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An Introduction To Pythogorean Neutrosophic Refined Sets And Some Of Their Basic Operations

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Abstract :

In this paper a new set called PYTHOGOREAN NEUTROSOPHIC REFINED SET is introduced by applying an refinement idea to pythogorean nuetrosophic set and some of the basic operations are defined and explained with a suitable examples. Also we examine some of the desired properties of Pythogarean Neutrosophic Refined set with respect to the definitions introduced.

Keywords:

Neutrosophic refined set, Neutrosophic pythogorean set, Basic operations

1.INTRODUCTION :

In recent times many ideas have been introduced to deal with indeterminacy, uncertainty, vagueness. Fuzzy set theory[1], intuitionistic fuzzy sets[2],rough set theory[3] play different measures in handling inconsistent datas. However, all these above theories failed to deal with inconsistent information which exist in beliefs system.

In 1995, Smarandache [4] developed a new concept called neutrosophic set (NS) which generalizes probability set, fuzzy set and intuitionistic fuzzy set. Neutrosophic set can be described by membership degree, indeterminacy degree and non-membership degree. Smarandache[5] gave n-valued refined neutrosophic logic and its applications. Then, Ye and Ye [44] gave single valued neutrosophic sets and operations laws. R. Jhansi [6] introduced the concept of Pythagorean Neutrosophic set with T and F as dependent components. In this paper Pythogarean Nuetrosophic Refined Set is introduced and some of the basic concepts are explained .

This paper is arranged in the following manner. In section 2, some definitions and notion about neutrosophic set , neutrosophic refined set and neutrosophic pythogorean set theory are given. These definitions will help us in later section. In section 3 we study the concept of Pythogorean Neutrosophic Refined (multi) sets and their operations with examples.In section 4, some the set properties and laws are discussed. Finally we conclude the paper.

2.PRELIMINARIES:

In this section the basic definitions and results about Neutrosophic sets which are used in this paper are given .

DEFINITION:2.1

Let U be a universe. A Neutrosophic set A on U can be defined as follows:

A = { < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > : x ∈ U }

Where T_A, I_A, F_A : U → [0,1] and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership.

DEFINITION:2.2

Let U be a Universe, a Neutrosophic refined set on can be defined as follows:

A ={ <x , ($T_A^1(X), T_A^2(X), T_A^3(X), \ldots\ldots T_A^P(X)$) , ($I_A^1(X), I_A^2(X), I_A^3(X), \ldots\ldots I_A^P(X)$) ,

($F_A^1(X), F_A^2(X), F_A^3(X), \ldots\ldots F_A^P(X)$): x ∈ U}

Where $T_A^1(X), T_A^2(X), T_A^3(X), \ldots\ldots T_A^P(X)$: U → [0,1] , $I_A^1(X), I_A^2(X), I_A^3(X), \ldots\ldots I_A^P(X)$: U → [0,1] and

$F_A^1(X), F_A^2(X), F_A^3(X), \ldots\ldots F_A^P(X)$: U → [0,1] such that $0 \leq T_A^j(X) + I_A^j(X) + F_A^j(X) \leq 3$ for

j = 1,2,3,…p and for any x ∈ U . ($T_A^1(X), T_A^2(X), T_A^3(X), \ldots\ldots T_A^P(X)$) , ($I_A^1(X), I_A^2(X), I_A^3(X), \ldots\ldots I_A^P(X)$) ,

($F_A^1(X), F_A^2(X), F_A^3(X), \ldots\ldots F_A^P(X)$) is the Truth-membership sequence, Indeterminate membership sequence & Falsity-membership sequence of the element x , respectively. Also, p is called the dimension of Neutrosophic refined set (NRS) A .

DEFINITION:2.3

Let U be a universe. A Pythagorean Neutrosophic set with T and F are dependent Neutrosophic components A on U is an object of the form

A = { < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > : x ∈ U }

Where T_A, I_A, F_A : U → [0,1] and $0 \leq (T_A(X))^2 + (I_A(X))^2 + (F_A(X))^2 \leq 2$

$T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership.

