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Second Order Dual Minimax Fractional Programming Under Binvexity

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Abstract: In this paper, we derive some theorems and duality theorems on Second Order DualMinimax Fractional Programming under binvexity

Keywords: Binvex functions and second order Duality.

. Introduction

In this Chapter, we consider the following minimax fractional programming problem.

(p) minimize $\Psi(x) = \operatorname*{Sup}_{y \in Y} \dfrac{f_i\,(x, y)}{g_i\,(x, y)}$

s.t $\quad h(x) \leq 0 \;, x \in R^n$

where Y is a compact subset of R^l, $f\,(\cdot,\cdot): R^n \times R^l \to R$, $g\,(\cdot,\cdot): R^n \times R^l \to R$ are twice continuously differentiable on $R^n \times R^l$ and $h\,(\cdot,\cdot): R^n \to R^m$ is twice continuously differentiable on R^n. It is assumed that for each (x, y) in $R^n \times R^l$, $f_i\,(x, y) \geq 0$ and $g_i\,(x, y) \geq 0$.

Since minimax fractional programming has wide applications, much attention has been paid optimality conditions and duality theorems for minimax fractional programming problems. For the case of convex differentiable minimax fractional programming, Yadav and Mukherjee [10] formulated two dual models for (p) and derived duality theorems. Chandra and Kumar [3] pointed out certain omissions in the dual formulation of Yadav and Mukherjee [9], they constructed two modified dual problems for minimiax fractional programming problem and proved duality results. Liu and Wu [5,6], and Yang and Hou [10] discussed optimality conditions and duality results for (p) involving generalized convexity assumptions.

Mangasarian [7] introduced the notation of second order duality for nonlinear programs by introducing an additional vector $p \in R^n$. He has indicated a possible computational advantage of the second order dual over the first order dual. Instead of imposing explicit condition on p, Mond [8] included p in a second order type convexity. Bector et al [2] discussed second order duality results for minimax programming problems under generalized B-Convexity. Later on, Liu [4] extended these results involving second order generalized B-Convexity. Recently, Husain et al [3] have formulated two types of second order dual models for minimax fractional programming problems, and derived weak, strong and strict coverse duality theorems under binvex assumptions.

In this paper, two types of second order duality in minimax fractional programming are formulated by introducing an additional vector r. The weak, strong and converse duality theorems are proved for these programes under binvexity assumptions. Our result generalizes these existing dual formulations which were discussed by the authors in but they not consider this in binvexity. Hence in this Paper an attempt to made to full this gap in the aim of research by developing some theorems and duality theorems in second order duality for minimax fractional programming.

2. Preliminaries

Let $S = \{x \in R^n : h(x) \le 0\}$ denote the set of all feasible solutions of (p). For each $(x, y) \in R^n \times R^l$, we define

$$J(x) = \{j \in M = \{1, 2, \} : h_j(x) \le 0\}$$

$$Y(x) = \left\{ y \in Y : \frac{f_i(x, y)}{g_i(x, y)} = \operatorname*{Sup}_{Z \in Y} \frac{f_i(x, z)}{g_i(x, z)} \right\},$$

and $k(x) = \left\{ (s, t, \tilde{y}) \in N \times R_s^+ X R^{ls} : 1 \le s \le n+1, t = (t_1, t_2, \ldots\ldots, t_s) \in R_+^s, \sum_{t=1}^{s} t_i = 1, \tilde{y} = (\tilde{y}_1, \tilde{y}_2, \ldots\ldots., \tilde{y}_s); \; \tilde{y}_i \in Y(x), i = 1, 2, \ldots\ldots\ldots s \right\}$

let $f : R^n \to R$ be a twice differentiable function.

