IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Generalized Pre–closed Sets in Pythagorean Fuzzy Topological Spaces

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Abstract: In this paper a Pythagorean Fuzzy generalized pre-closed sets and a Pythagorean Fuzzy generalized pre-open sets are introduced. Some of its properties are also studied. Also we have provided some applications of Pythagorean Fuzzy generalized pre-closed sets namely Pythagorean Fuzzy $_{p}T_{1/2}$ space and Pythagorean Fuzzy $_{gp}T_{1/2}$ space.

***Key Words* - Pythagorean Fuzzy topology, Pythagorean Fuzzy generalized pre-closed sets, Pythagorean Fuzzy generalized pre-open sets, Pythagorean Fuzzy $_{p}T_{1/2}$ space and Pythagorean Fuzzy $_{gp}T_{1/2}$ space.**

I. Introduction

The concept of Fuzzy sets was introduced by Zadeh in 1965. After the introduction of Intuitionistic Fuzzy set by Atanassov in 1986, R. R. Yager generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean Fuzzy set. In 1991, A. S. Binshahan introduced and investigated the notions of Fuzzy pre-open and Fuzzy pre-closed sets. In 2003, T. Fukutake, R. K. Saraf, M. Caldas and S. Mishra introduced generalized pre-closed Fuzzy sets in Fuzzy topological space. P. Rajarajeswari and L. Senthil Kumar introduced generalized pre-closed sets in Intuitionistic Fuzzy topological spaces. In this paper we have introduced Pythagorean Fuzzy generalized pre-closed sets and studied some of their properties.

II. Preliminaries

Definition 2.1: A Pythagorean Fuzzy set (PFS in short) A in X is an object having the form A $=\{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ where the functions $\lambda_A: X \to [0,1]$ and $\mu_A: X \to [0,1]$ denote the degree of membership (namely $\lambda_A(a)$) and the degree of non-membership (namely $\mu_A(a)$) of each element $a \in X$ to the set A respectively, $0 \le \lambda_A{}^2(a)+ \mu_A{}^2(a) \le 1$ for each $a \in X$.

Definition 2.2: Let A and B be PFSs of the forms $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ and $B = \{< a, \lambda_B(a), \mu_B(a) > / a \in X\}$. Then

a) $A \subseteq B$ if and only if $\lambda_A(a) \le \lambda_B(a)$ and $\mu_A(a) \ge \mu_B(a)$ for all $a \in X$
b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
c) $A^c =\{< a, \mu_A(a), \lambda_A(a) > / a \in X\}$
d) $A \cap B =\{< a, \lambda_A(a) \wedge \lambda_B(a), \mu_A(a) \vee \mu_B(a) > / a \in X\}$
e) $A \cup B =\{< a, \lambda_A(a) \vee \lambda_B(a), \mu_A(a) \wedge \mu_B(a) > / a \in X\}$

For the sake of simplicity, we shall use the notation $A = < a, \lambda_A, \mu_A>$ instead of $A = \{< a, \lambda_A(a), \mu_A(a) >/ a \in X\}$. Also for the sake of simplicity, we shall use the notation $A = < a, (\lambda_A, \lambda_B), (\mu_A, \mu_B) >$ instead of $A = < a,(A/\lambda_A, B/\lambda_B), (A/\mu_A, B/\mu_B) >$. The Pythagorean Fuzzy sets $0 = \{< a, 0, 1 > / a \in X\}$ and $1 = \{< a, 1, 0 > / a \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: A Pythagorean Fuzzy topology by subsets of a non - empty set X is a family τ of PFSs satisfying the following axioms.

a) $0, 1 \in \tau$
b) $G_1 \cap G_2 \in \tau$ for every $G_1, G_2 \in \tau$ and
c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a Pythagorean Fuzzy topological space (PFTS in short) and any PFS G in τ is called a Pythagorean Fuzzy open set (PFOS in short) in X. The complement A^c of a Pythagorean Fuzzy open set A in a PFTS (X, τ) is called a Pythagorean Fuzzy closed set (PFCS in short).

Definition 2.4: Let (X, τ) be a PFTS and $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ be a PFS in X. Then the interior and the closure of A are denoted by PFint(A) and PFcl(A) and are defined as follows.

