Geometrical Representation and Analysis of Lissajous Figures using wxMaxima

Vijay Jadhav[1], Aparna Jaware[1], Harish Kumar Dubey[1], Dattatray E. Kshirsagar[1*]

[1] Department of Physics, B. K. Birla College, Kalyan(MS),India .

Abstract

The undergraduate students find it difficult to solve trigonometric functions used in Maxwell's equation because they lack in mathematical skills. Therefore, the use of mathematical tools and graphs for plotting will make this task easy and interesting. Nowadays, wxMaxima is open-source mathematical software for algebraic equations, calculus operations, and graph plotting used by many students. wxMaxima is used in a physical system to imagine the results of various parameters. It can act as a tool for simulation. We use wxMaxima in this article to solve an example of Lissajous figures and a damped harmonic oscillator.

Keywords: Lissajous figures, harmonic motion, wave equations, wxMaxima.

1. Introduction:

For electronic communications, the exact calibration must be carried out. The device is used to compare a very exact crystal-controlled oscillator with the oscillator of the transmitter. This is achieved by a Lissajous figure and the perfect circle is obtained [1]. It is very interesting for students to study Lissajous figures, the nature and shape of the figures with their phases. Proving it with complex equations is truly a herculean task. students learn more with illustrations, images, and simulations, i.e. visual effects improve students' ability to comprehend physics. wxMaxima is free software for simplifying and solving complex equations and a perfect tool for graphics modeling and display. Here we will use this software as an instrument to help students to begin their understanding of physics. A lot of researchers used wxMaxima to solve problems in physics. M Ikhsan and Suhartati et al used wxMaxima for the study of the vectors [2]. D Afriza1 et al used wxMaxima in electrodynamics to solve the Gauss problems [3]. Timberlake, T.K., and J.W. Mixon have solved various problems with wxMaxima through his lecture notes on physics and classical mechanics [4]. SK Velten et al performed simulation and developed a mathematical model using wxMaxima [5]. F. L. Braga, Wesley Spalenza, Kc Lemos Filhot conducted orbital simulations for a celestial mechanical system using the Lagrangean relativistic formalism using wxMaxima [6].

Lissajous figures can be obtained only by solving wave equations. Due to various complex wave equations, diagramming them on board is difficult. With wxMaxima, these equations can be solved and simplified. However, still, it is hard to imagine and understand the plot with one wave equation on X-axis and the other on Y-axis. wxMaxima plots that with certain commands easily. The parameters can also be easily modified in the instruction, and the effect of changing the parameters can be easily displayed on the screen itself.

2. Method:

This paper investigated the effectiveness of using wxMaxima, and a set of commands used to display Lissajous figures. The waveform and the Lissajous patterns can be easily illustrated on the computer by adjusting parameters. When two simple harmonic motions of the same frequencies are acting along the X-axis and Y-axis respectively and if their displacements at any time t are x and y, then they can be represented by amplitude and frequency ratios.

$$x = A sin(\omega_1 t + \alpha) \qquad \qquad \text{......(1)} \quad \text{and}$$
$$y = B sin(\omega_2 t + \beta) \qquad \qquad \text{......(2)}$$

where A and B are the amplitudes of the component vibrations having frequencies ω_1 and ω_2 the phase of the first vibration is α and the second vibration is β. The first vibration is ahead of the second by a phase angle $(\beta - \alpha)$, i.e.the phase difference between the two vibrations is $(\beta - \alpha)$. When ω_1 is equal to ω_2, the above equations can be solved using wxMaxima and we can obtain,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - 2\frac{xy}{AB}\cos(\beta - \alpha) = sin^2(\beta - \alpha) \qquad \qquad \text{..... (3)}$$

⬜Corresponding author at: Department of Physics, B. K. Birla College, Kalyan(MS),India Tel.: +91 8652 431 228.
E-mail address: kesagar2008@gmail.com (Dattatray E. Kshirsagar).

This expression represents the resultant motion of the two vibrations represented by (1) and (2). It is an ellipse inclined to the axes of coordinates.

The nature of the graph depends on the frequency ratio ω_1/ω_2. When $\omega_1/\omega_2 = 1$, the graph is an ellipse, furthermore if A = B the graph is a circle. The frequency ratio can be found by the ratio of the number of times the graph touches the x and y axes, respectively [7,8,9].

3. Results and Discussion:

Equations (1) and (2) are two simple harmonic motions plotted in figure 1 and figure 2 using wxMaxima with commands shown.

