



# FUZZY SUPRA STRONGLY HYPER CONNECTED SPACES AND FUZZY SUPRA BAIRE SPACES

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**Abstract:** In this paper, the concept of fuzzy supra strongly hyperconnected spaces are studied. Finding the relations between fuzzy supra strongly hyperconnected spaces and other fuzzy supra topological spaces. Illustrate the concept with suitable examples.

**Keywords -** Fuzzy supra nowhere dense set, Fuzzy supra first category, Fuzzy supra P-space, Fuzzy supra submaximal space, Fuzzy supra hyperconnected spaces, Fuzzy supra Baire spaces.

## I. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by L.A.Zadeh [11] in 1965. By applying the fuzzy set notions to general topology, C.L.Chang [4] introduced the theory of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of hyperconnectedness in topological spaces has been introduced by Steen and Seebach [10] in 1970. In 2002, Caldas et.al.[3] defined fuzzy hyperconnectedness in fuzzy topological space. In 2017, T.M.Al-shami [2] introduced the concept of strongly hyperconnected spaces in classical topology. The concept of fuzzy supra hyperconnected spaces was introduced and studied by this author in [6]. In this paper, introduce the concept of fuzzy supra strongly hyperconnected spaces. Finding the relations between fuzzy supra strongly hyperconnected spaces and other fuzzy supra topological spaces. Illustrate the concept with suitable examples.

## II. PRELIMINARIES

### Definition 2.1 [8]

A fuzzy set  $\lambda$  in a fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra dense if there exists no fuzzy supra closed set  $\mu$  in  $(X, T^*)$  such that  $\lambda < \mu < 1$ . That is,  $cl^*(\lambda) = 1$ , in  $(X, T^*)$ .

### Definition 2.2 [8]

A fuzzy set  $\lambda$  in fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy supra open set  $\mu$  in  $(X, T^*)$  such that  $\mu < cl^*(\lambda)$ . That is,  $int^* cl^*(\lambda) = 0$ , in  $(X, T^*)$ .

### Lemma 2.1 [1]

For a fuzzy set  $\lambda$  of a fuzzy supra topological space  $(X, T^*)$ ,

- (i)  $1 - int^*(\lambda) = cl^*(1 - \lambda)$ ,
- (ii)  $1 - cl^*(\lambda) = int^*(1 - \lambda)$ .

### Definition 2.3 [6]

A fuzzy set  $\lambda$  in a fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra first category set if  $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy supra nowhere dense set in  $(X, T^*)$ . Any other fuzzy set in  $(X, T^*)$  is said to be fuzzy supra second category.

### Definition 2.4 [6]

A fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra P-space if every non zero fuzzy supra  $G_{\delta}$ -set in  $(X, T^*)$  is fuzzy supra open in  $(X, T^*)$ . That is, if  $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$  where  $\lambda_i \in (X, T^*)$  for  $i \in \mathbb{N}$ .

### Definition 2.5 [7]

A fuzzy set  $\lambda$  in a fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra  $G_{\delta}$ -set if  $\lambda = \bigwedge_{i=1}^{\infty} (\lambda_i)$ , where  $\lambda_i \in T^*$  for  $i \in \mathbb{N}$

### Definition 2.6 [7]

A fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra Baire spaces if  $int^*(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0$ , where  $(\lambda_i)$ 's are fuzzy supra nowhere dense set in  $(X, T^*)$ .

### III. FUZZY SUPRA HYPERCONNECTED SPACE

#### Definition 3.1 [6]

A fuzzy supra topological space  $(X, T^*)$  is said to be fuzzy supra hyper connected if every non- null fuzzy supra open set of  $X$  is fuzzy supra dense in  $(X, T^*)$ .

#### Definition 3.2 [5]

A fuzzy set  $\lambda$  in fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra semiopen if there exists a fuzzy supra open set  $\mu \in T^*$  such that  $\mu \leq \lambda \leq cl^*(\mu)$ .

#### Definition 3.3

In a fuzzy supra topological space  $(X, T^*)$  is said to be fuzzy supra connected if  $X$  cannot be represented as the union of two non empty disjoint open fuzzy sets.