- $PFint(A) = \cup\{G|G$ is a PFOS in X and $G \subseteq A\}$
- $PFcl(A) = \cap \{K|K$ is a PFCS in X and $A \subseteq K\}$

Note that for any PFS A in (X, τ), we have $PFcl(A^c) = (PFint(A))^c$ ant $PFint(A^c) = (PFcl(A))^c$.

Definition 2.5: A PFS A = < a, λ_A, μ_A> in a PFTS (X, τ) is said to be an

1. Pythagorean Fuzzy semi closed set (PFSCS in short) if PFint(PFcl(A)) ⊆ A,
2. Pythagorean Fuzzy semi open set (PFSOS in short) if A ⊆ PFcl(PFint(A))
3. Pythagorean Fuzzy pre-closed set (PFPCS in short) if PFcl(PFint(A)) ⊆ A
4. Pythagorean Fuzzy pre-open set (PFPOS in short) if A ⊆ PFint(PFcl(A))
5. Pythagorean Fuzzy α-closed set (PFαCS in short) if PFcl(PFint(PFcl(A))⊆ A
6. Pythagorean Fuzzy α-open set (PF αOS in short) if A ⊆ PFint(PFcl(PFint(A)))

Definition 2.6: Let A be a PFS of a PFTS (X, τ). Then the semi closure of a (PFscl(A) in short) is defined as PFscl(A) = ∩{K|K is a PFSCS in X and A ⊆ K}.

Definition 2.7: Let A be a PFS of a PFTS (X, τ). Then the semi interior of a (PFsint(A) in short) is defined as PFsint(A) = ∪{G|G is a PFOS in X and G ⊆ A}.

Result 2.8: Let A be a PFS in (X, τ), then

1. PFscl(A) = A ∪ PFint(PFcl(A))
2. PFsint(A) = A ∩ PFcl(PFint(A))

Definition 2.9: A PFS A = < a, λ_A, μ_A> in a PFTS (X, τ) is said to be an

1. Pythagorean Fuzzy regular open set (PFROS in short) if A = PFint(PFcl(A))
2. Pythagorean Fuzzy regular closed set (PFRCS in short) if A = PFcl(PFint(A))

Definition 2.10: A PFS A of a PFTS (X, τ) is a Pythagorean Fuzzy generalized closed set (PFGCS in short) if PFcl(A) ⊆ U whenever A ⊆ U and U is a PFOS in X.

Definition 2.11: Let a PFS A of a PFTS (X, τ). Then the alpha closure of A (PFαcl(A)in short) is defined as PFαcl(A) = ∩ {K|K is a PFCS in X and A ⊆ K}.

Definition 2.12: Let a PFS A of a PFTS (X, τ). Then the alpha interior of A (PFαint(A)in short) is defined as PFαint(A) = ∩ {K|K is a PFCS in X and A ⊆ K}.

Result 2.13: Let A be a PFS in (X, τ), then

1. PFαcl(A) = A ∪ PFcl(PFint(PFcl(A)))
2. PFαint(A) = A ∩ PFint(PFcl(PFint(A)))

Definition 2.14: A PFS A of a PFTS (X, τ) is said to be a Pythagorean Fuzzy alpha generalized closed sets (PFαGCS in short) if PFαcl(A) ⊆ U whenever A ⊆ U and U is a PFOS in X.

Definition 2.15: Let (X, τ) be a PFTS and A = < a, λ_A, μ_A> be a PFS in X. The pre interior of A is denoted by PFpint(A) and is defined by the union of all Fuzzy pre-open sets of X which are contained in A. The intersection of all Fuzzy pre-closed sets containing A is called the pre closure of A and is denoted by PFpcl(A).

- PFpint(A) = ∪{G|G is a PFPOS in X and G ⊆ A}
- PFpcl(A) = ∩ {K|K is a PFPCS in X and A ⊆ K}

Result 2.16: If A is a PFS in X, then PFpcl(A) = A ∪ PFcl(PFint(A)).

III. Pythagorean Fuzzy Generalized Pre-closed Sets

In this section we introduce Pythagorean Fuzzy generalized pre-closed set and studied some of its properties.

