



“Optimal Portfolio Construction using Sharpe’s Single Index Model”

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ABSTRACT

Sharpe’s Single Index Model method involves selecting a cut-off rate for inclusion of securities in a portfolio. For this purpose, excess return to Beta ratio given above has to be calculated for each stock and rank them from highest to lowest. Then only those securities which have $R_i - R_f/\beta_i$, greater than cut-off point, fixed in advance can be selected. Optimal Portfolio of Sharpe is Called the Single Index Model. The Optimal Portfolio Is Directly Related to the Beta. By this method, selection of the portfolio has become easier due to the ranking of the securities in the order of their excess return and applying the yardstick of a required cut-off point for selection of securities. stock return is influenced by the market (beta), has a firm specific expected value (alpha) and firm-specific unexpected component (residual). Each stock's performance is in relation to the performance of a market index. Security analysts often use the functions as computing stock betas, evaluating stock selection skills, and conducting event studies.

KEYWORDS: Portfolio, Sharpes, Index, ICICI Bank, Auto sector.

1. INTRODUCTION

Capital market comprising the primary market and secondary markets is one of the most sensitive markets in the economy. The secondary market enables investors to continuously rearrange their assets if they so desire by divesting themselves of such assets while others can use their surplus funds to acquire them. This rearrangement is not a product of instant decisions but a thorough research.

The major tools used for this are fundamental analysis and technical analysis of which fundamental analysis requires a large amount of inside data regarding the companies concerned and also requires lot of calculations and deep knowledge. Whereas technical analysis is comparatively a simpler tool for an investor to decide his short or medium term investment decisions. By watching and understanding the price changes, its trend can

be analysed and the timings of entry and exit can be decided. In short this decisions such as when to buy or when to sell particular scrip or when to recognize the portfolio can be influenced by the technical analysis, moving averages method wherein the daily prices are compared with average of certain number of days.

This work “Optimal Portfolio Construction using Sharpe’s Single Index Model” attempts to construct an optimal portfolio by applying Sharpe’s Single Index Model. For this paper BSE, SENSEX and the 6 stocks have been used as market index for preparing portfolio. The monthly data for all the stocks have been considered. The proposed method formulates a unique cut off point (Cut-off rate of return) and select stocks having excess of their expected return over risk free rate of return surpassing this cut-off point. Percentage of investment in each of selected stocks is then decided on the basis of respective weights assigned to each stock depending on respective beta value, stock movement variance, unsystematic risk, and return on stock and risk-free return viz-a-viz the cut off rate of return.

1.1 STATEMENT OF PROBLEM

Investing in a single stock associates with lot of risk factors around it and that’s where the Construction of portfolio of securities has an edge as it has an advantage of diversification which is being done using Sharpe single index model.

1.2 OBJECTIVES OF THE STUDY

1. To evaluate performance of security through constructing a portfolio.
2. To allocate investment in different stocks considering risk-return criteria.
3. To study different type of risk such as systematic risk, unsystematic risk.
4. To provide direction to investors regarding performance of securities.

1.3 SCOPE OF THE STUDY

1. Selection of companies is restricted to SENSEX index only.
2. The companies are chosen and analyzed based on their performance in the past three year.
3. No other factors other than the share price movements, index movements, rate of return on government securities and beta value for the securities for the past three year are taken for analysis.

1.4 RESEARCH METHODOLOGY

In this study Secondary Data sources is used.

1. Considered the movement of share prices, expected returns, standard deviation and beta values.
2. A brief profile of the company created for the application of Sharpe’s Model.
3. The stock price movements, closing index points of the companies and beta values for the past three year are collected for analysis.
4. The Treasury bill rate of return is considered as the risk-free rate of return.
5. All the values obtained above are interpreted and analyzed using Sharpe’s Single Index Model.

1.5 LIMITATIONS OF THE STUDY

1. Only three-year data has been considered for the construction of the optimal portfolio.
2. The portfolio is constructed purely on the basis of Sharpe's model which basically considers the stock price movements and does not take into consideration company specific factors, industry specific factors and economic specific factors.