DEFINITION:2.4

t-norms are associative, monotonic and commutative two valued functions t that map from [0, 1] × [0, 1] into [0, 1].

These properties are formulated with the following conditions: ∀a, b, c, d ∈ [0, 1],

1. t(0, 0) = 0 and t(a, 1) = t(1, a) = a,

2. If a ≤ c and b ≤ d, then t(a, b) ≤ t(c, d)

3. t(a, b) = t(b, a)

4. t(a, t(b, c)) = t(t(a, b), c).

DEFINITION:2.5

t-conorms (s-norm) are associative, monotonic and commutative two placed functions s which map from [0, 1]×[0, 1] into [0, 1].

These properties are formulated with the following conditions: ∀a, b, c, d ∈ [0, 1],

1. s(1, 1) = 1 and s(a, 0) = s(0, a) = a,

2. if a ≤ c and b ≤ d, then s(a, b) ≤ s(c, d)

3. s(a, b) = s(b, a)

4. s(a, s(b, c)) = s(s(a, b), c).

t-norm and t-conorm are related in a sense of logical duality. Typical dual pairs of non parametrized t-norm and t-conorm are compiled below:

1. Drastic product:

$$t_w(a,b) = \begin{cases} \min(a,b), & \max(ab) = 1 \\ 0 & otherwise \end{cases}$$

2. Drastic sum:

$$s_w(a,b) = \begin{cases} \max(a,b), & \min(ab) = 0 \\ 1 & otherwise \end{cases}$$

3. Bounded product:

t_1(a, b) = max{0, a + b − 1}

4. Bounded sum:

s_1 (a, b) = min{1, a + b}

5. Algebraic product:

t_2(a, b) = a.b

6. Algebraic sum:

s_2(a, b) = a + b − a.b

7. Minimum:

t_3(a, b) = min{a, b}

8. Maximum:

s_3 (a, b) = max{a, b}

3. PYTHOGOREAN NEUTROSOPHIC REFINED SET

DEFINITION: 3.1

Let U be a Universe. A Pythogorean Neutrosophic Refined Set can be defined as follows:

$$P_{PNR} =\{ < x , (T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^K(X)) , \big((I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X)\big) ,$$

$$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))>: x \in U\}$$

Where $T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X)$: U → [0,1] ,

$I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k(X)$: U → [0,1] and

$F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)$: U → [0,1] such that

And $0 \leq (T_P^k(X))^2 + (I_P^k(X))^2 + (F_P^k(X))^2 \leq 2$

for j = 1,2,3,…p and for any x ∈ U . $T_P^k(X)$ is the degree of membership sequence, $I_P^k(X)$ is the degree of indeterminacy membership sequence and $F_P^k(X)$ is the degree of non-membership sequence.

DEFINITION:3.2

Let P_{PNR} and Q_{PNR} be Pythogorean Nuetrosophic Refined sets (PNRS) in U. P_{PNR} is said to be Pythogorean Nuetrosophic Refined Subset of Q_{PNR},

If $T_P^k(X) \leq T_Q^k(X)$,$I_P^k(X) \geq I_Q^k(X)$, $F_P^k(X) \geq F_Q^k(X)$ for every x ∈ U.

It is denoted by $P_{PNR} \subseteq Q_{PNR}$

EXAMPLE: 3.3

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} are any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = {<x, [0.6,0.7,0.8],[0.4,0.5,0.6],[0.4,0.3,0.2]>}

Q_{PNR}= {<x, [0.9,0.8,0.8],[0.1,0.1,0.1],[0.1,0.2,0.2]>}

Then, the set P_{PNR} is subset of Q_{PNR} .

i.e) $P_{PNR} \subseteq Q_{PNR}$.

DEFINITION: 3.4

Let P_{PNR} and Q_{PNR} be Pythogorean Neutrosophic Refined sets (PNRS) in U. P_{PNR} is said to Pythogorean Neutrosophic Refined equal set of Q_{PNR},

If $T_P^k(X) = T_Q^k(X)$, $I_P^k(X) = I_Q^k(X)$, $F_P^k(X) = F_Q^k(X)$ for every x ∈ U.