Definition 3.1: A PFS A is said to be a Pythagorean Fuzzy generalized pre-closed set (PFGPCS in short) in (X, τ) if PFpcl(A) ⊆ U whenever A ⊆ U and U is a PFOS in X. The family of all PFGPCSs of a PFTS (X, τ) is denoted by PFGPC(X).

Example 3.2: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.2, 0.8 >, < a, 0.3, 0.7 >}. Then the PFS A = {< a, 0.2, 0.8 >, < a, 0.2, 0.7 >} is a PFGPCS in X.

Theorem 3.3: Every PFCS is a PFGPCS but not conversely.

Proof: Let A be a PFCS in X and let A ⊆ U and U is a PFOS in (X, τ). Since PFpcl(A) ⊆ PFcl(A) and A is a PFCS in X, PFpcl(A) ⊆ PFcl(A) = A ⊆ U. Therefore A is a PFGPCS in X.

Example 3.4: Let X = {a, b} and let τ = 0, T, 1} be a PFT on X, where T = {< a, 0.2, 0.8 >, < a, 0.3, 0.7 >}. Then the PFS A = {< a, 0.2, 0.8 >, < a, 0.2, 0.7 >} is a PFGPCS in X but not a PFCS in X.

Theorem 3.5: Every PFαCS is a PFGPCS but not conversely.

Proof: Let A be a PFαCS in X and let A ⊆ U and U is a PFOS in (X, τ). By hypothesis, PFcl(PFint(PFcl(A))) ⊆ A. Since A ⊆ PFcl(A), PFcl(PFint(A)) ⊆ PFcl(PFint(PFcl(A))) ⊆ A. Hence PFpcl(A) ⊆ A ⊆ U. Therefore A is a PFGPCS in X.

Example 3.6: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.4, 0.6 >, < a, 0.2, 0.7 >}. Then the PFS A = {< a, 0.3, 0.7 >, < a, 0.1, 0.8 >} is a PFGPCS in X but not a PFαCS in X since PFcl(PFint(PFcl(A))) = {< a, 0.6, 0.4 >, < a, 0.7, 0.2 >} ⊈ A.

Theorem 3.7: Every PFGCS is a PFGPCS but not conversely.

Proof: Let A be a PFGCS in X and let A ⊆ U and U is a PFOS in (X, τ). Since PFpcl(A) ⊆ PFcl(A) and by hypothesis, PFpcl(A) ⊆ U. Therefore A is a PFGPCS in X.

Example 3.8: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.4, 0.6>, < a, 0.5, 0.5 Then the PFS A = {< a, 0.3, 0.7 >, < a, 0.4, 0.6 >} is a PFGPCS in X but not a PFGCS in X since A ⊆ T but PFcl(A) = {< a, 0.6, 0.4 >, < a, 0.5, 0.5 >} ⊈ T.

Theorem 3.9: Every PFRCS is a PFGPCS but not conversely.

Proof: Let A be a PFRCS in X. By Definition 2.9, A = PFcl(PFint(A)). This implies PFcl(A) = PFcl(PFint(A)). Therefore PFcl(A) = A. That is A is a PFCS in X. By Theorem 3.3, A is a PFGPCS in X.

Example 3.10: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.6, 0.4 >, < a, 0.7, 0.2 >}. Then the PFS A = {< a, 0.3, 0.7 >, < a, 0.2, 0.8 >} is a PFGPCS but not a PFRCS in X since PFcl(PFint(A)) = 0 ≠ A.

Theorem 3.11: Every PFPCS is a PFGPCS but not conversely.

Proof: Let A be a PFPCS in X and let A ⊆ U and U is a PFOS in (X, τ). By Definition 2.5, PFcl(PFint(A)) ⊆ A. This implies that PFpcl(A) = A ∪ PFcl(PFint(A)) ⊆ A. Therefore PFpcl(A) ⊆ U. Hence A is a PFGPCS in X.

Example 3.12: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.6, 0.4 >, < a, 0.3, 0.7 >}. Then the PFS A = {< a, 0.8, 0.2 >, < a, 0.3, 0.7 >} is a PFGPCS but not a PFPCS in X since PFcl(PFint(A)) = 1 ⊈ A.

Theorem 3.13: Every PFαGCS is a PFGPCS but not conversely.