2. LITERATURE REVIEW

According to Saravanan and Natarajan in a study conducted on "Optimal portfolio construction with nifty stocks" (2012) showed that returns on either individual securities or on portfolio comprises of securities of different companies listed in Nifty stocks under various sectors are asymmetrical and heterogeneous. The optimal portfolio consists of four stocks selected out of 50 short listed scrip's, giving the return of 0.116. Further it helps to elicit that return on securities of different portfolio is independent of the systematic risk prevailing in the market.

According to Radhika Desai and Manisha Surti in a study conducted on "Optimum Portfolio Construction: Sharpe Single Index Model" showed that Optimum portfolio carry

SI No:	Company	Sector
1	Bajaj Auto Limited	Auto
2	Hero Motocorp Ltd:	Auto
3	Axis Bank Ltd	Banking
4	ICICI Bank Ltd	Banking
5	ACC Cement Ltd	Cement
6	Ambuja Cement Ltd	Cement

According to Suresh Chandra Das & Dr. Bishnupriya Mishra in a study conducted on "Optimal Portfolio – Does number of scrip's matter" showed that one can able to forecast individual security's return through the market movement and can make use of it. It is found that Indian security market information context Sharpe's single index market model will hold well. Further it helps to elicit that return on securities of different portfolio is independent on the systematic risk prevailing in the market.

According to Son-Nan Chen and Stephen J. Brown in a study conducted on "Estimation Risk and Simple Rules for Optimal Portfolio Selection" showed that the presence of estimation risk reduces the relative impact of estimated systematic risk on optimal portfolio choices. In addition, investors can be hurt by not taking estimation risk into account.

Financial services are the economic services provided by the finance industry, which encompasses a broad range of organizations that manage money, including credit unions, banks, creditcard companies, insurance

companies, accountancy companies, consumer finance companies, stock brokerages, investment funds and some government sponsored enterprises.

Financial Products:

The survey also revealed that in the past couple of years, apart from trading, the firms have started offering various investment related value added services. The sustained growth of the economy in the past couple of years has resulted in broking firms offering many diversified services related to IPOs, mutual funds, company research etc. However, the core trading activity is still the predominant form of business, forming 90% of the firms in the sample. 67% firms are engaged in offering IPO related services. The broking industry seems to have capitalised on the growth of the mutual fund industry, which was pegged at 40% in 2006. More than 50% of the sample broking houses deal in mutual fund investment services. The average growth in assets under management in the last two years is almost 48%. Company research is another lucrative area where the broking firms offer their services; more than 33% of the firms are engaged in providing company research services. Additionally, a host of other value added services such as fundamental and technical analysis, investment banking, arbitrage etc are offered by the firms at different levels.

Capital Market:

Capital market is a market of securities (debt or equity), where business enterprises (companies) and governments can raise long term funds. Capital market may be classified as primary markets and secondary markets. In primary market new stocks or bond issues are sold to investors via a mechanism known as underwriting. In secondary markets, existing securities are sold and brought among investors or traders, usually on a security exchange, over the counter or elsewhere. The capital market includes stock market (equity securities) and bond market (debt). Market participants include individual retail investors, institutional investors such as mutual funds, banks, insurance companies and hedge funds, and also publicly traded corporations trading in their own shares.

Stock Market:

A stock market or equity market is the aggregation of buyers and sellers (a loose network of economic transactions, not a physical facility or discrete entity) of stocks (shares); these are securities listed on a stock exchange as well as those only traded privately.

A stock exchange is a form of exchange which provides services for stock brokers and traders to trade stocks, bonds, and other securities. Stock exchanges also provide facilities for issue and redemption of securities and other financial instruments, and capital events including the payment of income and dividends. Securities traded on a stock exchange include stock issued by companies, unit trusts, derivatives, pooled investment products and bonds.

Indian Stock Broking Industry:

The Indian broking industry is one of the oldest trading industries that have been around even before the establishment of the BSE in 1875. Despite passing through a number of changes in the post liberalisation period, the industry has found its way towards sustainable growth.

