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On Some Relations OF Area and Volume of Rectangular Prism and the Diophantine Equation $\frac{1}{n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$

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Abstract:

In this paper, a relation between volume V and surface area S of a rectangular prism has been taken as $V = \frac{n}{2}S$, n is a positive integer. Diophantine equation $\frac{1}{n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ has been discussed for positive integer solutions in different cases. The Diophantine equation $\frac{q}{p} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ has also been discussed for positive integer solutions.

Key words: Rectangular prism and Diophantine equation

Introduction:

Hari Kishan et. al. (2011) discussed the Diophantine equations of second and higher degree of the form $3xy = n(x + y)$ and $3xyz = n(xy + yz + zx)$ etc. **Rabago, J. F.T. & Tagle, R.P.** (1913) discussed the area and volume of a certain regular solid and the Diophantine equation $\frac{1}{2} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$. **Sander, J.** (1913) discussed the Diophantine equation $\frac{1}{2} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ and obtained solutions of this Diophantine equation.

In this paper, the relation between area and volume of rectangular prism and the Diophantine equation $\frac{1}{n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ has been discussed. A rectangular prism has base as a rectangle. The Diophantine equation $\frac{q}{p} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ has also been discussed for positive integer solutions.

Analysis:

Let l, w and h be the length, width and height (in positive integer) of a rectangular prism respectively. Then volume V and surface area S of rectangular prism are given by:

$$V = lwh \text{ and } S = 2(lw + wh + lh). \qquad \ldots(1)$$

Now we consider the following relation between V and S:

$$V = \frac{n}{2}S. \qquad \ldots(2)$$

This implies that

$$lwh = n(lw + wh + lh),$$

or $$\frac{1}{n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}. \qquad \ldots(3)$$

This is the given Diophantine equation.

Now we consider the following cases:

Case 1: Rectangular prism is a cube: In this case $l = w = h$. Therefore from (3), we have

$$l^3 = 3nl^2.$$

This gives $l = w = h = 3n$. Thus in this case, the given Diophantine equation is given by

$$(l, w, h) = (3n, 3n, 3n).$$

Few examples may be $(l, w, h) = (3,3,3), (6,6,6), (9,9,9)$ and $(12,12,12)$ etc.

Case 2: Two dimensions of the prism are equal and third is different:

Let $l = w \neq h$.

In this case, equation (3) implies that

$$l^2h = n(l^2 + 2lh),$$

or $$l(l(n-h) + 2nh) = 0,$$

or $$l = w = \frac{2nh}{h-n}. \quad \ldots(4)$$

If $h = 2n$ then from (4), we have $l = w = 4n$. Thus we have

$$(l, w, h) = (4n, 4n, 2n).$$

If $h = 3n$ then from (4), we have $l = w = 3n$. Thus we have

$$(l, w, h) = (3n, 3n, 3n).$$

This is the same as in case 1.

If $h = 4n$ then from (4), we have $l = w = \frac{8}{3}n$. Thus we have

$$(l, w, h) = \left(\frac{8}{3}n, \frac{8}{3}n, 4n\right). \quad \ldots(5)$$

From (5), it is clear that the rectangular prism has integral dimensions when n is a multiple of 3. Let $n = 3m$. Then

$$(l, w, h) = (8m, 8m, 12m).$$

If $h = 5n$ then from (4), we have $l = w = \frac{5}{2}n$. Thus we have

$$(l, w, h) = \left(\frac{5}{2}n, \frac{5}{2}n, 5n\right). \quad \ldots(6)$$

From (6), it is clear that the rectangular prism has integral dimensions when n is a multiple of 2. Let $n = 2m$. Then

$$(l, w, h) = (5m, 5m, 10m).$$

If $h = 6n$ then from (4), we have $l = w = \frac{12}{5}n$. Thus we have

$$(l, w, h) = \left(\frac{12}{5}n, \frac{12}{5}n, 6n\right). \quad \ldots(7)$$

From (7), it is clear that the rectangular prism has integral dimensions when n is a multiple of 3. Let $n = 5m$. Then

$$(l, w, h) = (12m, 12m, 30m).$$

In the same way, the other solutions can be obtained.

There may be two other cases given by $l = h \neq w$ and $h = w \neq l$. Solutions can be found for these cases also.

Case 3: All dimensions are unequal: Let $l \neq w \neq h$. We have to find the values of l, w, and h such that

$$\frac{1}{n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}. \quad \ldots(8)$$

L.H.S=$\frac{1}{n} = \frac{1}{2n} + \frac{1}{3n} + \frac{1}{6n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ =R.H.S.

This gives $l = 2n, w = 3n, h = 6n$. Since l, w and h are symmetric in the Diophantine equation, we have $3! = 6$ different solutions.

Further suppose we have

$$\frac{q}{p} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}. \quad \ldots(9)$$

L.H.S.= $\frac{q}{p} = \frac{1}{\frac{p+1}{q}} + \frac{1}{p\left(\frac{p+1}{q}\right)}$

$= \frac{1}{p} + \frac{1}{\frac{p+1}{q-1}} + \frac{1}{p\left(\frac{p+1}{q-1}\right)}$ =R.H.S.

This gives $l = p, w = \frac{p+1}{q-1}$ and $h = p\left(\frac{p+1}{q-1}\right)$. Now if p, q are arbitrary positive integers with $p + 1 \equiv 0\big(mod(q-1)\big)$ then $(l, w, h) = \left(p, \frac{p+1}{q-1}, p\left(\frac{p+1}{q-1}\right)\right)$ is the solution of Diophantine equation (9). Thus we have the following theorem:

Theorem: If p, q are arbitrary positive integers such that $p + 1 \equiv 0\big(mod(q-1)\big)$ then $(l, w, h) = \left(p, \frac{p+1}{q-1}, p\left(\frac{p+1}{q-1}\right)\right)$ is the solution of the Diophantine equation

$$\frac{q}{p} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}.$$

Since l, w and h are symmetric in the Diophantine equation, we have $3! = 6$ different solutions.

For example if $p = 5$ and $q = 3$ then the Diophantine equation is given by

$\frac{3}{5} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ and its solution is given by $(l, w, h) = (5,3,15)$.

Concluding Remarks:

Here the relation between surface area S and volume V has been taken as $V = \frac{n}{2}S$ which provide the Diophantine equation $\frac{1}{n} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$. Then its integer solutions have been obtained in three different cases. The Diophantine equation $\frac{q}{p} = \frac{1}{l} + \frac{1}{w} + \frac{1}{h}$ has also been discussed for positive integer solutions.

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