Proof: Let A be a PFαGCS in X and let A ⊆ U and U is a PFOS in (X, τ). By Definition 2.14, A ∪ PFcl(PFint(PFcl(A))) ⊆ U. This implies PFcl(PFint(PFcl(A))) ⊆ U and PFcl(PFint(A)) ⊆ U. Therefore PFpcl(A) = A ∪ PFcl(PFint(A)) ⊆ U. Hence A is a PFGPCS in X.

Example 3.14: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.5, 0.5 >, < a, 0.6, 0.4 >}. Then the PFS A = {< a, 0.4, 0.6 >, < a, 0.5, 0.5 >} is a PFGPCS but not a PFαGCS in X since PFαcl(A) = 1 ⊈ T.

Proposition 3.15: PFSCS and PFGPCS are independent to each other.

Example 3.16: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.5, 0.5 >, < a, 0.2, 0.6 >}. Then the PFS A = T is a PFSCS but not a PFGPCS in X since A ⊆ T but PFpcl(A) = {< a, 0.5, 0.5 >, < a, 0.6, 0.2 >} ⊈ T.

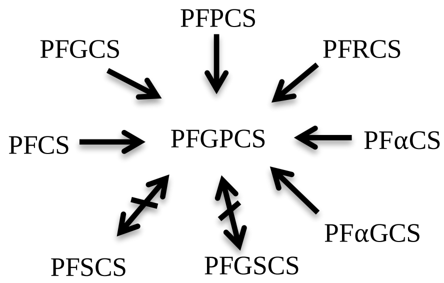
Example 3.17: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.8, 0.2 >, < a, 0.8, 0.2 >}. Then the PFS A = {< a, 0.8, 0.2 >, < a, 0.7, 0.2 >} is a PFGPCS but not a PFSCS in X since PFint(PFcl(A)) ⊈ A.

Proposition 3.18: PFGSCS and PFGPCS are independent to each other.

Example 3.19: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.5, 0.5 >, < a, 0.2, 0.6 >}. Then the PFS A = T is a PFSCS but not a PFGPCS in X since A ⊆ T but PFpcl(A) = {< a, 0.5, 0.5 >, < a, 0.6, 0.2 >} ⊈ T.

Example 3.20: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.7, 0.3 >, < a, 0.9, 0.1 >}. Then the PFS A = {< a, 0.6, 0.4 >, < a, 0.7, 0.3 >} is a PFGPCS but not a PFGSCS in X since A ⊆ T but PFscl(A) = 1 ⊈ T.

The following implications are true.



In this diagram by "A → B" we mean A implies B but not conversely and "A ↮ B" means A and B are independent of each other. None of them is reversible.

Remark 3.21: The union of any two PFGPCSs is not a PFGPCS in general as seen in the following example.

Example 3.22: Let X = {a, b} be a PFTS and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.6, 0.4 >, < a, 0.8, 0.2 >}. Then the PFSs A = {< a, 0.1, 0.9 >, < a, 0.8, 0.2 >}, B = {< a, 0.6, 0.4 >, < a, 0.7, 0.3 >} are PFGPCSs but A ∪ B is not a PFGPCS in X.

IV. PYTHAGOREAN FUZZY GENERALIZED PRE-OPEN SETS

In this section we introduce Pythagorean Fuzzy generalized pre-open sets and studied some of its properties.

Definition 4.1: A PFS A is said to be a Pythagorean Fuzzy generalized pre-open set (PFGPOS in short) in (X, τ) if the complement A^c is a PFGPCS in X.

The family of all PFGPOSs of a PFTS (X,τ) is denoted by PFGPO(X).

Example 4.2: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.7, 0.2 >, < a, 0.6, 0.3 >}. Then the PFS A = {< a, 0.8, 0.2 >, < a, 0.7, 0.2 >} is a PFGPOS in X.

Theorem 4.3: For any PFTS (X,τ), we have the following:

- Every PFOS is a PFGPOS
- Every PFSOS is a PFGPOS
- Every PFαOS is a PFGPOS
- Every PFPOS is a PFGPOS.

The converse of the above statements need not be true which can be seen from the following examples.

Example 4.4: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.2, 0.8 >, < a, 0.3, 0.7 >}. Then the PFS A = {< a, 0.8, 0.2 >, < a, 0.7, 0.2 >} is a PFGPOS in (X, τ) but not a PFOS in X.