(%i1)   x=A*sin(ω*t+α);

(%o1)   x=A*sin(t*ω+α)

(%i2)   %o1,A=5,ω=π,α=π/4;

(%o2)   x=5*sin(%pi*t+%pi/4)

(%i3)   x(t):=5*sin(π*t+π/4)$
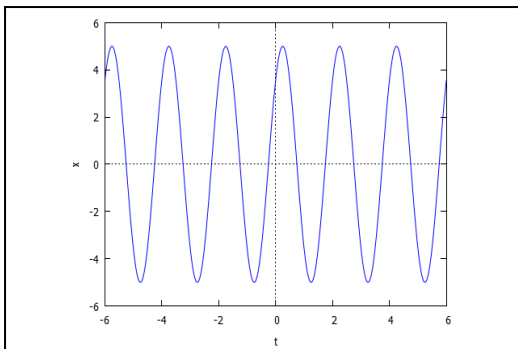
(%i5)   plot2d([x],[t,-10,10],[x,-6,6])$

Figure-1. Plot of $x = A sin(\omega_1 t + \alpha)$

(%i7)   y=B*sin(ω*t+β);

(%o7)   y=B*sin(t*ω+β)

(%i8)   %o7,B=0.5,ω=π,β=π/4;

(%o8)   y=0.5*sin(%pi*t+%pi/4)

(%i9)   y(t):=0.5*sin(π*t+π/4)$
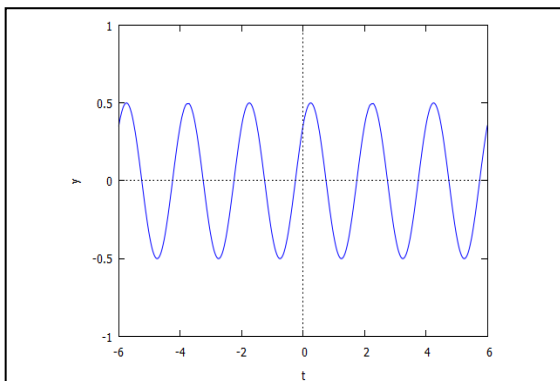
(%i15) plot2d([y],[t,-6,6],[y,-1,1])$

Figure-2. Plot of $y = B sin(\omega_2 t + \beta)$

Equation (3) is obtained using following commands

(x/A)=sin(ω*t+α);

(%o1) x/A=sin(t*ω+α)

--> (x/A)=sin(ω*t)*cos(α)+cos(ω*t)*sin(α);

(%o2) x/A=cos(α)*sin(t*ω)+sin(α)*cos(t*ω)

--> (y/B)=sin(ω*t+β);

(%o3) y/B=sin(t*ω+β)

--> (y/B)=sin(ω*t)*cos(β)+cos(ω*t)*sin(β);

(%o4) y/B=cos(β)*sin(t*ω)+sin(β)*cos(t*ω)

--> (x/A=cos(α)*sin(t*ω)+sin(α)*cos(t*ω))^2;

(%o5) x^2/A^2=(cos(α)*sin(t*ω)+sin(α)*cos(t*ω))^2

--> (y/B=cos(β)*sin(t*ω)+sin(β)*cos(t*ω))^2;

(%o6) y^2/B^2=(cos(β)*sin(t*ω)+sin(β)*cos(t*ω))^2

--> (x^2/A^2=(cos(α)*sin(t*ω)+sin(α)*cos(t*ω))^2)+(y^2/B^2=(cos(β)*sin(t*ω)+sin(β)*cos(t*ω))^2);

(%o7) y^2/B^2+x^2/A^2=(cos(β)*sin(t*ω)+sin(β)*cos(t*ω))^2+(cos(α)*sin(t*ω)+sin(α)*cos(t*ω))^2

--> (x/A)^2+(y/B)^2-(2*x*y*cos(β-α)/(A*B)) =(sin(β-α))^2;

(%o8) -(2*x*y*cos(β-α))/(A*B)+y^2/B^2+x^2/A^2=sin(β-α)^2

(%i3) (x/A)^2+(y/B)^2-(2*x*y*cos(β-α)/(A*B)) =(sin(β-α))^2;

(%o3) -(2*x*y*cos(β-α))/(A*B)+y^2/B^2+x^2/A^2=sin(β-α)^2

To plot equation (3) the condition $(\beta - \alpha) = 0$ and $\omega_1 = \omega_2$ is considered and it is plotted in figure 3. It is a straight line with slope $\frac{B}{A}$ with an inclination of $45°$. However, when $(\beta - \alpha) = \pi$ and $\omega_1 = \omega_2$,it is a straight line with slope $\frac{B}{A}$ having an inclination of $135°$.