#### Definition 3.4

A fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra extremally disconnected space if  $cl^*(\lambda) = \lambda$ , where  $\lambda \in T^*$ .

#### Example 3.1:

Let  $X = \{a, b, c\}$ . Then the fuzzy sets  $\alpha, \beta$  and  $\gamma$  are defined on  $X$  as follows:

$\alpha : X \rightarrow [0, 1]$  defined as  $\alpha(a) = 0.4, \alpha(b) = 0.6, \alpha(c) = 0.7$ .

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.3, \beta(b) = 0.2, \beta(c) = 0.7$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.6, \gamma(b) = 0.4, \gamma(c) = 0.3$ .

Then,  $T^* = \{0, \alpha, \gamma, \alpha \vee \beta, 1\}$  is a fuzzy supra open sets in  $(X, T^*)$ . Now, we see that  $cl^*(\alpha) = \alpha, cl^*(\gamma) = \gamma$  and  $cl^*(\alpha \vee \beta) = \alpha \vee \beta$ . Therefore  $(X, T^*)$  is fuzzy supra extremally disconnected space.

#### Proposition 3.1

Every fuzzy supra hyper connected space  $(X, T^*)$  is fuzzy supra extremally disconnected.

#### Proof.

Suppose that  $(X, T^*)$  is fuzzy supra hyper connected space. Then for any fuzzy supra open set  $\beta, cl^*(\beta) = 1_x$ , which implies that  $cl^*(\beta)$  is fuzzy supra dense and as a consequence the space  $(X, T^*)$  is fuzzy supra extremally disconnected.

#### Remark 3.1

The converse of the above proposition need not be true. That is, Every fuzzy supra extremally disconnected space  $(X, T^*)$  is need not be a fuzzy supra hyper connected space.

For consider the example 3.1,  $(X, T^*)$  is fuzzy supra extremally disconnected space. But  $cl^*(\alpha) = cl^*(\gamma) = cl^*(\alpha \vee \beta) \neq 1$ . Therefore,  $\alpha, \gamma, (\alpha \vee \beta)$  are fuzzy supra open set but not fuzzy supra dense set in  $(X, T^*)$ . Hence,  $(X, T^*)$  is not a fuzzy supra hyperconnected space.

#### Proposition 3.2

Let  $(X, T^*)$  be a fuzzy supra topological space. Then the following conditions are equivalent:

- $(X, T^*)$  is fuzzy supra hyperconnected spaces,
- Every fuzzy supra preopen set is fuzzy supra dense set.

#### Proof.

##### (i) $\Rightarrow$ (ii)

Let us assume that  $\alpha$  is fuzzy supra pre-open set in  $(X, T^*)$ , this implies that  $\alpha \leq int^* cl^*(\alpha)$ . By hypothesis,  $(X, T^*)$  is fuzzy supra hyper connected space, we have  $cl^*(\alpha) = cl^*(int^* cl^*(\alpha)) = 1_x$ . Therefore  $\alpha$  is fuzzy supra dense set.

##### (ii) $\Rightarrow$ (i)

Let  $\alpha$  be any fuzzy supra pre-open set, thus  $\alpha \leq int^* cl^*(\alpha)$ . By hypothesis, given that  $\alpha$  is fuzzy supra dense set. Therefore  $cl^*(\alpha) = cl^*(int^* cl^*(\alpha)) = 1_x$ . It follows that  $(X, T^*)$  is fuzzy supra hyper connected space.

#### Proposition 3.3

Arbitrary union of fuzzy supra hyper connected set of  $X$  is fuzzy supra hyper connected set.

#### Proof.

Let  $\{\alpha_i / i \in \Lambda\}$  be a collection of fuzzy supra hyper connected sets of  $X$ .