Example 4.5: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.6, 0.4 >, < a, 0.4, 0.6 >}. Then the PFS A = {< a, 0.2, 0.8 >, < a, 0.7, 0.3 >} is a PFGPOS but not a PFSOS in X.

Example 4.6: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.4, 0.6 >, < a, 0.2, 0.7 >}. Then the PFS A = {< a, 0.7, 0.3 >, < a, 0.8, 0.1 >} is a PFGPOS but not a PFαOS in X.

Example 4.7: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.6, 0.4 >, < a, 0.5, 0.5 >}. Then the PFS A = {< a, 0.7, 0.3 >, < a, 0.6, 0.4 >} is a PFGPOS but not a PFPOS in X.

Theorem 4.8: Let (X, τ) be a PFTS. If A ∈ PFGPO(X) then V ⊆ PFint(PFcl(A)) whenever V ⊆ A and V is PFCS in X.

Proof: Let A ∈ PFGPO(X). Then A^c is a PFGPCS in X. Therefore PFpcl(A^c) ⊆ U whenever A^c ⊆ U and U is a PFOS in X. That is PFcl(PFint(A^c)) ⊆ U. This implies U^c ⊆ PFint(PFcl(A)) whenever U^c ⊆ A and U^c is PFCS in X. Replacing U^c by V, we get V ⊆ PFint(PFcl(A)) whenever V ⊆ A and V is PFCS in X.

Theorem 4.9: Let (X, τ) be a PFTS. Then for every A ∈ PFGPO(X) and for every B ∈ PFS(X), PFpint(A) ⊆ B ⊆ A implies B ∈ PFGPO(X).

Proof: By hypothesis A^c ⊆ B^c ⊆ $(\text{PFpint}(A))^c$. Let B^c ⊆ U and U be a PFOS. Since A^c ⊆ B^c , A^c ⊆ U. But A^c is a PFGPCS, PFpcl(A^c) ⊆ U. Also B^c ⊆ $(\text{PFpint}(A))^c$ = PFpcl(A^c). Therefore PFpcl(B^c) ⊆ PFpcl(A^c) ⊆ U. Hence B^c is a PFGPCS. Which implies B is a PFGPOS of X.

Remark 4.10: The intersection of any two PFGPOSs is not a PFGPOS in general.

Example 4.11: Let X = {a, b} and let τ = {0, T, 1} be a PFTS be a PFT on X, where T = {< a, 0.6, 0.4 >, < a, 0.8, 0.2 >}. Then the PFSs A = {< a, 0.9, 0.1 >, < a, 0.2, 0.8 >} and B = {< a, 0.4, 0.6 >, < a, 0.3, 0.7 >} are PFGPOSs but A ∩ B is not a PFGPOS in X.

Theorem 4.12: A PFS A of a PFTS (X, τ) is a PFGPOS if and only if F ⊆ PFpint(A) whenever F is a PFCS and F ⊆ A.

Proof: Necessity: Suppose A is a PFGPOS in X. Let F be a PFCS and F ⊆ A. Then F^c is a Pythagorean Fuzzy Open set in X such that $A^c \subseteq F^c$. Since A^c is a PFGPCS, we have PFpcl(A^c) ⊆ F^c. Hence $(\text{PFpint}(A))^c \subseteq F^c$. Therefore F ⊆ PFpint(A).

Sufficiency: Let A be a PFS of X and let F ⊆ PFpint(A) whenever F is a PFCS and F ⊆ A. Then $A^c \subseteq F^c$ and F^c is a PFOS. By hypothesis, $(\text{PFpint}(A))^c \subseteq F^c$. Which implies PFpcl($A^c$) ⊆ F^c. Therefore A^c is a PFGPCS of X. Hence A is a PFGPOS of X.

Corollary 4.13: A PFS A of a PFTS (X, τ) is a PFGPOS if and only if F ⊆ PFint(PFcl(A)) whenever F is a PFCS and F ⊆ A.