Definition (1):

Function f is said to be binvex at $\bar{x} \in R^n$, such that for all $x, p \in R^n$, we have

$$\frac{f_i(x)}{g_i(x)} - \frac{f_i(\bar{x})}{g_i(\bar{x})} + \frac{1}{2} p^T \nabla^2 \frac{f_i(\bar{x})}{g_i(\bar{x})} p \ge (x - \bar{x})^t \left[\nabla f(\bar{x}) + \nabla^2 f(\bar{x}) p \right]$$

Definition (2) : Function f is said to be strictly binvex at $\bar{x} \in R^n$ such that for all $x, p \in R^n$, we have

$$\frac{f_i(x)}{g_i(x)} - \frac{f_i(\bar{x})}{g_i(\bar{x})} + \frac{1}{2} p^T \nabla^2 \frac{f_i(\bar{x})}{g_i(\bar{x})} p > (x - \bar{x})^t \left[\nabla \frac{f_i(\bar{x})}{g_i(\bar{x})} + \nabla^2 \frac{f_i(\bar{x})}{g_i(\bar{x})} p \right]$$

The following theorem will be needed in the proofs of strong duality theorems.

Theorem (1) (Necessary Conditions). Let x^* be a solution of (p) and let $\nabla g_j(x^*),\ j \in J(x^*)$ be linearly independent. There exist $\left(s^*, t^*, \bar{y}^*\right) \in k(x^*),\ \lambda^* \in R_+$ and $\mu^* \in R_+^m$ such that

$$\nabla \sum_{i=1}^{s^*} t_i^* \left(f_i\,(x^*, \bar{y}_i^*)\right) - \lambda^* g_i\,(x^*, \bar{y}_i^*) + \nabla \sum_{j=1}^{m} \mu_j^*\, h_j(x^*) = 0$$

$$\left(f_i\,(x^*, \bar{y}_i^*)\right) - \lambda^* g_i\,(x^*, \bar{y}_i^*) = 0,\ i = 1, 2, \ldots\ldots, s^*$$

$$\sum_{j=1}^{m} \mu_j^*\, h_j(x^*) = 0,$$

$$t_i^* \geq 0,\ \sum_{i=1}^{s^*} t_i^* = 1,\ \ \bar{y}_i^* \in y(x^*),\ i = 1, 2, \ldots\ldots, s^*$$

3. First Duality Model

By utilizing the necessary optimality conditions of the previous section, we formulate the following second order dual to (p) as follows.

$$\text{(MD)} \qquad \max_{(s,t,\bar{y}) \in k(z)} \ \sup_{(z,\mu,\lambda,r,p) \in H_1(s,t,\bar{y})} \lambda$$

where $H_1(s, t, \bar{y})$ denotes the set of all $(z, \mu, \lambda, \gamma, p) \in R^n \times R_+^m \times R_+ \times R^n \times R^n$

Satisfying $\nabla \sum_{i=1}^{s} t_i \left(f_i\,(z, \bar{y}_i)\right) - \lambda gi\,(z, \bar{y}_i) + \nabla^2 \sum_{i=1}^{s} t_i \left(f_i\,(z, \bar{y}_i) - \lambda gi\,(z, \bar{y}_i)\right) p$

$$+ \nabla \sum_{j=1}^{m} \mu_j g_j(z) + \nabla^2 \sum_{j=1}^{m} \mu_j g_j(z) p = 0 \qquad (1)$$

$$\sum_{i=1}^{s} t_i \left(f_i\,(z, \bar{y}_i)\right) - \lambda gi\,(z, \bar{y}_i) + \left(\nabla \sum_{i=1}^{s} t_i \left(f_i\,(z, \bar{y}_i) - \lambda gi\,(z, \bar{y}_i)\right) r\right)$$

$$-\frac{1}{2} p^T \nabla^2 \sum_{i=1}^{s} t_i \left(f_i\,(z, \bar{y}_i) - \lambda gi\,(z, \bar{y}_i)\right) p \geq 0 \qquad (2)$$

$$\sum_{j=1}^{m} \mu_j g_j(z) + \left(\nabla \sum_{j=1}^{m} \mu_j g_j(z)\right) r - \frac{1}{2} p^T \nabla^2 \sum_{j=1}^{m} \mu_j g_j(z) p \geq 0 \qquad (3)$$

$$\left(\nabla \sum_{i=1}^{s} t_i \left(f_i\,(z, \bar{y}_i) - \lambda gi\,(z, \bar{y}_i)\right)\right) r + \left(\nabla \sum_{j=1}^{m} \mu_j g_j(z)\right) r \leq 0 \qquad (4)$$

if, for a triplet $(s, t, \bar{y}) \in k(z)$, the set $H_1(s, t, \bar{y}) = \phi$, then we define the supremum over it to be $-\infty$.