Then, for each  $i \in \Lambda$ , we have  $\alpha_i$ ,

$$\alpha_i \leq cl^* int^*(\alpha_i) \vee int^* cl^*(\alpha_i)$$

Then,  $\forall \alpha_i \leq \vee (cl^* int^*(\alpha_i) \vee int^* cl^*(\alpha_i))$

$$= cl^* int^*(\vee \alpha_i) \vee int^* cl^*(\vee \alpha_i) \quad [\text{Since, } \vee cl^*(\alpha_i) \leq cl^*(\vee \alpha_i) \text{ and } int^*(\alpha_i) \leq int^*(\vee \alpha_i)]$$

Now,  $cl^*(\vee \alpha_i) \leq cl^*[(cl^* int^*(\vee \alpha_i) \vee int^* cl^*(\vee \alpha_i))]$

$$= [cl^* cl^* int^*(\vee \alpha_i) \vee (cl^* int^* cl^*(\vee \alpha_i))] \quad [\text{Inequality holds in respect of closure property}]$$

$$= cl^* int^*(\vee \alpha_i) \vee (cl^* int^* cl^*(\vee \alpha_i))$$

Therefore,  $cl^* int^*(\vee \alpha_i) \vee cl^* int^* cl^*(\vee \alpha_i) = 1$ .

Hence,  $cl^*(\vee \alpha_i) = 1$ .

Implies that  $(\vee \alpha_i)$  is a fuzzy supra hyperconnected in a fuzzy supra topological spaces  $(X, T^*)$ .

#### Proposition 3.4

A fuzzy supra topological spaces  $(X, T^*)$  is fuzzy supra hyper connected space, for every non null fuzzy supra open set  $\alpha$  of  $X$ , if  $cl^* int^*(\alpha) \vee cl^* int^* cl^*(\alpha) \leq 1$ .

#### Proof.

Let  $\alpha$  be a fuzzy set in  $(X, T^*)$ . For every non null fuzzy supra open subset  $\alpha \leq cl^* int^*(\alpha) \vee int^* cl^*(\alpha)$ . Since  $\alpha$  is hyper connected in  $(X, T^*)$ . Therefore,  $cl^*(\alpha) = 1$ .  $cl^*(\alpha) \leq cl^*(cl^* int^*(\alpha) \vee int^* cl^*(\alpha))$ ,  $1 = cl^*(cl^* int^*(\alpha) \vee int^* cl^*(\alpha))$ . This implies  $cl^*(cl^* int^*(\alpha) \vee int^* cl^*(\alpha)) = 1$  [ $\alpha$  is finite fuzzy supra open set,  $cl^*(\vee \alpha) \geq \vee cl^* \alpha$ ]. Hence,  $cl^* int^*(\alpha) \vee cl^* int^* cl^*(\alpha) \leq 1$ .

#### Proposition 3.5

A fuzzy supra topological spaces  $(X, T^*)$  is fuzzy supra hyper connected for non null finite fuzzy supra closed sets  $\beta$  of  $X$  if  $cl^* int^*(\beta) \wedge cl^* int^* cl^*(\beta) \leq 1$ .

#### Proof.

Let  $\beta$  is fuzzy supra closed set. Then by definition, we have  $\beta \geq cl^* int^*(\beta) \wedge int^* cl^*(\beta)$ ,  $1 - \beta \leq 1 - [cl^* int^*(\beta) \wedge int^* cl^*(\beta)]$ . Therefore,  $int^*(1 - \beta) \leq int^*[1 - (cl^* int^*(\beta) \wedge int^* cl^*(\beta))]$ ,  $1 - cl^*(\beta) \leq 1 - cl^*[cl^* int^*(\beta) \wedge int^* cl^*(\beta)]$ . Since  $\beta$  is fuzzy supra hyper connected,

$cl^*(\beta) = 1$  and  $cl^*(\wedge\beta_i) \subseteq \wedge cl^*(\beta_i)$ . Therefore,  $1-1 \leq 1-cl^*cl^*int^*(\beta) \wedge cl^*int^*cl^*(\beta)$ ,  $0 \leq 1-cl^*int^*(\beta) \wedge cl^*int^*cl^*(\beta)$ . This implies,  $cl^*int^*(\beta) \wedge cl^*int^*cl^*(\beta) \leq 1$ .

#### IV. FUZZY SUPRA STRONGLY HYPERCONNECTED SPACE

##### Definition :4.1

In a fuzzy supra topological spaces  $(X, T^*)$ , a fuzzy set  $\lambda$  is said to be fuzzy supra strongly nowhere dense set, if  $\lambda \wedge (1-\lambda)$  is a fuzzy supra nowhere dense set in  $(X, T^*)$ . That is  $int^*\{cl^*[\lambda \wedge (1-\lambda)]\} = 0$ , in  $(X, T^*)$ .