The equity broking industry in India has several unique features like it is more than a century old, dynamic, forward looking and service providers, well conversant, highly innovative and even adoptable. The regulations and reforms been laid down in the equity market has resulted in rapid growth and development. Basically the growth in the equity market is largely due to the effective intermediaries. The broking houses not only act as an intermediate link for equity market but also for the commodity market, the foreign currency exchange market and many more. The broking houses have also made impact on foreign investors to invest in India to certain extent. In the last decade, the Indian brokerage industry has undergone a dramatic transformation.

2.1 COSTRUCTION OF THE OPTIMUM PORTFOLIO

Portfolio is a combination of securities such as stocks, bonds and money market instruments. The process of blending together the broad asset classes so as to obtain optimum return with minimum risk is called portfolio construction. Diversification of investments helps to spread risk over money assets. A diversification of securities gives the assurance of obtaining the anticipated return on portfolio. In a diversified portfolio, some securities may not perform as expected, but others may exceed the expectation and making the actual return of the reasonably close to the anticipated one.

Keeping a portfolio a single security may lead to a greater likelihood of the actual return somewhat different from that of the expected return. Hence, it is a common practice to diversify securities in the portfolio.

The steps followed in constructing optimal portfolio are as follows:

Determination of objectives

Selection of securities based on the objectives

Choose a suitable approach for construction portfolio

Apply the approach and concert the portfolio

Assessment of risk and return

VARIOUS METHODS OF CONSTRUCTING OPTIMAL PORTFOLIO

1. Some of the famous methods for constructing optimal portfolio are:
2. Markowitz model
3. Sharpe's single index model
4. Markowitz model:

The foundation of Modern Portfolio Theory is laid by Markowitz in 1951. Markowitz theory advise investor to invest in multiple securities rather than put all eggs in one basket because efficient diversification of the portfolio involves combining securities with less than positive correlation in order to reduce risk in the portfolio without sacrificing any of the portfolio return.

SINGLE INDEX MODEL:

A casual observation of stock prices over a period reveals that most stock prices move with the market index. When the Sensex increases, stock prices also tend to increase and vice-versa. This indicates that some underlying factors influence the market index as well as the stock prices. Stock prices are linked to the market index and this relationship can be used to determine the return on stock. The following equation can be used towards this purpose:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Where,

R_i = Expected return on security i

α_i = intercept of the straight line or alpha co-efficient

β_i = slope of the straight line or beta co-efficient

R_m = rate of return on the market index

e_i = error term

The single index model is based on the assumption that stocks vary together because of the common moment in stock market. There are no effects beyond the market (i.e., any fundamental factor effects) that account for the stocks co-moment. The expected return, standard deviation and co- variance of the single index model represent the joint moment of securities.

The variance of security's return $\sigma_{i = i m}^2 + \sigma_{ei}^2$

The covariance of returns between securities i and j is

$$\sigma_{ij} = \beta_i + \beta_j \sigma_m^2$$

Risk:

The variance of the security has two components namely; systematic risk or market risk and unsystematic risk or unique risk. The variance explained by the index is referred to as systematic risk. The unexplained variance is called residual variance or unsystematic risk.

Systematic risk = β^2 * variance of market index

$$= \beta^2 \sigma_m^2$$

i m

Unsystematic risk = Total variance – Systematic risk

$$e_i^2 = \sigma_i^2 - \text{Systematic risk}$$

i i

Thus, total risk = Systematic risk + Unsystematic risk

$$= \beta^2 \sigma_m^2 + e_i^2$$

i m i

Portfolio Variance:

From this point, the portfolio variance can be derived

$$\sigma_p^2 = \left[\left(\sum_{i=1}^N x_i \beta_i \right)^2 \sigma_m^2 \right] + \left[\sum_{i=1}^N x_i^2 e_i^2 \right]$$

Where,

σ_p^2 = Variance of portfolio

σ_m^2 = Expected variance of index

e_i^2 = Variation in a security's return not related to the market index

x_i = The portion of stock i in the portfolio

Expected return of Portfolio:

Likewise, the expected return on the portfolio can also be estimated. For each security, α_i and β_i should be estimated.

$$R_p = \sum_{i=1}^N x_i (\alpha_i + \beta_i R_m)$$

Portfolio return is the weighted average of the estimated return for each security in the portfolio. The weights are the respective stocks' proportions in the portfolio.