It is denoted by $P_{PNR} = Q_{PNR}$.

EXAMPLE:3.5

Let X be a non – empty set in U.If P_{PNR} and Q_{PNR} are any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = {<x, [0.6,0.7,0.8],[0.4,0.5,0.6],[0.4,0.3,0.2]>}

Q_{PNR} = {<x, [0.6,0.7,0.8],[0.4,0.5,0.6],[0.4,0.3,0.2]>}

Then the sets P_{PNR} and Q_{PNR} are equal .

i.e) $P_{PNR} = Q_{PNR}$.

DEFINITION:3.6

Let P_{PNR} be Pythogorean Neutrosophic Refined sets (PNRS) in U. It's compliment is defined as follows:

$P_{PNR}{}^c$ = { <x , ($F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)$),

$\left(1 - I_P^1(X), 1 - I_P^2(X), 1 - I_P^3(X), \ldots\ldots\ldots\ldots 1 - I_P^k(X)\right)$,

$T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X)$ >: x ϵ U }.

It is denoted as $P_{PNR}{}^c$

EXAMPLE: 3.7

Let X be a non – empty set in U. The Pythogorean Neutrosophic Refined set P_{PNR} is defined as follows :

P_{PNR} = {<x, [0.2,0.3,0.4],[0.1,0.2,0.3],[0.8,0.7,0.6]>}

The compliment of the given set is,

$P_{PNR}{}^c$ = {<x, [0.8,0.7,0.6],[0.9,0.8,0.7],[0.2,0.3,0.4]>}

DEFINITION : 3.8

1. If $T_P^k(X)$ = 0 and $I_P^k(X) = F_P^k(X)$ = 1 for all j = 1,23,…..p ,then the set P_{PNR} is called null – Pythogorean Neutrosophic Refined Set . It is denoted as $\emptyset_{PNR}$

2. If $T_P^k(X)$ = 1 and $I_P^k(X) = F_P^k(X)$ = 0 for all j = 1,23,…..p ,then the set P_{PNR} is called Universal – Pythogorean Neutrosophic Refined Set . It is denoted as U_{PNR}

EXAMPLE:3.9

Let X be a non – empty set in U. The null –Pythogorean Neutrosophic Refined Set is ,

$\emptyset_{PNR}$ = { <x, [0,0,0],[1,1,1],[1,1,1] >}.

The Universel – Pythogorean Neutrosophic Refined Set is ,

U_{PNR} = { <x, [1,1,1],[0,0,0],[0,0,0] >}.

DEFINITION: 3.10

Let X be a non empty set in U,

P_{PNR} = { < x , $(T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^K(X))$, $\left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X\right)$,

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))$>: x ∈ U}

Q_{PNR} = { < x , $(T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X))$, $\left(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X\right)$,

$(F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X))$>: x ∈ U}

are Pythogorean Neutrosophic Refined sets (PNRS) in U.

The union of P_{PNR} and Q_{PNR} is defined as Follows :

$P_{PNR} \cup Q_{PNR}$ = { < x, s(($T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X))$,

t$\left(\left(I_P^1(X), I_Q^1(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)\right)\right.$,

t(($F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))$> : x ∈ U}.

EXAMPLE: 3.11

Let X be a non – empty set in U. . If P_{PNR} and Q_{PNR} are any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = { <x, [0.3,0.4,0.5],[0.2,0.2,0.2],[0.7,0.6,0.5] >}

Q_{PNR} = { <x, [0.5,0.6,0.9],[0.4,0.4,0.5],[0.5,0.4,0.1] >}

Then the union of two set is ,

$P_{PNR} \cup Q_{PNR}$ = { <x, [0.5,0.6,0.9],[0.2,0.2,0.2],[0.5,0.4,0.1] >}.

DEFINITION: 3.12

Let X be a non empty set in U,

P_{PNR} = { < x , $(T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^K(X))$, $\left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X\right)$,

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))$>: x ∈ U}

Q_{PNR} = { < x , $(T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X))$, $\left(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X\right)$,

$(F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X))$>: x ∈ U}

are Pythogorean Neutrosophic Refined sets (PNRS) in U.