##### Definition :4.2

In a fuzzy supra topological spaces  $(X, T^*)$  a fuzzy set  $\lambda$  is said to be a fuzzy supra strongly first category set if  $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy supra strongly nowhere dense sets in  $(X, T^*)$ . Any other fuzzy set in  $(X, T^*)$  is said to be a fuzzy supra strongly second category set in  $(X, T^*)$ .

##### Definition :4.3

A fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra strongly Baire space if  $cl^*(\wedge_{i=1}^{\infty} (\lambda_i))=1$ , where  $(\lambda_i)$ 's are fuzzy supra strongly nowhere dense sets in  $(X, T^*)$ .

##### Definition 4.4

A fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra strongly hyper connected space, if the following conditions hold:

- (i) if  $\lambda$  is a fuzzy supra dense set in  $(X, T^*)$ , then  $\lambda$  is a fuzzy supra open set in  $(X, T^*)$  and
- (ii) if  $\lambda$  is a fuzzy supra open set in  $(X, T^*)$ , then  $\lambda$  is a fuzzy supra dense set in  $(X, T^*)$ .

##### Example 4.1:

Let  $X = \{a, b, c\}$ . Then the fuzzy sets  $\alpha, \beta$  and  $\gamma$  are defined on  $X$  as follows:

$\alpha : X \rightarrow [0, 1]$  defined as  $\alpha(a) = 0.7, \alpha(b) = 0.8, \alpha(c) = 0.9$ .

$\beta : X \rightarrow [0, 1]$  defined as  $\beta(a) = 0.6, \beta(b) = 0.9, \beta(c) = 0.8$ .

$\gamma : X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.6, \gamma(b) = 0.7, \gamma(c) = 0.8$ .

Then,  $T^* = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \wedge \beta, 1\}$ , is a fuzzy supra open sets in  $(X, T^*)$ . All the fuzzy supra open sets are fuzzy supra dense sets in  $(X, T^*)$  and all the fuzzy supra dense sets are fuzzy supra open sets in  $(X, T^*)$ . Hence  $(X, T^*)$  is a fuzzy supra strongly hyperconnected space.

##### Definition 4.5

A fuzzy supra topological space  $(X, T^*)$  is called a fuzzy supra submaximal space if for each fuzzy set in  $(X, T^*)$  such that  $cl^*(\lambda) = 1$ , then  $\lambda \in T^*$  in  $(X, T^*)$ .

##### Proposition 4.1:

If  $(X, T^*)$  is a fuzzy supra strongly hyper connected space, then

- (i)  $(X, T^*)$  is a fuzzy supra hyper connected space,
- (ii)  $(X, T^*)$  is a fuzzy supra submaximal space.

##### Proof.

(i) Let  $\mu$  be a fuzzy supra open set in  $(X, T^*)$ . Since  $(X, T^*)$  is a fuzzy supra strongly hyper connected space, the fuzzy supra open set  $\mu$  is a fuzzy supra dense set in  $(X, T^*)$  and hence  $(X, T^*)$  is a fuzzy supra hyper connected space.

(ii) Let  $\lambda$  be a fuzzy supra dense set in  $(X, T^*)$ . Since  $(X, T^*)$  is a fuzzy supra strongly hyper connected space, the fuzzy supra dense set  $\lambda$  is a fuzzy supra open set in  $(X, T^*)$  and hence  $(X, T^*)$  is a fuzzy supra submaximal space.

##### Proposition 4.2

If  $\lambda$  is a fuzzy supra dense set in a fuzzy supra strongly hyper connected space, then  $1-\lambda$  is a fuzzy supra nowhere dense set in  $(X, T^*)$

##### Proof.

Let  $\lambda$  be a fuzzy supra dense set in  $(X, T^*)$ . Then,  $cl^*(\lambda) = 1$  in  $(X, T^*)$ . Since  $(X, T^*)$  is a fuzzy supra strongly hyper connected space,  $\lambda$  is a fuzzy supra open set in  $(X, T^*)$  and hence  $1-\lambda$  is a fuzzy supra closed set in  $(X, T^*)$ . Then,  $cl^*(1-\lambda)=1-\lambda$ , in  $(X, T^*)$ . This implies that  $int^*cl^*(1-\lambda) = int^*(1-\lambda) = 1-cl^*(\lambda) = 1-1 = 0$ , in  $(X, T^*)$ . Hence  $1-\lambda$  is a fuzzy supra nowhere dense set in  $(X, T^*)$ .