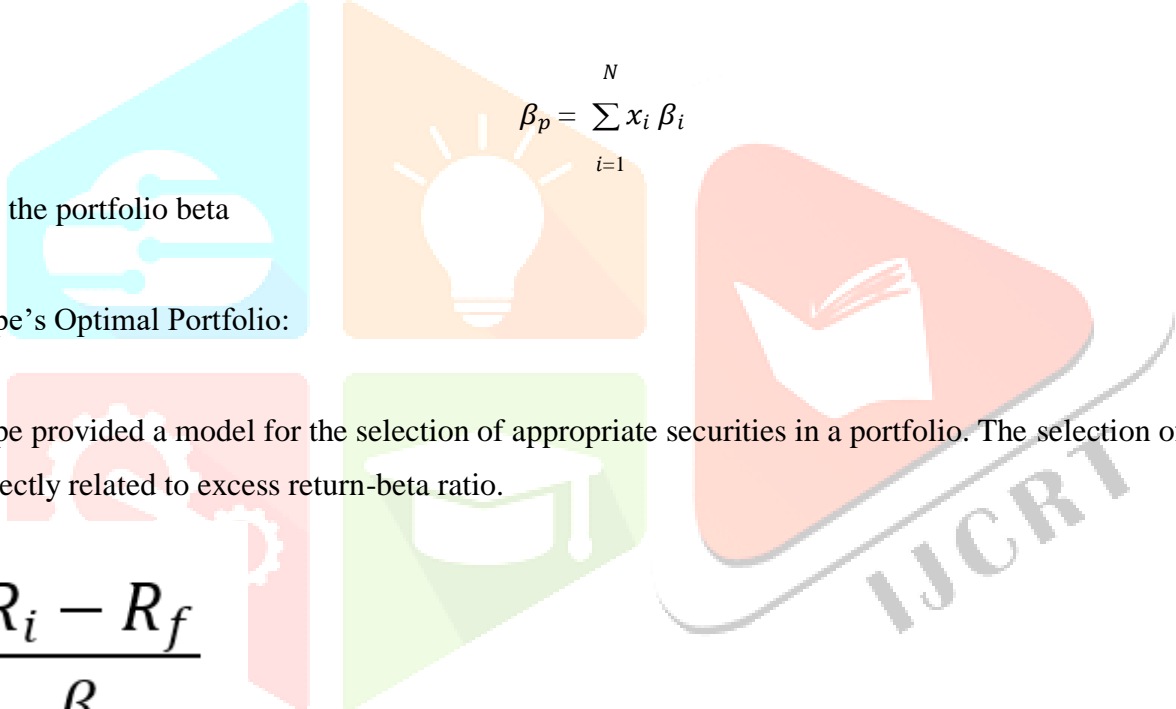
Portfolio Beta:

A portfolio's beta value is the weighted average of the beta values of its component stocks, with the weights being their relative shares in the portfolio.

β_p is the portfolio beta

$$\beta_p = \sum_{i=1}^N x_i \beta_i$$

Sharpe's Optimal Portfolio:



Sharpe provided a model for the selection of appropriate securities in a portfolio. The selection of any stock is directly related to excess return-beta ratio.

$$\frac{R_i - R_f}{\beta_i}$$

R_i = the expected return on stock i

R_f = the return on a risk-free asset

β_i = the expected change in the rate of return on stock i associated with one unit change in the market return.

The excess return is the difference between the expected return on the stock and the risk-free rate of interest such as the rate offered on a government security or Treasury bill. The excess return to beta ratio measures the additional return on a security (excess of the risk-free asset return) per unit of systematic risk or non-diversifiable risk. This ratio provides a relationship between potential risk and reward.

The ranking of stocks is done on the base of the excess return to beta. Portfolio manager would like to include stocks with higher ratios. The selection of stocks depends on a unique cut-off

rate such that all stocks with higher ratios of $R_i - R_f$

$$\beta_i$$

are included, and stocks with lower ratios are left out. The cut-off point is denoted by C^* .

