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"MHD HELE-SHAW FLOW OF AN ELASTICOVISCOUS FLUID THROUGH POROUS MEDIA"

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Abstract :-

Purpose of this paper is to study the MHD Hele-Shaw flow of an elasticoviscous fluid thorugh porous media. Here, an attempt is made to solve the unsteady Hele-Shaw flow of viscous-elastic fluid through Porous media, assuming the pressure gradient to the proportional to exp (-mt). the velocity components are obtained and the effect of visco-elasticity is discussed on velocity components. In the end vorticity is also discussed.

Introduction :-

Main aim of this paper is to study the MHD Hele-Shaw flow of an elasticoviscous fluid through Porous media. Many research workers have paid their attention towards the study of Hele-Shaw flow. The steady Hele-Shaw flows have been studied by Buckmaster, Lee and Fung, Thompson and Lamb, Gupta have discussed unsteady Hele-Shaw flow of a non-Newtonian fluid and of a viscoelastic fluid through Porous media.

2. Formulation of the problem:

Here, we have assumed the following notations

u, v, w = components of velocity

ν	=	Kinetic viscosity
β	=	Visco-elastic parameter
t	=	Time variable
ρ	=	density of fluid
d	=	Characteristic length
u_0	=	Velocity

Let us consider the flow of viscous elastic fluid passing through the Porous medium confined between two parallel plates located at Z = -d and Z = d (2d is very small quantity) Past a circular cylinder $x^2 + y^2 = b^2$, $-d \le z \le d$

The equations governing the flow are –

(2.1) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (continuity equation)

(2.2) $(1-\beta\nabla^2)\frac{\partial u}{\partial t} = \nu\nabla^2 u - \frac{1}{\rho}\frac{\partial p}{\partial x}$

(2.3) $(1-\beta\nabla^2)\frac{\partial v}{\partial t} = \nu\nabla^2 v - \frac{1}{\rho}\frac{\partial p}{\partial y}$

(2.4) $(1-\beta^2)\frac{\partial w}{\partial t} = \nu\nabla^2 w - \frac{1}{\rho}\frac{\partial p}{\partial z}$

Equations (2.2), (2.3) and (2.4) can be written as

(2.5) $\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial z^2} + \beta\frac{\partial^2}{\partial z^2}\left(\frac{\partial u}{\partial t}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x}$

(2.6) $\frac{\partial v}{\partial t} = \nu\frac{\partial^2 u}{\partial z^2} + \beta\frac{\partial^2}{\partial z^2}\left(\frac{\partial v}{\partial t}\right) - \frac{1}{\rho}\frac{\partial p}{\partial y}$

(2.7) $-\frac{1}{\rho}\frac{\partial p}{\partial z} = 0$

Parameter boundary conditions:

u = 0, v = 0 at z = $\pm$d

3. <u>Solution of the problem :</u>

The non-dimensional variables which are appropriate for fluid transients are

$$t^* = \frac{tv}{b^2}, u^* = \frac{u}{u_0}, v^* = \frac{v}{u_0}, p^* = \frac{bp}{v\rho u_0}, \beta^* = \frac{\beta}{b^2}$$

$$z^* = \frac{z}{b}, x^* = \frac{x}{b}, y^* = \frac{y}{b} \text{ and } d^* = \frac{d}{b}$$

Inserting all non-dimensional quantiting in (2.1), (2.5), (2.6) and (2.7) and dropping the asterisks, we obtain.

(3.1) $$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

(3.2) $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial z^2}+\beta\frac{\partial^2}{\partial z^2}\left(\frac{\partial u}{\partial t}\right)-\frac{\partial p}{\partial x}$$

(3.3) $$\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial z^2}+\beta\frac{\partial^2}{\partial z^2}\left(\frac{\partial v}{\partial t}\right)-\frac{\partial p}{\partial y}$$

(3.4) $$\frac{\partial p}{\partial z}=0$$

The boundary conditions are:

(3.5) $Z=\pm d$, $u=o=v$

By virtue of equations (3.2) and (3.3), we have-

(3.6) $$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=0$$

Remark (3.1): It is note-worthy that the relations (3.4) indicates that p is independent of z. therefore, p is the function of x, y and t.

Let,

(3.7) $$u=F(z,t)\frac{\partial f}{\partial x}$$

(3.8) and $$v=F(z,t)\frac{\partial f}{\partial y}$$

Where f is some function of x and y. inserting equation (3.7) and equation (3.8) into equation (3.1), we obtain.