##### Proposition 4.3

If  $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy supra dense sets in a fuzzy supra strongly hyper connected space  $(X, T^*)$ , then  $1-\lambda$  is a fuzzy supra first category set in  $(X, T^*)$ .

##### Proof.

Let  $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's ( $i = 1$  to  $\infty$ ) are fuzzy supra dense sets in  $(X, T^*)$ . Then, by proposition 4.2,  $(1-\lambda_i)$ 's are fuzzy supra nowhere dense sets in  $(X, T^*)$ . Now  $\vee_{i=1}^{\infty} (1-\lambda_i)$  is a fuzzy supra first category set in  $(X, T^*)$ . But  $\vee_{i=1}^{\infty} (1-\lambda_i) = 1 - \wedge_{i=1}^{\infty} (\lambda_i) = 1-\lambda$ . Thus  $1-\lambda$  is a fuzzy supra first category set in  $(X, T^*)$ .

##### Proposition 4.4

If  $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy supra dense sets in a fuzzy supra strongly hyper connected space  $(X, T^*)$ , then  $\lambda$  is a fuzzy supra residual set in  $(X, T^*)$ .

##### Proof.

Let  $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's fuzzy supra dense sets in  $(X, T^*)$ . Since  $(X, T^*)$  is a fuzzy supra strongly hyper connected space, by proposition 4.3,  $1-\lambda$  is a fuzzy supra first category set and hence  $\lambda$  is a fuzzy supra residual set in  $(X, T^*)$ .

##### Proposition 4.5

If  $\lambda$  and  $\mu$  are any two non-zero fuzzy open sets in a fuzzy supra strongly hyper connected space  $(X, T^*)$ , then  $\lambda \wedge \mu \neq 0$ , in  $(X, T^*)$

##### Proof.

Suppose that  $\lambda \wedge \mu = 0$ , where  $\lambda, \mu \in T^*$ . Then,  $\lambda \leq 1-\mu$  in  $(X, T^*)$ . This implies that  $cl^*(\lambda) \leq cl^*(1-\mu)$ , in  $(X, T^*)$ . Since  $(X, T^*)$  is a fuzzy supra strongly hyper connected space, the fuzzy supra open set  $\lambda$  is a fuzzy supra dense set in  $(X, T^*)$ . Then  $1 \leq cl^*(1-\mu)$  in  $(X, T^*)$ . That is,  $cl^*(1-\mu)=1$  and then by lemma 2.1,  $1-int^*(\mu)=1$  in  $(X, T^*)$ . This implies that  $int^*(\mu)=0$ , a contradiction to  $\mu \in T^*$  for which  $int^*(\mu)=\mu \neq 0$ . Hence,  $\lambda \wedge \mu \neq 0$ , in  $(X, T^*)$ .

**Proposition 4.6**

If  $\text{int}^*(\lambda)=0$  and  $\text{int}^*(\mu)=0$  for any two non-zero fuzzy sets defined on  $X$  in a fuzzy supra strongly hyperconnected space  $(X, T^*)$  then  $\text{int}^*(\lambda \vee \mu) = 0$ , in  $(X, T^*)$ .