The intersection of P_{PNR} and Q_{PNR} is defined as Follows:

$P_{PNR} \cap Q_{PNR}$ = { < x, t(($T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots .. (T_P^k(X), T_Q^k(X))$,

s(($I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots .. (I_P^k(X), I_Q^k(X))$,

s(($F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots .. (F_P^k(X), F_Q^k(X))$> : x ∈ U}.

EXAMPLE:3.13

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} are any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = { <x, [0.3,0.4,0.5],[0.2,0.2,0.2],[0.7,0.6,0.5] >}

Q_{PNR} = { <x, [0.5,0.6,0.9],[0.4,0.4,0.5],[0.5,0.4,0.1] >}

Then the intersection of two set is ,

$P_{PNR} \cap Q_{PNR}$ = { <x, [0.3,0.4,0.5].[0.4,0.4,0.5].[,0.7,0.6,0.5] >}.

DEFINITION:3.14

Let P_{PNR} and Q_{PNR} are Pythogorean Neutrosophic Refined sets (PNRS) in U.

Then $P_{PNR} \setminus Q_{PNR}$ is defined as follows:

$P_{PNR} \setminus Q_{PNR}$ = { < x, t($T_P^k(X),\ F_Q^k(X)$) , t($I_P^k(X), 1 - I_Q^k(X)$), s($F_P^k(X),\ T_Q^k(X)$) >: x ∈ U}

For every j = 1,2,3,…..p.

EXAMPLE:3.15

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} be any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = { <x, [0.3,0.4,0.5],[0.2,0.2,0.2],[0.7,0.6,0.5] >}

Q_{PNR} = { <x, [0.5,0.6,0.9],[0.4,0.4,0.5],[0.5,0.4,0.1] >}

The difference of two set is ,

$P_{PNR} \setminus Q_{PNR}$ = { <x, [0.3,0.4,0.1],[0.6,0.6,0.5],[0.7,0.6,0.9] >}.

4.ALGEBRIC PROPERTIES OF PYTHOGOREAN NUETROSOPHIC REFINED SETS

PROPOSITION: 4.1 (Commutative Law)

Let P_{PNR} , Q_{PNR} ∈ PNRS (U) . Then,

1. $P_{PNR} \cup Q_{PNR} = Q_{PNR} \cup P_{PNR}$
2. $P_{PNR} \cap Q_{PNR} = Q_{PNR} \cap P_{PNR}$

Thus the Pythogorean Neutrosophic Refined Union and Intersection satisfies commutative property.

EXAMPLE: 4.2

Let X be a non – empty set in U. . If P_{PNR} and Q_{PNR} are any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = { <x, [0.3,0.4,0.5],[0.2,0.2,0.2],[0.7,0.6,0.5] >}

Q_{PNR} = { <x, [0.5,0.6,0.9],[0.4,0.4,0.5],[0.5,0.4,0.1] >}

$P_{PNR} \cup Q_{PNR}$ = { <x, [0.5,0.6,0.9],[0.2,0.2,0.2],[0.5,0.4,0.1] >}

$Q_{PNR} \cup P_{PNR}$ = { <x, [0.5,0.6,0.9],[0.2,0.2,0.2],[0.5,0.4,0.1] >}

Thus, $P_{PNR} \cup Q_{PNR} = Q_{PNR} \cup P_{PNR}$.

$P_{PNR} \cap Q_{PNR}$ = { <x, [0.3,0.4,0.5],[0.4,0.4,0.5],[0.7,0.6,0.5] >}

$Q_{PNR} \cap P_{PNR}$ = { <x, [0.3,0.4,0.5],[0.4,0.4,0.5],[0.7,0.6,0.5] >}

Thus, $P_{PNR} \cap Q_{PNR} = Q_{PNR} \cap P_{PNR}$

PREPOSITION: 4.3 (Associative Law)

Let P_{PNR} , Q_{PNR} , R_{PNR} are any Pythogarean Nuetrosophic Refined sets PNRS (U) defined as follows : Then,

1. $P_{PNR} \cup (Q_{PNR} \cup R_{PNR}) = (P_{PNR} \cup Q_{PNR}) \cup R_{PNR}$
2. $P_{PNR} \cap (Q_{PNR} \cap R_{PNR}) = (P_{PNR} \cap Q_{PNR}) \cap R_{PNR}$

Thus the Pythogorean Neutrosophic Refined Union and Intersection satisfies Associative property.