The steps involved in finding out the stocks to be included in the optimal portfolio are as follows:

1. Find out the excess returns to beta ratio for each stock under consideration.
2. Rank them from the highest to lowest.
3. Calculate C_i for all stocks according to ranked order using the following formula

$$C_i = \frac{\frac{(R_i - R_f) \beta_i}{\sigma^2}}{1 + \frac{\sigma^2}{m} \sum_{i=1}^N \frac{\beta_i^2}{\sigma_{ei}^2}}$$

Where,

σ^2 = variance of the market index

σ_{ei}^2 = variance of a stock's movement that is not associated with the movement of market index, i.e., a stock's unsystematic risk.

4. The cumulated values of C_i start declining after a particular C_i and that point is taken as the cut-off point; also that stock ratio is the cut-off ratio C .

Construction of an optimal portfolio:

The portfolio manager has to find out how much will be invested in each security, once the securities are selected. The percentage of funds to be invested in each security can be estimated as follows:

$$X_i = \frac{Z_i}{\sum_{i=1}^N Z_i}$$

Sl No:	Company	Sector
1	Bajaj Auto Limited	Auto
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6	Ambuja Cement Ltd	Cement

Risk free rate of return:

Risk free security has zero variance and standard deviation. The risk free securities have no risk of default. The Government treasury bills or bonds are approximate example of risk free securities as they have no risk of default. The 364 days Treasury bill rate of return rate of return is considered as a risk free rate of return for Three years from Jan to Dec2012- 2014 is 8.27 %.

Hence, R_f 8.27%

Return on Market:

The return on the market is the return obtained by the companies consisting in the SENSEX index. The closing price of the S & P SENSEX index from Dec -2012 to Jan-2014 is recorded for calculation purposes.

4. ANALYSIS AND INTERPRETATION

Table 1: Bajaj Auto Limited

Month	Close Price	ri
12-Jan	1600.95	
12-Feb	1798.95	12.3677
12-Mar	1677.9	-6.7289
12-Apr	1623.1	-3.266
12-May	1513	-6.7833
12-Jun	1572	3.89954
12-Jul	1601.35	1.86705
12-Aug	1615.4	0.87739
12-Sep	1832.5	13.4394
12-Oct	1813.95	-1.0123
12-Nov	1930.55	6.42796
12-Dec	2131	10.3831
13-Jan	2027.8	-4.8428
13-Feb	1971.6	-2.7715
13-Mar	1794.9	-8.9623
13-Apr	1881.8	4.8415
13-May	1822.65	-3.1433
13-Jun	1917.45	5.20122
13-Jul	1935.7	0.95179
13-Aug	1839.5	-4.9698
13-Sep	1988.2	8.08372
13-Oct	2129	7.08178
13-Nov	1974.75	-7.2452
13-Dec	1910.55	-3.251
14-Jan	1902.8	-0.4056
14-Feb	1942.7	2.09691
14-Mar	2078.3	6.97998

14-Apr	1913.45	-7.932
14-May	1953.15	2.07479
14-Jun	2313.2	18.4343
14-Jul	2078.2	-10.159
14-Aug	2259.75	8.73593
14-Sep	2348.95	3.94734
14-Oct	2609.05	11.073
14-Nov	2639.6	1.17092
14-Dec	2438.45	-7.6205

Average return	1.491745
Risk	7.112909
Stock Variance	51.70041
Beta	1.135175

Interpretation:

The average return of the stock is 1.4917, while the beta of the stock is 1.1351 and stock variance happens to be 51.7004. This denotes that for the given risk, the company's stock is yielding a positive return. Accordingly, since the stock is giving a positive average rate of return, it is expected to be positive in the near future too.

Table 2: Hero Motocorp Ltd:

Date	Close Price	Ri
12-Jan	1863.9	
12-Feb	1950.45	4.64349
12-Mar	2054.85	5.35261
12-Apr	2234.9	8.7622
12-May	1837	-17.804
12-Jun	2142.1	16.6086
12-Jul	2005.7	-6.3676
12-Aug	1788.1	-10.849
12-Sep	1879.3	5.10039
12-Oct	1878.95	-0.0186
12-Nov	1827.9	-2.7169
12-Dec	1898.35	3.85415
13-Jan	1823.4	-3.9482
13-Feb	1667.8	-8.5335
13-Mar	1541.9	-7.5489
13-Apr	1645.1	6.69304
13-May	1730.7	5.20333
13-Jun	1662.25	-3.9551
13-Jul	1819.2	9.44202
13-Aug	2030.1	11.593
13-Sep	2008.8	-1.0492
13-Oct	2077.1	3.40004
13-Nov	2050.6	-1.2758
13-Dec	2074.8	1.18014
14-Jan	1970.45	-5.0294
14-Feb	1965.1	-0.2715
14-Mar	2272.85	15.6608
14-Apr	2193.85	-3.4758
14-May	2343.75	6.83274
14-Jun	2627.2	12.0939
14-Jul	2596.05	-1.1857
14-Aug	2608.15	0.46609
14-Sep	2842.05	8.96804
14-Oct	3061.6	7.72506
14-Nov	3142.2	2.63261
14-Dec	3103.4	-1.2348

Average return	1.70959
Risk	7.54079
Stock Variance	58.9929
Beta	0.98182

Interpretation :

The average return of the stock is 1.7095 %, while the beta of the stock is 0.9818 and stock variance happens to be 58.9929. This denotes that for the given risk, the company's stock is yielding a positive return. Accordingly, since the stock is giving a positive average rate of return, it is expected to be positive in the near future too.

Table 3: Axis Bank Ltd:

Month	Close Price	ri
12-Jan	1074.2	
12-Feb	1178.05	9.66766
12-Mar	1145.9	-2.7291
12-Apr	1106.6	-3.4296
12-May	971.55	-12.204
12-Jun	1015.8	4.55458
12-Jul	1042.7	2.64816
12-Aug	999.9	-4.1047
12-Sep	1136.75	13.6864
12-Oct	1183.65	4.1258
12-Nov	1316.35	11.2111
12-Dec	1356.5	3.0501
13-Jan	1505.2	10.962
13-Feb	1343.3	-10.756
13-Mar	1301.25	-3.1304
13-Apr	1493.1	14.7435
13-May	1430.65	-4.1826
13-Jun	1323.05	-7.5211
13-Jul	1034.45	-21.813
13-Aug	835.75	-19.208
13-Sep	1007.3	20.5265
13-Oct	1222.9	21.4038
13-Nov	1155.3	-5.5278
13-Dec	1299.65	12.4946
14-Jan	1119.55	-13.858
14-Feb	1265.9	13.0722
14-Mar	1460.45	15.3685
14-Apr	1519.2	4.02273
14-May	1836.15	20.863
14-Jun	1921.4	4.64287
14-Jul	391.85	-79.606
14-Aug	397.3	1.39084
14-Sep	377.8	-4.9081
14-Oct	438.75	16.1329

14-Nov	481.05	9.64103
14-Dec	502.05	4.36545

Average return	0.77783
Risk	16.3866
Stock Variance	329.807
Beta	2.31826

Interpretation:

The average return of the stock is 0.7778, while the beta of the stock is 2.3182 and stock variance happens to be 329.8073. This denotes that for the given risk, the company's stock is yielding a positive return. Accordingly, since the stock is giving a positive average rate of return, it is expected to be positive in the near future too.