(%i4) %o3, α=0, β=0;

(%o4) y^2/B^2-(2*x*y)/(A*B)+x^2/A^2=0

(%i5) (x/A)-(y/B)=0;

(%o5) x/A-y/B=0

(%i7) %o5, A=5, B=0.5;

(%o7) x/5-2.0*y=0

(%i8) load (implicit plot)$

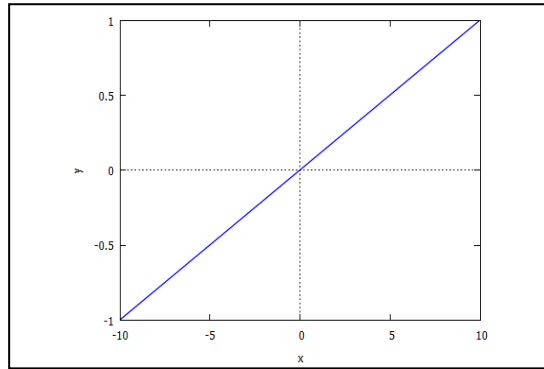(%i10) implicit plot (x/5-2.0*y=0,[x,-10,10],[y,-1,1])$

Figure-3. Plot of $\frac{x^2}{A^2} + \frac{y^2}{B^2} - 2\frac{xy}{AB}\cos(\beta - \alpha) = sin^2(\beta - \alpha)$

when $\alpha = 0, \beta = 0$.

However, when $(\beta - \alpha) = \frac{\pi}{4}$ and $\omega_1 = \omega_2$ is plotted in figure 3. It is an ellipse and the commands used to plot the graph are as follows,

(x/A)^2+(y/B)^2-(2*x*y*cos(π/4))/(A*B)=(sin(π/4))^2;

(%o27)   y^2/B^2-(sqrt(2)*x*y)/(A*B)+x^2/A^2=1/2

>   %o27,A=5,B=0.5;

(%o28)   4.0*y^2-0.4*sqrt(2)*x*y+x^2/25=1/2

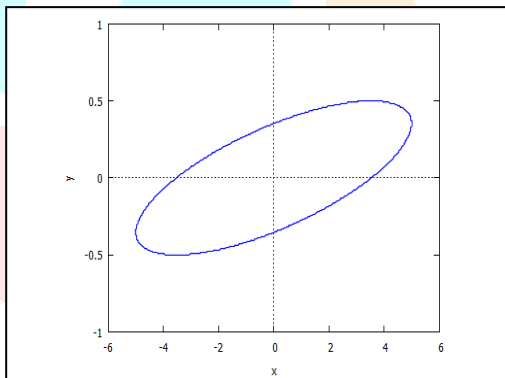>   implicit_plot(4.0*y^2-0.4*sqrt(2)*x*y+x^2/25=1/2, [x,-6,6], [y,-4,4])$

Figure-4. Plot of $\frac{x^2}{A^2} + \frac{y^2}{B^2} - 2\frac{xy}{AB}\cos(\beta - \alpha) = sin^2(\beta - \alpha)$

when $(\beta - \alpha) = \frac{\pi}{4}$

When $(\beta - \alpha) = \frac{\pi}{2}$ and $\omega_1 = \omega_2$ is plotted in figure 5. It represents a circle symmetric about the x and y-axis. If the Lissajous figure is circular, then the phase difference between the two sinusoidal signals will be $\frac{\pi}{2}$.