EXAMPLE: 4.4

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} be any Pythogorean Neutrosophic Refined sets defined as follows :

P_{PNR} = { <x, [0.3,0.4,0.5],[0.2,0.2,0.2],[0.7,0.6,0.5] >}

Q_{PNR} = { <x, [0.5,0.6,0.9],[0.4,0.4,0.5],[0.5,0.4,0.1] >}

R_{PNR} = { <x, [0.3,0.4,0.5],[0.1,0.1,0.1],[0.7,0.6,0.5] >}

$(Q_{PNR} \cup R_{PNR})$ = { <x, [0.5,0.6,0.9],[0.1,0.1,0.1],[0.5,0.4,0.1] >}

$P_{PNR} \cup (Q_{PNR} \cup R_{PNR})$ = { <x, [0.5,0.6,0.9],[0.1,0.1,0.1],[0.5,0.4,0.1] >} ----- (1)

$(P_{PNR} \cup Q_{PNR})$ = { <x, [0.5,0.6,0.9],[0.2,0.2,0.2],[0.5,0.4,0.1] >}

$(P_{PNR} \cup Q_{PNR}) \cup R_{PNR}$ = { <x, [0.5,0.6,0.9],[0.1,0.1,0.1],[0.5,0.4,0.1] >} ------ (2)

Thus from (1) and (2),

$P_{PNR} \cup (Q_{PNR} \cup R_{PNR}) = (P_{PNR} \cup Q_{PNR}) \cup R_{PNR}$ is satisfied.

$(Q_{PNR} \cap R_{PNR})$ = { <x, [0.3,0.4,0.5],[0.4,0.4,0.5],[0.7,0.6,0.5] >}

$P_{PNR} \cap (Q_{PNR} \cap R_{PNR})$ = { <x, [0.3,0.4,0.5],[0.4,0.4,0.5],[0.7,0.6,0.5] >} ------ (3)

$(P_{PNR} \cap Q_{PNR})$ = { <x, [0.3,0.4,0.5],[0.4,0.4,0.5],[0.7,0.6,0.5] >}

$(P_{PNR} \cap Q_{PNR}) \cap R_{PNR}$ = { <x, [0.3,0.4,0.5],[0.4,0.4,0.5],[0.7,0.6,0.5] >} ----- (4)

Thus from (3) and (4),

$P_{PNR} \cap (Q_{PNR} \cap R_{PNR}) = (P_{PNR} \cap Q_{PNR}) \cap R_{PNR}$ is satisfied.

PROPOSITION: 4.5 (Idempotent Law)

Let P_{PNR} be any Pythogorean Neutrosophic Refined set in U. Then,

1. $P_{PNR} \cup \emptyset_{PNR} = P_{PNR}$
2. $P_{PNR} \cap U_{PNR} = P_{PNR}$

PROOF: The proof is clear from the definition 3.11 and 3.12

PROPOSITION:4.6 (Domination Law)

Let P_{PNR} be any Pythogorean Neutrosophic Refined set in U. Then,

1. $P_{PNR} \cup U_{PNR} = U_{PNR}$
2. $P_{PNR} \cap \emptyset_{PNR} = \emptyset_{PNR}$

PROOF: It is clear from the definition 3.11 and 3.12

PREPOSITION: 4.7 (Double Compliment Law)

Let P_{PNR} be any Pythogorean Neutrosophic Refined set in U. Then,

1. $({P_{PNR}}^c)^c = P_{PNR}$

PROOF:

P_{PNR} = { <x, $T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X), \left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X \right)$,

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))$>: x ∈ U}

${P_{PNR}}^c$ = { <x , $(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))$,

$\left(1 - I_P^1(X), 1 - I_P^2(X), 1 - I_P^3(X), \ldots\ldots\ldots 1 - I_P^k(X) \right)$,

$T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X)$ >: x ϵ U }.