**Proof.**

Let  $\lambda$  and  $\mu$  be two non-zero fuzzy sets defined on  $X$  such that  $\text{int}^*(\lambda)=0$  and  $\text{int}^*(\mu)=0$  in  $(X, T^*)$ . We want to prove that  $\text{int}^*(\lambda \vee \mu)=0$ . Assume the contrary and suppose that  $\text{int}^*(\lambda \vee \mu) \neq 0$  in  $(X, T^*)$ . Then, there exists a fuzzy supra open set  $\delta$  in  $(X, T^*)$  such that  $\delta \leq \lambda \vee \mu$ . Then  $\text{cl}^*(\delta) \leq \text{cl}^*(\lambda \vee \mu)$  in  $(X, T^*)$ . Since  $(X, T^*)$  is a fuzzy supra strongly hyper connected space, for the fuzzy supra open set  $\delta$  in  $(X, T^*)$ ,  $\text{cl}^*(\delta) = 1$ , in  $(X, T^*)$ . This implies that  $1 \leq \text{cl}^*(\lambda \vee \mu)$  in  $(X, T^*)$ . That is,  $\text{cl}^*(\lambda \vee \mu) = 1$ , in  $(X, T^*)$ . If  $\text{cl}^*(\lambda) = 1$ , in the fuzzy strongly hyper connected space  $(X, T^*)$  then  $\lambda$  will be a fuzzy supra open set in  $(X, T^*)$  for which  $\text{int}^*(\lambda) = \lambda$  and this will contradict the fact that  $\text{int}^*(\lambda) = 0$ . Hence  $\text{cl}^*(\lambda) \neq 1$ , in  $(X, T^*)$ . Similarly,  $\text{cl}^*(\mu) \neq 1$ , in  $(X, T^*)$ . Now  $1 - \text{cl}^*(\lambda) \neq 0$  and  $1 - \text{cl}^*(\mu) \neq 0$  in  $(X, T^*)$  and  $[1 - \text{cl}^*(\lambda)] \wedge [1 - \text{cl}^*(\mu)] = 1 - [\text{cl}^*(\lambda) \vee \text{cl}^*(\mu)] = 1 - \text{cl}^*(\lambda \vee \mu) = 1 - 1 = 0$ . Thus,  $1 - \text{cl}^*(\lambda)$  and  $1 - \text{cl}^*(\mu)$  are non-zero fuzzy supra open sets in  $(X, T^*)$  with  $[1 - \text{cl}^*(\lambda)] \wedge [1 - \text{cl}^*(\mu)] = 0$ . But this is a contradiction [by proposition 4.5]. Hence  $\text{int}^*(\lambda \vee \mu) = 0$ , in  $(X, T^*)$ .

**V. FUZZY SUPRA HYPERCONNECTED SPACE AND FUZZY SUPRA BAIRE SPACE****Proposition 5.1**

If  $\text{cl}^*(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy supra dense set in a fuzzy supra submaximal space  $(X, T^*)$ , then  $(X, T^*)$  is a fuzzy supra Baire space .

**Proof.**

Let  $(\lambda_i)$ 's be fuzzy supra dense sets in a fuzzy supra submaximal space. Then  $\lambda_i \in T^*$  in  $(X, T^*)$ . Now  $\text{cl}^*(\lambda_i) = 1$  and  $\text{int}^*(\lambda_i) = \lambda_i$  implies that  $\text{cl}^* \text{int}^*(\lambda_i) = 1$  Then we have  $1 - \text{cl}^* \text{int}^*(\lambda_i) = 0$ . This implies that  $\text{int}^* \text{cl}^*(1 - (\lambda_i)) = 0$ . Hence  $(1 - (\lambda_i))$ 's are fuzzy supra nowhere dense sets in  $(X, T^*)$ . Now  $\text{cl}^*(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$ , implies that  $1 - \text{cl}^*(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 0$ . Then,  $\text{int}^*(\bigvee_{i=1}^{\infty}(1 - \lambda_i)) = 0$ , where  $(1 - (\lambda_i))$ 's are fuzzy supra nowhere dense sets in  $(X, T^*)$ . Hence,  $(X, T^*)$  is a fuzzy supra Baire space.

**Proposition 5.2**

If  $\text{cl}^*(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy supra dense set in a fuzzy supra strongly hyperconnected space  $(X, T^*)$ , then  $(X, T^*)$  is a fuzzy supra Baire space.