(%i1)   (x/A)^2+(y/B)^2-(2*x*y*cos(β-α)/(A*B)) =(sin(β-α))^2;

(%o1)   - (2*x*y*cos(β-α))/(A*B)+y^2/B^2+x^2/A^2=sin(β-α)^2

(%i2)   %o1, β-α=δ;

(%o2)   - (2*x*y*cos(δ))/(A*B)+y^2/B^2+x^2/A^2=sin(δ)^2

(%i3)   %o2, A=5, B=0.5,δ=π/2;

(%o3)   4.0*y^2+x^2/25=1

(%i4)   load(implicit_plot)$

(%i7)   implicit_plot(4.0*y^2+x^2/25=1,[x,-8,8],[y,-0.6,0.6])$
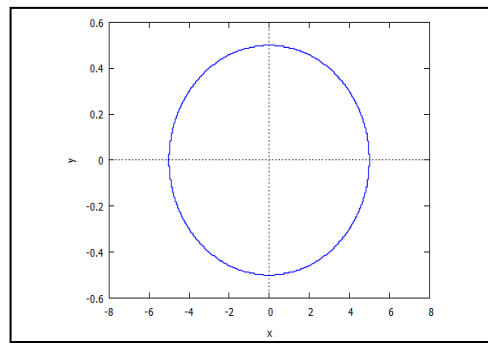
Figure-5. Plot of $\frac{x^2}{A^2}+\frac{y^2}{B^2}-2\frac{xy}{AB}\cos(\beta-\alpha)=sin^2(\beta-\alpha)$

$$\text{when } (\beta-\alpha)=\frac{\pi}{2}$$

Nevertheless, when $\omega_1/\omega_2. = 2$. Equation (1) and (2) can be solved using wxMaxima it simplifies to

$$\left[\frac{x}{A}-\sin(\alpha-\beta)\right]^2+\frac{4y^2}{B^2}\left[\frac{y^2}{B^2}+\frac{x}{A}\sin(\alpha-\beta)-1\right]=0$$

$$.....(4)$$

Equation (4) is obtained using wxMaxima using following commands

(%i1)   x=A*sin(2*ω*t+α);

(%o1)   x=A*sin(2*t*ω+α)

(%i2)   y=B*sin(2*ω*t+β);

(%o2)   y=B*sin(2*t*ω+β)

(%i3)   ((x/A)-sin(α-β))^2+(4*y^2/B^2)*((y^2/B^2)+((x*sin(α-β))/A)-1)=0;

(%o3)   (sin(β-α)+x/A)^2+(4*y^2*(-(x*sin(β-α))/A+y^2/B^2-1))/B^2=0

(%i4)   %o3,β-α=δ;

(%o4)   (sin(δ)+x/A)^2+(4*y^2*(-(x*sin(δ))/A+y^2/B^2-1))/B^2=0

If, $(\beta-\alpha)=0$, the above equation is plotted using wxMaxima and it is shown in figure 6. It shows symmetric lobes about the x-axis. The commands used are

(%i5)   %o4,A=5,B=0.5,δ=0;

(%o5)   16.0*y^2*(4.0*y^2-1)+x^2/25=0

(%i6)   load(implicit_plot)$

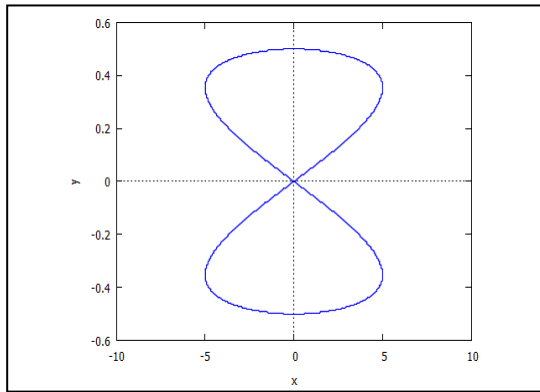(%i11)   implicit_plot(16.0*y^2*(4.0*y^2-1)+x^2/25=0,[x,-10,10],[y,-0.6,0.6])$

Figure-6 Plot of $\left[\frac{x}{A} - \sin(\alpha - \beta)\right]^2 + \frac{4y^2}{B^2}\left[\frac{y^2}{B^2} + \frac{x}{A}\sin(\alpha - \beta) - 1\right] = 0$

when $(\alpha - \beta) = 0$

If, $(\alpha - \beta) = \pi$, the equation is plotted using wxMaxima and it is shown in figure 7. It shows symmetric lobes about x-axis similar to that in figure 6. The commands used are

(%i12)   %o4, A=5,B=0.5,δ=π;