(3.9) $$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$$

Inserting equations (3.7) and (3.8) into (3.2) and (3.3) and integrating, we obtain

(3.10) $$p=\left(\frac{\partial^2 F}{\partial z^2}+\beta\frac{\partial^2 F}{\partial z^2\partial t}-\frac{\partial F}{\partial t}\right)f+A$$

Where A is some arbitrary function of time. Let pressure gradient be proportional to exp (-mt). In equation (3.10), we assume that

(3.11) $$\frac{\partial^2 F}{\partial z^2}+\beta\frac{\partial^3 F}{\partial z^2 \partial t}-\frac{\partial F}{\partial t}=-Ce^{-mt}$$

Where m is positive integer and C is given constant. To solve (3.11), we try assuming $F(Z,t)=e^{-mt}\Phi(Z)$, and we have

(3.12) $$\Phi(Z)=-\frac{C}{m}\left(1-\frac{\cos b_1 Z}{\cos b_1 d}\right)$$ Where $b_1^2=\frac{m}{1-Bm}$

(3.13) Thus, $$F(Z,t)=\frac{-Ce^{-mt}}{m}\left(1-\frac{\cos b_1 Z}{\cos b_1 d}\right)$$

Now, the function f(x, y) can be evaluated by (3.9) subject to the condition.

$u\cos\theta+v\sin\theta=0$, When r = b

(3.14) or $\frac{\partial u}{\partial r}=0$ When r = b

(3.15) We have, $x=r\cos\theta$

(3.16) and $y=r\sin\theta$

and $\frac{\partial f}{\partial n}\to 1, \frac{\partial f}{\partial y}=0$

squaring equations (3.15) and (3.16) and adding, then we get

(3.17) $r^2=x^2+y^2$

Differentiation partially with respect to "x", then

$$\frac{\partial r}{\partial x}=\frac{x}{r}$$

(3.18) $$\frac{\partial r}{\partial x}=\cos\theta$$

Again, differentiation partially with respect to "y"

(3.19) i.e. $\frac{\partial r}{\partial y} = \sin\theta$

Now, dividing equation (3.16) by equation (3.15), then

(3.20) $\theta = \tan^{-1}\left({}^{y}\!/_{x} \right)$

Differentiation equation (3.20) partially with respect to "x" and "y"

$$\frac{\partial \theta}{\partial x} = -\frac{y}{x^2 + y^2} = -\frac{\sin\theta}{r}$$

and $\frac{\partial \theta}{\partial y} = -\frac{x}{x^2 + y^2} = \frac{\cos\theta}{r}$ Since $|x|,|y| \to \infty$ Therefore,

(3.21) $\therefore f(x, y) = \left(r + \frac{1}{r} \right)\cos\theta$

Differentiating partially with respect to "x"

$$\frac{\partial f}{\partial x} = \left(r + \frac{1}{r} \right)\frac{\sin^2\theta}{r} + \cos^2\theta\left(1 - \frac{1}{r^2} \right)$$

(3.22) $\frac{\partial f}{\partial x} = \left[1 + \left(\frac{x^2 - y^2}{x^2 + y^2} \right) \right]$

Again, differentiating equation (3.21) partially with respect to "y"

$$\frac{\partial f}{\partial y} = -\left(r + \frac{1}{r} \right)\sin\theta\frac{\partial \theta}{\partial y} + \cos\theta\left(\frac{\partial r}{\partial y} - \frac{1}{r^2}\frac{\partial r}{\partial y} \right)$$

$$\frac{\partial f}{\partial y} = -\left(r + \frac{1}{r} \right)\frac{\sin\theta\cos\theta}{r} + \cos\theta\left(\sin\theta - \frac{\sin\theta}{r^2} \right)$$

(3.23) $\frac{\partial f}{\partial y} = -\frac{2xy}{\left(x^2 + y^2 \right)^2}$

Putting the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ from equations (3.22) and (3.23) into equations (3.7) and (3.8), then we get

(3.24) $u = A_1 e^{-mt}\left(1 - \frac{\cos b_1 Z}{\cos b_1 d} \right)\left[1 - \frac{\left(x^2 - y^2 \right)}{\left(x^2 + y^2 \right)^2} \right]$

$$v = A_1 e^{-mt}\left(1 - \frac{\cos b_1 Z}{\cos b_1 d}\right)\left[\frac{-2xy}{\left(x^2+y^2\right)^2}\right]$$

(3.25) and

Now introducing a new function as vorticity function ξ and given by

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Putting the value of $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}$ from equations (3.25) and (3.24), then we get

$$\xi = 4A_1 e^{-mt}\left(1 - \frac{\cos b_1 Z}{\cos b_1 d}\right)\left[\frac{y}{\left(x^2+y^2\right)^2}\right]$$

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