**Proof.**

Let  $(\lambda_i)$ 's be fuzzy supra dense sets in a fuzzy supra strongly hyperconnected space  $(X, T^*)$ . By proposition 4.1, Every fuzzy supra strongly hyperconnected space is fuzzy supra submaximal space. Therefore  $(\lambda_i)$ 's be fuzzy supra dense sets in a fuzzy supra submaximal space with  $\text{cl}^*(\bigwedge_{i=1}^{\infty}(\lambda_i)) = 1$ . Hence by proposition 5.1,  $(X, T^*)$  is a fuzzy supra Baire space

**Remark 5.1.**

The converse of the above proposition is need not be true. That is, A fuzzy supra topological spaces  $(X, T^*)$  is a fuzzy supra Baire space then  $(X, T^*)$  need not be a fuzzy supra strongly hyperconnected space. For consider the example.

**Example 5.1.**

Let  $X = \{a, b, c\}$ . Then the fuzzy sets  $\lambda$ ,  $\mu$  and  $\gamma$  are defined on  $X$  as follows:

$\lambda: X \rightarrow [0, 1]$  defined as  $\lambda(a) = 1, \lambda(b) = 0, \lambda(c) = 0.3$ .

$\mu: X \rightarrow [0, 1]$  defined as  $\mu(a) = 0.6, \mu(b) = 1, \mu(c) = 0.7$ .

$\gamma: X \rightarrow [0, 1]$  defined as  $\gamma(a) = 0.8, \gamma(b) = 0, \gamma(c) = 1$ .

Then,  $T^* = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \mu \vee \gamma, \lambda \wedge \mu, \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \vee \gamma), 1\}$ , is a fuzzy supra open sets in  $(X, T^*)$ . The fuzzy sets  $1 - \lambda, 1 - \gamma, 1 - (\lambda \vee \mu), 1 - (\mu \vee \gamma), 1 - (\lambda \wedge \mu), 1 - [\lambda \vee (\mu \wedge \gamma)], 1 - [\mu \vee (\lambda \vee \gamma)]$  are fuzzy supra nowhere dense sets in  $(X, T^*)$ . Also,  $\text{int}^*[(1 - \lambda) \vee (1 - \gamma) \vee 1 - (\lambda \vee \mu) \vee 1 - (\mu \vee \gamma) \vee 1 - (\lambda \wedge \mu) \vee 1 - (\lambda \vee (\mu \wedge \gamma)) \vee 1 - (\mu \vee (\lambda \vee \gamma))] = 0$ . Therefore  $(X, T^*)$  is a fuzzy supra Baire space. But, the fuzzy supra open sets  $\{\lambda, \mu, \gamma, \lambda \vee \mu, \mu \vee \gamma, \lambda \wedge \mu, \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \vee \gamma)\}$  are not a fuzzy supra dense sets in  $(X, T^*)$ . Therefore  $(X, T^*)$  is not a fuzzy supra strongly hyperconnected space.

**Proposition 5.3**

If a fuzzy supra P-space in a fuzzy supra hyper connected space  $(X, T^*)$  is a fuzzy supra Baire space.

**Proof.**

Let  $\alpha$  be a fuzzy supra  $G_\delta$ - set in fuzzy supra P- space in  $(X, T^*)$ . Since,  $(X, T^*)$  is fuzzy supra hyper connected space. Therefore fuzzy supra open set  $\alpha$  is a fuzzy supra  $G_\delta$ - set in  $(X, T^*)$ . That is,  $\text{cl}^*(\alpha) = 1$ . Since  $\alpha$  is fuzzy supra dense set in a fuzzy supra P- space. Therefore,  $(1 - \alpha)$  is a fuzzy supra first category set in  $(X, T^*)$ . Therefore,  $(1 - \alpha) = \bigvee_{i=1}^{\infty}(\alpha_i)$ , where  $(\alpha_i)$ 's are fuzzy supra nowhere dense sets in  $(X, T^*)$ . Then  $\text{int}^*(\bigvee_{i=1}^{\infty}(\alpha_i)) = \text{int}^*(1 - \alpha) = 1 - \text{cl}^*(\alpha) = 1 - 1 = 0$ . Hence,  $\text{int}^*(\bigvee_{i=1}^{\infty}(\alpha_i)) = 0$ . Where  $(\alpha_i)$ 's are fuzzy supra nowhere dense set in  $(X, T^*)$ . Then  $(X, T^*)$  is fuzzy supra Baire space.

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