(%o12)   16.0*y^2*(4.0*y^2-1)+x^2/25=0

(%i13) implicit_plot(16.0*y^2*(4.0*y^2-1)+x^2/25=0,[x,-10,10], [y,-0.6,0.6])$
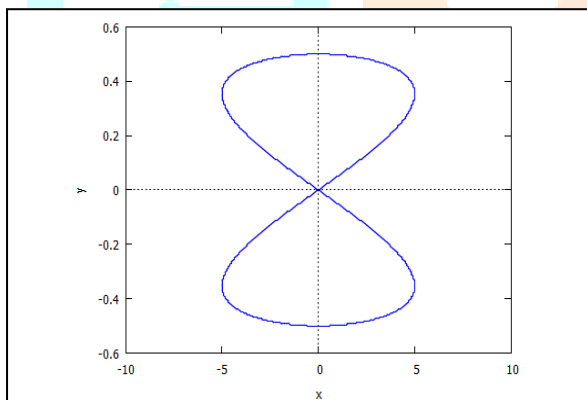

Figure-7 Plot of $\left[\frac{x}{A} - \sin(\alpha - \beta)\right]^2 + \frac{4y^2}{B^2}\left[\frac{y^2}{B^2} + \frac{x}{A}\sin(\alpha - \beta) - 1\right] = 0$ when $(\alpha - \beta) = \pi$

When $(\beta - \alpha) = \frac{\pi}{4}$, the equation is plotted using wxMaxima with the following commands and the plot is shown in Figure8.

(%i6)   ((x/A)-sin(δ))^2+(4*y^2/B^2)*((y^2/B^2)+((x*sin(δ))/A)-1)=0;

(%o6)   (4*y^2*((x*sin(δ))/A+y^2/B^2-1))/B^2+(x/A-sin(δ))^2=0

(%i11)   %o6,A=5,B=0.5,δ=π/4;

(%o11) 16.0*y^2*(4.0*y^2+x/(5*sqrt(2))-1)+(x/5-1/sqrt(2))^2=0

(%i9)   load(implicit_plot)$

(%i12)   implicit_plot(16.0*y^2*(4.0*y^2+x/(5*sqrt(2))-1)+(x/5-1/sqrt(2))^2=0,[x,-10,10],[y,-1,1])$
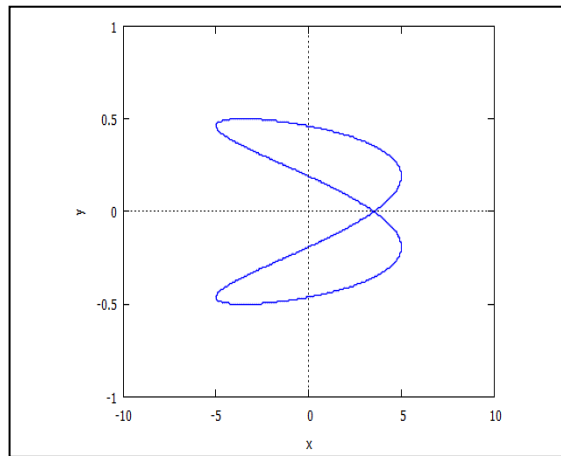
Figure-8 Plot of $\left[\frac{x}{A} - \sin(\alpha - \beta)\right]^2 + \frac{4y^2}{B^2}\left[\frac{y^2}{B^2} + \frac{x}{A}\sin(\alpha - \beta) - 1\right] = 0$

when $(\alpha - \beta) = \frac{\pi}{4}$

If, $(\beta - \alpha) = \frac{2\pi}{4}$, the equation is plotted using wxMaxima and the plot is shown in figure 9. The commands employed are as follows

(%i14) %o6, A=5, B=0.5,δ=(-π/4);

(%o14) 16.0*y^2*(4.0*y^2-x/(5*sqrt(2))-1)+(x/5+1/sqrt(2))^2=0

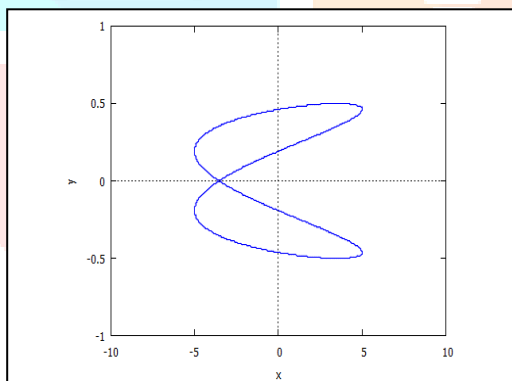(%i15) implicit_plot(16.0*y^2*(4.0*y^2-x/(5*sqrt(2))-1)+(x/5+1/sqrt(2))^2=0,[x,-10,10],[y,-1,1])$