Theorem1 (Weak Duality): Let x and $(z, \mu, \lambda, s, t, \bar{y}, r, p)$ be feasible solutions of (p) and (MD), respectively. Assume that

(i) $\sum_{i=1}^{s} t_i \left(f_i\,, \bar{y}_i\right) - \lambda gi\,(\cdot\,, \bar{y}_i)$ is η-bonvex at z;

(ii) $\sum_{j=1}^{m} \mu_j g_j(\cdot)$ is η-bonvex at z.

Then $\sup_{y \in Y} \dfrac{f_i\,(x, y)}{g_i\,(x, y)} \geq \lambda$

Proof: by the feasibility of x for (p), $\mu \geq 0$ and (3), we get

$$\sum_{j=1}^{m} \mu_j g_j(x) \leq 0 \leq \sum_{j=1}^{m} \mu_j g_j(z) + \left(\nabla \sum_{j=1}^{m} \mu_j g_j(z)\right) r - \frac{1}{2} p^T \nabla^2 \sum_{j=1}^{m} \mu_j g_j(z) p$$

The above inequality together with hypothesis (ii) implies

$$\left(\nabla\sum_{j=1}^{m}\mu_j g_j(x)+\nabla^2\sum_{j=1}^{m}\mu_j g_j(z)\,p\right)(x-z)^t \le \left(\nabla\sum_{j=1}^{m}\mu_j g_j(z)\right)r$$

From (9.3.1), (9.3.4) and (9.3.5), we have

$$\left(\nabla\sum_{i=1}^{s}t_i\left(f_i(z,\bar{y}_i)-\lambda gi(z,\bar{y}_i)\right)\right)+\left(\nabla^2\sum_{i=1}^{s}t_i\left(f_i(z,\bar{y}_i)-\lambda gi(z,\bar{y}_i)\right)p\right)(x-z)^t \ge -\left(\nabla\sum_{j=1}^{m}\mu_j g_j(z)\right)r$$

$$\ge \left(\nabla\sum_{i=1}^{s}t_i\left(f_i(z,\bar{y}_i)-\lambda gi(z,\bar{y}_i)\right)\right)r$$

which in view of hypothesis (i) and (2) fields

$$\nabla\sum_{i=1}^{s}t_i\left(f_i(x,\bar{y}_i)\right)-\lambda gi(x,\bar{y}_i) \ge \sum_{i=1}^{s}t_i\left(f_i(z,\bar{y}_i)-\lambda gi(z,\bar{y}_i)\right)+$$

$$\left(\nabla\sum_{i=1}^{s}t_i\left(f_i(z,\bar{y}_i)-\lambda gi(z,\bar{y}_i)\right)r\right)$$

$$-\frac{1}{2}p^T\nabla^2\left(\sum_{i=1}^{s}t_i\left(f(z,\bar{y}_i)-\lambda h(z,\bar{y}_i)\right)\right)p \ge 0$$

Therefore, there exists a certain i_o such that

$$f_i(x,\bar{y}_{io})-\lambda gi(x,\bar{y}_{io}) \ge 0$$

Hence $\sup\limits_{y\in Y}\dfrac{f_i(x,y)}{g_i(x,y)} \ge \dfrac{f_i(x,\bar{y}_{io})}{g_i(x,\bar{y}_{io})} \ge \lambda$

Theorem 2 (Strong Duality)

Assume that x^* is an optimal solution of (p) and $\nabla g_i(x^*),\ j \in J(x^*)$ are linearly independent. Then there exist $(s^*, t^*, y^*) \in k(x^*)$ and $(x^*, \mu^*, \lambda^*, r^* = 0, p^* = 0) \in H_1(s^*, t^*, y^*)$ such that $\left(x^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^* = 0, p^* = 0\right)$ is a feasible solution of (MD) and the two objectives have the same values. If, in addition, the assumptions of weak duality hold for all feasible solutions $\left(x, \mu, \lambda, s, t, \bar{y}, r, p\right)$ of (MD), then $\left(x^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^* = 0, p^* = 0\right)$ is an optimal solution of (MD).