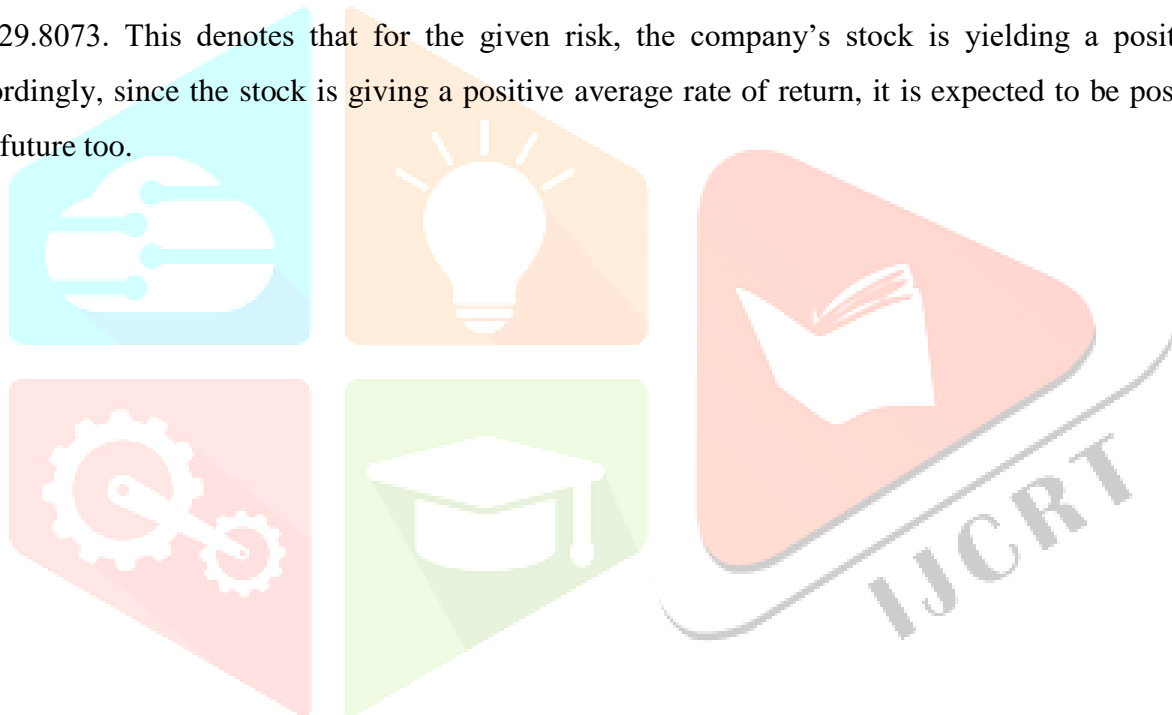


Table 4: ICICI Bank Ltd:

Month	Close Price	ri
12-Jan	902	
12-Feb	906.5	0.49889
12-Mar	887.25	-2.1236
12-Apr	881.45	-0.6537
12-May	784.3	-11.022
12-Jun	899.6	14.701
12-Jul	960.75	6.79747
12-Aug	909.3	-5.3552
12-Sep	1057.25	16.2708
12-Oct	1050.05	-0.681
12-Nov	1098.6	4.62359
12-Dec	1137.3	3.52267
13-Jan	1190.85	4.70852
13-Feb	1040.3	-12.642
13-Mar	1045.35	0.48544
13-Apr	1163.15	11.269
13-May	1154.3	-0.7609
13-Jun	1069.75	-7.3248
13-Jul	909.3	-14.999
13-Aug	804.3	-11.547
13-Sep	884.95	10.0274
13-Oct	1121.05	26.6795
13-Nov	1067.9	-4.7411
13-Dec	1098.45	2.86076
14-Jan	988.5	-10.01
14-Feb	1043.7	5.58422
14-Mar	1245.45	19.3303
14-Apr	1244.8	-0.0522
14-May	1418.4	13.946
14-Jun	1418.45	0.00353
14-Jul	1473	3.84575
14-Aug	1556.55	5.6721
14-Sep	1435.25	-7.7929
14-Oct	1625.45	13.2521
14-Nov	1754.6	7.94549
14-Dec	353	-79.882

Average return	0.13484
Risk	15.2892
Stock Variance	292.183
Beta	3.33599

Interpretation:

The average return of the stock is 0.1348, while the beta of the stock is 3.3359 and stock variance happens to be 292.183. This denotes that for the given risk, the company's stock is yielding a positive return. Accordingly, since the stock is giving a positive average rate of return, it is expected to be positive in the near future too.

