	(0.4,0.8)	(0.1,0.6)
R_3	(0.1,0.3)	(0.2,0.4)	(0.7,0.2)
R_4	(0.4,0.2)	(0.3,0.5)	(0.3,0.2)

Table (ii)

The relation between Benefits and Automobile

S_2	Yamaha	Hero	Bajaj	TVS
R_1	(0.4,0.1)	(0.1,0.2)	(0.3,0.2)	(0.1,0.3)
R_2	(0.4,0.3)	(0.3,0.4)	(0.2,0.1)	(0.4,0.5)
R_3	(0.7,0.1)	(0.2,0.7)	(0.4,0.3)	(0.2,0.5)

Table (iii)

The cotangent similarity measure between S_1 and S_2

Cotangent similarity measure	Yamaha	Hero	Bajaj	TVS
R_1	**0.8432**	0.6966	0.7657	0.7524
R_2	0.5898	0.6974	0.6206	**0.7416**
R_3	**0.8905**	0.7947	0.8434	0.8094
R_4	0.8397	0.8295	**0.8837**	0.8715

Weight information

$W=(w_1, w_2, w_3)^T = (0.25,0.35,0.4)^T$ such that $\sum_{j=1}^{n} w_j=1$

Table (iv)

The weighted cotangent similarity measure between S_1 and S_2

Weighted cotangent similarity measure	Yamaha	Hero	Bajaj	TVS
R_1	**0.8496**	0.6806	0.7624	0.7422
R_2	0.5806	0.7193	0.6262	**0.7638**
R_3	**0.90047**	0.7613	0.8322	0.7787
R_4	0.8195	0.8155	**0.8787**	0.8704

The highest correlation measure reflects the best automobile selection

Therefore, R_1 selects Yamaha

R_2 selects TVS

R_3 selects Yamaha

R_4 selects Bajaj

V.CONCLUSION

In this paper we have proposed cotangent similarity measures for Pythagorean fuzzy sets and proved some of its properties. We proposed cotangent similarity measures for Pythagorean fuzzy sets can be used in the field of practical decision making pattern recognition ,medical diagnosis ,data mining clustering analysis.

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