Proof: Since x^* is an optimal solution of (p) and $\nabla g_i(x^*),\ j \in J(x^*)$ are linearly independent, then by Theorem (1), thee exist $(s^*, t^*, y^*) \in k(x^*)$ and $(x^*, \mu^*, \lambda^*, r^* = 0, p^* = 0) \in H_1(s^*, t^*, \bar{y}^*)$ such that

$(x^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^* = 0, p^* = 0)$ is a feasible solution of (MD) and the two objectives have the same values. Optimality of $(x^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^*, p^* = 0)$ for (MD), thus follows weak duality (Theorem 9.3.1).

Theorem 3 (Strict Converse Duality)

Let x^* and $(z^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^*, p^*)$ be optimal solutions of (p) and (MD), respectively, suppose that

(i) $\nabla g_j(x^*),\ j \in J(x^*)$ are linearly independent,

(ii) $\sum_{i=1}^{s} t_i^*\left(f_i(\cdot, \bar{y}_i^*) - \lambda^* gi(\cdot, \bar{y}_i^*)\right)$ is strictly binvex at z*,

(iii) $\sum_{j=1}^{m}\mu_j^* g_j(\cdot)$ is binvex at z*. Then $z^* = x^*$.

Proof: Suppose to the contrary that $z^* \neq x^*$, and we will derive a contradiction. Since x^* and $(z^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^*, p^*)$ be optimal solutions of (p) and (MD), respectively, and $\nabla g_j(x^*),\ j \in J(x^*)$ are linearly independent, therefore, by theorem (2), we have

$$\sup_{y^* \in Y} \frac{f_i(x^*, y^*)}{g_i(x^*, y^*)} = \lambda^* \tag{6}$$

The feasibility of x^* for (p), $\mu^* \geq 0$ and (3) imply

$$\sum_{j=1}^{m}\mu_j^* g_j(x^*) \leq 0 \leq \sum_{j=1}^{m}\mu_j^* g_j(z^*) + \left(\sum_{j=1}^{m}\mu_j^* g_j(z^*)\right) r^* -$$

$$-\frac{1}{2} p^{*T} \nabla^2 \sum_{j=1}^{m}\mu_j^* g_j(z^*)\, p^*,$$

Which along with hypothesis (iii) gives

$$\left(\nabla \sum_{i=1}^{m}\mu_j^* g_j(z^*) + \nabla^2 \sum_{j=1}^{m}\mu_j^* g_j(z^*) p\right)(x^* - z^*)^t \leq \left(\nabla \sum_{i=1}^{m}\mu_j^* g_j(z^*)\right)$$

Therefore, inequality (9.3.1) along with (9.3.7) yields

$$\nabla \sum_{i=1}^{s} t_i^* \left(f(z^*, \bar{y}_i^*)\right) - \lambda^* h(z^*, \bar{y}_i^*) + \left(\nabla^2 \sum_{i=1}^{s} t_i^* \left(f_i(z^*, \bar{y}_i^*) - \lambda g_i(z^*, \bar{y}_i^*)\right) p\right)(x^* - z^*)^t$$

$$\geq \left(\nabla \sum_{i=1}^{s} t_i^* \left(f_i(z^*, \bar{y}_i^*) - \lambda^* g_i(z^*, \bar{y}_i^*)\right)\right) r^*,$$

which by hypothesis (ii) and inequality (9. 3.2) gives

$$\sum_{i=1}^{s} t_i^* \left(f(x^*, \bar{y}_i^*) - \lambda^* h(x^*, \bar{y}_i^*) \right) > 0$$

For a certain i_o, this implies

$$\sup_{y^* \in Y} \frac{f_i(x^*, y^*)}{g_i(x^*, y^*)} \geq \frac{f_i(x^*, \bar{y}_{io}^*)}{g_i(x^*, \bar{y}_{io}^*)} > \lambda^*,$$

which is a contradiction to (6). Hence $z^* = x^*$.