$({P_{PNR}}^c)^c$ = { <x, $T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X), \left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X \right)$,

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))$>: x ∈ U}

HENCE PROVED.

PROPOSITION: 4.8 (De Morgan's Law)

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} be any Pythogorean Neutrosophic Refined sets then, De Morgan's law is valid.

1. $(P_{PNR} \cup Q_{PNR})^c = {P_{PNR}}^c \cap {Q_{PNR}}^c$
2. $(P_{PNR} \cap Q_{PNR})^c = {P_{PNR}}^c \cup {Q_{PNR}}^c$

PROOF:

$(P_{PNR} \cup Q_{PNR})^c = (\{<x, T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X), \left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X\right)),$

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))>: x \in U\} \cup$

$\{<x, (T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X)), \left(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X\right)),$

$(F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X))>: x \in U))^c$

$= \{<x, s((T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X)),$

$t(\left(I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)\right),$

$t((F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))> : x \in U\}^c$

$= \{<x, t((F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^j(X), F_Q^j(X))$

$s(\left(1 - I_P^1(X), 1 - I_Q^1(X), (1 - I_P^2(X), 1 - I_Q^2(X), \ldots\ldots (1 - I_P^j(X), 1 - I_Q^j(X)\right),$

$t((T_P^1(X), T_Q^1(X), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^j(X), T_Q^j(X)) >\}$

$= {P_{PNR}}^c \cap {Q_{PNR}}^c$

$(P_{PNR} \cap Q_{PNR})^c = (\{<x, T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X), \left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X\right)),$

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))>: x \in U\} \cap$

$\{<x, (T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X)), \left(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X\right)),$

$(F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X))>: x \in U))^c$

$= (\{<x, t((T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X)),$

$s(\left(I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)\right),$

$s((F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))> : x \in U\}^c$

$= \{<x, s((F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^j(X), F_Q^j(X))$

$t(\left(1 - I_P^1(X), 1 - I_Q^1(X), (1 - I_P^2(X), 1 - I_Q^2(X), \ldots\ldots (1 - I_P^j(X), 1 - I_Q^j(X)\right),$

$t((T_P^1(X), T_Q^1(X), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^j(X), T_Q^j(X)) >\}$

$= {P_{PNR}}^c \cup {Q_{PNR}}^c$

Hence Proved .

THEOREM: 4.9

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} be any Pythogorean Neutrosophic Refined sets then,

$(P_{PNR} \subseteq Q_{PNR}) \Leftrightarrow {P_{PNR}}^c \subseteq {Q_{PNR}}^c$

PROOF: The proof is clear from the definition 3.3 and 3.6

PROPOSITION:4.10

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} be any Pythogorean Neutrosophic Refined sets then,

1. $P_{PNR} \setminus Q_{PNR} = P_{PNR} \cap Q_{PNR}$
2. $Q_{PNR} \setminus P_{PNR} = Q_{PNR} \cap P_{PNR}$
3. $P_{PNR} \setminus Q_{PNR} = P_{PNR}$ if $P_{PNR} \cap Q_{PNR} = \emptyset_{PNR}$

PROOF: The proof is clear from the definitions 3.12 and 3.14

PREPOSITION:4.11

Let X be a non – empty set in U. If P_{PNR} and Q_{PNR} be any Pythogorean Neutrosophic Refined sets then,

1. $(P_{PNR} \setminus Q_{PNR}) \cup Q_{PNR} = P_{PNR} \setminus Q_{PNR}$
2. $(P_{PNR} \setminus Q_{PNR}) \cap Q_{PNR} = \emptyset_{PNR}$
3. $(P_{PNR} \setminus Q_{PNR}) \cup (Q_{PNR} \setminus P_{PNR}) = (P_{PNR} \cup Q_{PNR}) \setminus (P_{PNR} \cap Q_{PNR})$

PROOF: The proof is obvious.