Proof: Necessity: Suppose A is a PFGPOS in X. Let F be a PFCS and F ⊆A.Then F^c is a PFOS in X such that $A^c \subseteq F^c$. Since A^c is a PFGPCS, we have PFpcl(A^c) ⊆ F^c.Therefore PFcl(PFint(A^c) ⊆ F^c. Hence $(\text{PFint}(\text{PFcl}(A)))^c \subseteq F^c$. Therefore F ⊆ PFint(PFcl(A)).

Sufficiency: Let A be a PFS of X and let F ⊆ PFint(PFcl(A)) whenever F is a PFCS and F ⊆ A. Then $A^c \subseteq F^c$ and F^c is a PFOS. By hypothesis, $(\text{PFint}(\text{PFcl}(A)))^c \subseteq F^c$. Hence PFcl(PFint($A^c$)) ⊆ F^c , which implies pcl(A^c) ⊆ F^c. Hence A is a PFGPOS of X.

Theorem 4.14: For a PFS A, A is a PFOS and a PFGPCS in X if and only if A is a PFROS in X.

Proof: Necessity: Let A be a PFOS and a PFGPCS in X. Then pcl(A) ⊆ A. This implies PFcl(PFint(A)) ⊆ A. Since A is a PFOS, it is a PFPOS. Hence A ⊆ PFint(PFcl(A)). Therefore A = PFint(PFcl(A)). Hence A is a PFROS in X.

Sufficiency: Let A be a PFROS in X. Therefore A = PFint(PFcl(A)). Let A ⊆ U and U is a PFOS in X. This implies PFpcl(A) ⊆ A. Hence A is a PFGPCS in X.

V. APPLICATIONS OF PYTHAGOREAN FUZZY GENERALIZED PRE-CLOSED SETS

In this section we provide some applications of Pythagorean Fuzzy generalized pre-closed sets.

Definition 5.1: A PFTS (X, τ) is said to be a Pythagorean Fuzzy $_pT_{1/2}$ ($PF_pT_{1/2}$ in short) space if every PFGPCS in X is a PFCS in X.

Definition 5.2: A PFTS (X, τ) is said to be a Pythagorean Fuzzy $_{gp}T_{1/2}$ ($PF_{gp}T_{1/2}$ in short) space if every PFGPCS in X is a PFPCS in X.

Theorem 5.3: Every $PF_pT_{1/2}$ space is a $PF_{gp}T_{1/2}$ space. But the converse is not true in general.

Proof: Let X be a $PF_pT_{1/2}$ space and let A be a PFGPCS in X. By hypothesis A is a PFCS in X. Since every PFCS is a PFPCS, A is a PFPCS in X. Hence X is a $PF_{gp}T_{1/2}$ space.

But the converse need not be true which can be seen in the following example.

Example 5.4: Let X = {a, b} and let τ = {0, T, 1} be a PFT on X, where T = {< a, 0.9, 0.1 >, < a, 0.9, 0.1 >}.Then (X, τ) is a $PF_{gp}T_{1/2}$ space. But it is not a $PF_pT_{1/2}$ space since the PFS A = {< a, 0.2, 0.8 >, < a, 0.3, 0.7 >} is PFGPCS but not a PFCS in X.

Theorem 5.5: Let (X, τ) be a PFTS and X is a $PF_pT_{1/2}$ space then
(i) Any union of PFGPCSs is a PFGPCS.
(ii) Any intersection of PFGPOSs is a PFGPOS.

Proof: (i) Let $\{A_i\}_{i\in J}$ is a collection of PFGPCSs in a $PF_pT_{1/2}$ space (X, τ). Therefore every PFGPCS is a PFCS. But the union of PFCS is a PFCS. Hence the union of PFGPCS is a PFGPCS in X.
(ii) It can be proved by taking complement in (i).

Theorem 5.6: A PFTS X is a $PF_{gp}T_{1/2}$ space if and only if PFGPO(X) = PFPO(X).

Proof: Necessity: Let A be a PFGPOS in X, then A^c is a PFGPCS in X. By hypothesis A^c is a PFPCS in X. Therefore A is a PFPOS in X. Hence PFGPO(X) = PFPO(X).

Sufficiency: Let A be a PFGPCS in X. Then A^c is a PFGPOS in X. By hypothesis A^c is a PFPOS in X. Therefore A is a PFPCS in X. Hence X is a $PF_{gp}T_{1/2}$ space..

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