4. Second duality model

In this section, we formulate the following second order dual to (p) as follows.

$$\text{(GMD)} \qquad \max_{(s,t,\bar{y}) \in k(z)} \quad \sup_{(z,\mu,\lambda,r,p) \in H_2(s,t,\bar{y})} \quad \lambda$$

where $H_2(s, t, \bar{y})$ denotes the set of all $(z, \mu, \lambda, r, p) \in R^n \times R_+^m \times R_+ \times R^n \times R^n$

Satisfying

$$\nabla \sum_{i=1}^{s} t_i \left(f_i(z, \bar{y}_i) \right) - \lambda gi(z, \bar{y}_i) + \nabla^2 \sum_{i=1}^{s} t_i \left(f_i(z, \bar{y}_i) - \lambda h(z, \bar{y}_i) \right) p$$

$$+ \nabla \sum_{j=1}^{m} \mu_j g_j(z) + \nabla^2 \sum_{j=1}^{m} \mu_j g_j(z) p = 0 \qquad (4.1)$$

$$\sum_{i=1}^{s} t_i f_i(z, \bar{y}_i) - \lambda g_i(z, \bar{y}_i) + \sum_{j \in J_0} \mu_j g_j(z) + \left(\nabla \sum_{i=1}^{s} t_i f_i(z, \bar{y}_i) - \lambda g_i(z, \bar{y}_i) + \sum_{j \in J_0} \mu_j g_j(z) \right) r$$

$$-\frac{1}{2} p^T \nabla^2 \sum_{i=1}^{s} t_i \left(f_i(z, \bar{y}_i) - \lambda g_i(z, \bar{y}_i) + \sum_{j \in J_0} \mu_j g_j(z) \right) p \geq 0 \qquad (4.2)$$

$$\sum_{j\in J_\alpha} \mu_j g_j(z) + \left(\sum_{j\in J_\alpha} \mu_j g_j(z)\right) r - \frac{1}{2} p^T \nabla^2 \sum_{j\in J_\alpha} \mu_j g_j(z) p \geq 0, \quad \alpha = 1, 2, \dots. r$$

(4.3)

$$\left(\nabla \sum_{j\in J_\alpha} t_i \left(f_i\,(z, \bar{y}_i) - \lambda g_i\,(z, \bar{y}_i)\right) + \left(\nabla \sum_{j=1}^{m} \mu_j g_j(z)\right)\right) r \leq 0 \qquad (4.4)$$

where $J_\alpha \subseteq M$, $\alpha = 0, 1, 2, \dots, k$ with $\bigcup_{\alpha=0}^{k} J_\alpha = M$ and $J_\alpha \cap J_\beta = \phi$, if $\alpha \neq \beta$. If, for a triplet $(s, t, \bar{y}) \in k(z)$, the set $H_1(s, t, \bar{y}) = \phi$, then we define the supremum over it to be $-\infty$.

Theorem 4.1 (Weak Duality): let x and $(z, \mu, \lambda, s, t, \bar{y}, r, p)$ be feasible solutions of (p) and (GMD), respectively. Assume that

(i) $\sum_{i=1}^{s} t_i \left(f_i\,(\cdot, \bar{y}_i) - \lambda gi\,(\cdot, \bar{y}_i)\right) + \sum_{j\in J_0} \mu_i g_j(\cdot)$ is binvex at z;

(ii) $\sum_{j\in J_\alpha} \mu_i g_j(\cdot), \quad \alpha = 1, 2, \dots.. r$ binvex at z.

Then $\sup_{y\in Y} \dfrac{f_i\,(x, y)}{g_i\,(x, y)} \geq \lambda$

Proof: By the feasibility of x for (p), $\mu \geq 0$ and (9.4.3), we get

$$\sum_{j\in J_\alpha} \mu_i g_j(x) \leq 0 \leq \sum_{j\in J_\alpha} \mu_i g_j(z) + \left(\nabla \sum_{j\in J_\alpha} \mu_i g_j(z)\right) r - \frac{1}{2} p^T \nabla^2 \sum_{j\in J_\alpha} \mu_i g_j(z) p, \ \alpha = 1, 2, \dots. r$$

(4.5)