Figure-9 Plot of $\left[\frac{x}{A} - \sin(\alpha - \beta)\right]^2 + \frac{4y^2}{B^2}\left[\frac{y^2}{B^2} + \frac{x}{A}\sin(\alpha - \beta) - 1\right] = 0$

when $(\alpha - \beta) = \frac{2\pi}{4}$

Also, when $(\beta - \alpha) = \frac{\pi}{2}$, the equation is plotted using wxMaxima and shown in figure10. This demonstrates symmetric inverted parabola along the x-axis in the first and second quadrants without lobes. The commands used are

(%i1)   ((x/A)-sin(δ))^2+(4*y^2/B^2)*((y^2/B^2)+((x*sin(δ))/A)-1)=0;

(%o1)   (4*y^2*((x*sin(δ))/A+y^2/B^2-1))/B^2+(x/A-sin(δ))^2=0

(%i2)   %o1,A=5,B=0.5,δ=π/2;

(%o2)   16.0*y^2*(4.0*y^2+x/5-1)+(x/5-1)^2=0

(%i3)   load(implicit_plot)$

(%i8)   implicit_plot(16.0*y^2*(4.0*y^2+x/5-1)+(x/5-1)^2=0,[x,0,10],[y,-0.4,0.4])$
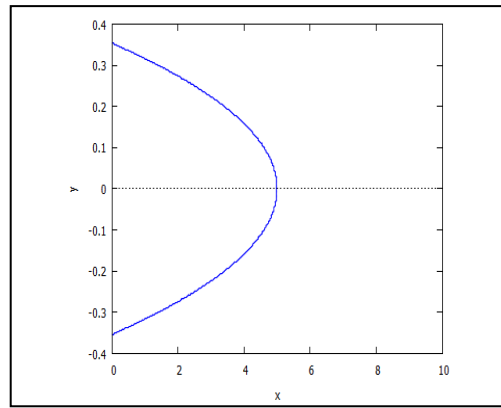
Figure-10 Plot of $\left[\frac{x}{A} - \sin(\alpha - \beta)\right]^2 + \frac{4y^2}{B^2}\left[\frac{y^2}{B^2} + \frac{x}{A}\sin(\alpha - \beta) - 1\right] = 0$ when $(\alpha - \beta) = \frac{\pi}{2}$

4. Conclusion:

This paper investigated the effectiveness of using wxMaxima, and a set of commands used to display Lissajous figures. The waveform and the Lissajous patterns can be easily illustrated on the computer by adjusting parameters such as amplitude and frequency ratios.

References:

[1]. A. Albertazzi Jr., M. R. Viotti, C. L. N. Veiga, and A. V. Fantin, A non-linear piezoelectric actuator calibration using N-dimensional Lissajous figure, Proc. SPIE 9960, Interferometry XVIII, 99600L (28 August 2016); https://doi.org/10.1117/12.2237367

[2]. Khairina, M Ikhsan and Suhartati ,The validity of vector analysis module using wxMaxima software, Journal of Physics: Conference Series, Volume 1088, The 6th South East Asia Design Research International Conference (6th SEA-DR IC)27–28 June 2018, Banda Aceh, Indonesia.

[3]. D Afriza, C M Zubainur and A Veloo ,The use of the wxMaxima linear algebra module on Gauss elimination lesson for mathematics education students, Journal of Physics: Conference Series, Volume 1088, The 6th South East Asia Design Research International Conference (6th SEA-DR IC)27–28 June 2018, Banda Aceh, Indonesia.

[4]. Timberlake T.K., Mixon J.W. (2016) Basic Newtonian Physics with Maxima. In: Classical Mechanics with Maxima. Undergraduate Lecture Notes in Physics. Springer, New York, NY pp 1-2.

[5]. Kai Velten,, Mathematical Modeling and Simulation: Introduction for Scientists and Engineers, John Wiley & Sons.

[6]. Braga FL, Spalenza W, Filhot KCL. Nonlinear celestial mechanics orbits revisited: an effective relativistic lagrangean approach. Phys Astron Int J.2017;1(1):26-30. DOI: 10.15406/paij.2017.01.00004

[7]. Thomas B. Greenslade Jr., All about Lissajous figures, The Physics Teacher 31, 364 (1993).

[8]. P. P. Ong and S. H. Tang , Fourier analysis with Lissajous figures, American Journal of Physics 53, 252 (1985).

[9]. B A Smith, Lissajous' figures, Physics Education, Volume 16, Number 1.l