PROPOSITION: 4.12 (DISTRIBUTIVE LAW)

Let P_{PNR} , Q_{PNR} , R_{PNR} are any Pythogorean Neutrosophic Refined sets PNRS (U)

defined as follows : Then,

1. $P_{PNR} \cup (Q_{PNR} \cap R_{PNR}) = (P_{PNR} \cup Q_{PNR}) \cap (P_{PNR} \cup R_{PNR})$
2. $P_{PNR} \cap (Q_{PNR} \cup R_{PNR}) = (P_{PNR} \cap Q_{PNR}) \cup (P_{PNR} \cap R_{PNR})$

PROOF:

$P_{PNR} \cup (Q_{PNR} \cap R_{PNR})$ = ({<x , $T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X), \left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X\right))$,

$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X))$>: x ∈ U} ∪

({ < x , $(T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X))$, $\left(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X\right))$,

$(F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X))$>: x ∈ U} ∩

{ < x , $(T_R^1(X), T_R^2(X), T_R^3(X), \ldots\ldots T_R^K(X))$, $\left(I_R^1(X), I_R^2(X), I_R^3(X), \ldots\ldots I_R^k X\right))$,

$(F_R^1(X), F_R^2(X), F_R^3(X), \ldots\ldots F_R^k(X))$>: x ∈ U})

= ({ < x, s(($T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X))$,

t($\left(I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)\right)$,

t(($F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))$> }) ∩

({ < x, s(($T_Q^1(X), T_R^1(X)), (T_Q^2(X), T_R^2(X), \ldots\ldots (T_Q^k(X), T_R^k(X))$, t($\Big($ $I_Q^1(X), I_R^2(X), (I_Q^2(X), I_R^2(X), \ldots\ldots (I_Q^k(X), I_R^k(X)\Big)$,

t(($F_Q^1(X), F_R^1(X)), (F_Q^2(X), F_R^2(X)), \ldots\ldots (F_Q^k(X), F_R^k(X))$> })

= $(P_{PNR} \cup Q_{PNR}) \cap (P_{PNR} \cup R_{PNR})$

$P_{PNR} \cap (Q_{PNR} \cup R_{PNR})$ = ({<x , $T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X), \big(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X)\big)$,

($F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)$)>: x ∈ U} ∩

({ < x , ($T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X)$) , $\big(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X)\big)$,

($F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X)$)>: x ∈ U} ∪

{ < x , ($T_R^1(X), T_R^2(X), T_R^3(X), \ldots\ldots T_R^K(X)$) , $\big(I_R^1(X), I_R^2(X), I_R^3(X), \ldots\ldots I_R^k X)\big)$,

($F_R^1(X), F_R^2(X), F_R^3(X), \ldots\ldots F_R^k(X)$)>: x ∈ U})

= ({ < x, t(($T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X))$,

s($\Big($ $I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)\Big)$,

s(($F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))$> }) ∪

({ < x, t(($T_Q^1(X), T_R^1(X)), (T_Q^2(X), T_R^2(X), \ldots\ldots (T_Q^k(X), T_R^k(X))$,

s($\Big($ $I_Q^1(X), I_R^2(X), (I_Q^2(X), I_R^2(X), \ldots\ldots (I_Q^k(X), I_R^k(X)\Big)$,

s(($F_Q^1(X), F_R^1(X)), (F_Q^2(X), F_R^2(X)), \ldots\ldots (F_Q^k(X), F_R^k(X))$> })

= $(P_{PNR} \cap Q_{PNR}) \cup (P_{PNR} \cap R_{PNR})$

Hence Proved.

5. CONCLUSION:

This paper ensures the work of introducing the new set namely Pythogorean Neutrosophic Refined Set by developing the concepts of Nuetrosophic Refined set and Pythogorean Neutrosophic sets. Some of the basic operations and laws are defined and illustrated with an examples.

Conflicts Of Interest: The authors declare no conflict of interest.

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