The above inequality (4.5) with hypothesis (ii) implies

$$\left(\nabla\sum_{j\in J_\alpha}\mu_j g_j(z)+\nabla^2\sum_{j\in J_\alpha}\mu_j g_j(z)p\right)(x-z)^t \le \left(\nabla\sum_{j\in J_\alpha}\mu_j g_j(z)\right)r,\ \ \alpha=1,2,\ldots r$$

which together with (4.1), (4.4) yields

$$\nabla\sum_{i=1}^{s}t_i\left(f_i\ (z,\bar{y}_i)\right)-\lambda gi\,(z,\bar{y}_i)+\nabla^2\sum_{i=1}^{s}t_i\left(f_i\ (z,\bar{y}_i)-\lambda g_i\ (z,\bar{y}_i)\right)p$$

$$+\ \nabla\sum_{j\in J_0}\mu_j g_j(z)+\left(\nabla^2\sum_{j\in J_0}\mu_j g_j(z)p\right)(x-z)^t$$

$$\ge\nabla\sum_{i=1}^{s}t_i\left(f_i\ (z,\bar{y}_i)\right)-\lambda gi\,(z,\bar{y}_i)+\left(\sum_{j\in J_0}\mu_j g_i\,(z)p\right)r$$

In view of hypothesis (i) and the above inequality implies

$$\sum_{i=1}^{s}t_i\left(f_i\ (x,\bar{y}_i)\right)-\lambda gi\,(x,\bar{y}_i)+\sum_{j\in J_0}\mu_j g_i\,(x)$$

$$\ge\sum_{i=1}^{s}t_i\left(f_i\ (z,\bar{y}_i)\right)-\lambda g_i\ (z,\bar{y}_i)+\sum_{j\in J_0}\mu_j g_j(z)+\left(\nabla\sum_{i=1}^{s}t_i\left(f_i\ (z,\bar{y}_i)\right)-\lambda g_i\ (z,\bar{y}_i)+\sum_{j\in J_0}\mu_j g_j(z)\right)r$$

$$-\frac{1}{2}p^T\left(\nabla^2\sum_{i=1}^{s}t_i\left(f_i\ (z,\bar{y}_i)-\lambda g_i\,(z,\bar{y}_i)\right)+\sum_{j\in J_0}\mu_j g_j(z)\right)p$$

By $\mu\ge 0,\ g(x)\le 0$ and (4.2) it follows that

$$\sum_{i=1}^{s}t_i\left(f_i\ (x,\bar{y}_i)-\lambda g_i\,(x,\bar{y}_i)\right)\ge 0$$

Therefore, there exists a certain i_o such that

$$f_i\,(x, \bar{y}_{io}) - \lambda g_i\,(x, \bar{y}_{io}) \geq 0$$

$$\sup_{y \in Y} \frac{f_i\,(x, y)}{g_i\,(x, y)} \geq \frac{f_i\,(x, \bar{y}_{io})}{g_i\,(x, \bar{y}_{io})} > \lambda$$

The proof of the following theorem is similar to that of Theorem (9.3.2) and hence, is omitted.

Theorem 4.2 Strong Duality: Assume that x^* is an optimal solution of (p) and $\nabla g_j(x^*),\ j \in J(x^*)$ are linearly independent. Then there exist $(s^*, t^*, \bar{y}^*) \in k(x^*)$ and $(x^*, \mu^*, \lambda^*, r^* = 0, p^* = 0)$ $\in H_1(s^*, t^*, \bar{y}^*)$ such that

$(x^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^* = 0, p^* = 0)$ is a feasible solution of (GMD) and the two objectives have the same values. If, in addition, the assumptions of weak duality hold for all feasible solutions $(x, \mu, \lambda, s, t, \bar{y}, r, p)$ of (GMD), then $(x^*, \mu^*, \lambda^*, s^*, t^*, \bar{y}^*, r^* = 0, p^* = 0)$ is an optimal solution of (GMD).

Theorem 4.3 (Strict Converse Duality)

Let x^* and $(z^*, s^*, t^*, \bar{y}^*, r^*, p^*)$ be optimal solutions of (p) and (GMD), respectively, suppose that

(i) $\nabla g_j(x^*),\ j \in J(x^*)$ are linearly independent.

(ii) $\sum_{i=1}^{s} t_i^* \left(f_i\,(\cdot, \bar{y}_i^*) - \lambda^* g_i\,(\cdot, \bar{y}_i^*) \right) + \sum_{j \in J_0} \mu_j^* g_j(\cdot)$ is strictly binvex at z^*.

(iii) $\sum_{j \in J_\alpha} \mu_j^* g_j(\cdot),\quad \alpha = 1, 2, \ldots.. k$ binvex at z^*.

Then $z^* = x^*$.

Proof: It can be proved similarly to Theorem (3.3.).

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