Table 5: ACC Cement Ltd

Month	Close Price	ri
12-Jan	1189.9	
12-Feb	1304.9	9.66468
12-Mar	1356.35	3.94283
12-Apr	1234.65	-8.9726
12-May	1140.2	-7.6499
12-Jun	1267.55	11.1691
12-Jul	1319.6	4.10635
12-Aug	1312.45	-0.5418
12-Sep	1469.2	11.9433
12-Oct	1379.3	-6.119
12-Nov	1383.95	0.33713
12-Dec	1429.4	3.28408
13-Jan	1322.55	-7.4752
13-Feb	1271.1	-3.8902
13-Mar	1161.25	-8.6421
13-Apr	1237.1	6.53176
13-May	1218.85	-1.4752
13-Jun	1223.45	0.37741
13-Jul	1171.35	-4.2585
13-Aug	961.8	-17.89
13-Sep	1112.45	15.6633
13-Oct	1136.7	2.17987
13-Nov	1096.55	-3.5322
13-Dec	1108.2	1.06242
14-Jan	1010.85	-8.7845
14-Feb	1103.45	9.16061
14-Mar	1391.55	26.109
14-Apr	1291.7	-7.1755
14-May	1339.5	3.70055
14-Jun	1468.05	9.59687
14-Jul	1394.5	-5.0101
14-Aug	1491.7	6.97024
14-Sep	1404.15	-5.8691
14-Oct	1500.9	6.89029
14-Nov	1470.35	-2.0355



14-Dec	1399.4	-4.8254
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Average return	0.84632
Risk	8.56556
Stock Variance	74.6377
Beta	1.49727

Interpretation:

The average return of the stock is 0.8463, while the beta of the stock is 1.4972 and stock variance happens to be 74.6377. This denotes that for the given risk, the company's stock is yielding a positive return. Accordingly, since the stock is giving a positive average rate of return, it is expected to be positive in the near future too.

Table 6: Ambuja Cement Ltd

Month	Close Price	ri
12-Jan	160.7	
12-Feb	160.05	-0.4045
12-Mar	171.25	6.99781
12-Apr	150.9	-11.883
12-May	151.05	0.0994
12-Jun	173.95	15.1605
12-Jul	180.25	3.62173
12-Aug	185.45	2.88488
12-Sep	201.95	8.89728
12-Oct	202.7	0.37138
12-Nov	207.7	2.4667
12-Dec	200.95	-3.2499
13-Jan	202.3	0.67181
13-Feb	191.85	-5.1656
13-Mar	173.45	-9.5908
13-Apr	186.45	7.49496
13-May	183.25	-1.7163
13-Jun	186.7	1.88267
13-Jul	167.3	-10.391
13-Aug	170.8	2.09205
13-Sep	183.15	7.23068
13-Oct	188.45	2.8938
13-Nov	185.35	-1.645
13-Dec	182.75	-1.4028
14-Jan	159.3	-12.832
14-Feb	167.35	5.05336
14-Mar	201.15	20.1972
14-Apr	197.45	-1.8394
14-May	217.3	10.0532



14-Jun	220.7	1.56466
14-Jul	206.45	-6.4567
14-Aug	206.3	-0.0727
14-Sep	213.7	3.58701
14-Oct	227.45	6.43425
14-Nov	229.85	1.05518
14-Dec	228.7	-0.5003

Average return	1.27305
Risk	6.94731
Stock Variance	49.3504
Beta	1.03391

Interpretation:

The average return of the stock is 1.2730, while the beta of the stock is 1.0339 and stock variance happens to be 49.3504. This denotes that for the given risk, the company's stock is yielding a positive return. Accordingly, since the stock is giving a positive average rate of return, it is expected to be positive in the near future too.

Sl.No	Company	t R	t σ	t Q	σ^2 t	Q^2 t	Sys. Risk	σ^2 e t
1	Bajaj Auto	1.492	7.1129	1.135	50.593	1.2886	19.6336	32.067
2	Hero Motocorp	1.71	7.5407	0.982	56.862	0.9639	14.6872	44.306
3	Axis Bank Ltd	0.778	16.387	2.318	268.52	5.3743	81.8842	247.92
4	ICICI Bank Ltd	0.135	15.289	3.336	233.76	11.129	169.56	122.62
5	ACC	0.846	8.5655	1.497	73.368	2.2418	34.1567	40.481
6	Ambuja	1.273	6.9473	1.034	48.265	1.0689	16.2868	33.064

Where,

R_i = Return

σ_i = Risk

β_i = Beta

Systematic Risk = $\beta_i^2 * \sigma^2$

Unsystematic Risk ($\sigma^2 - \text{Systematic Risk}$) = σ_i^2

σ_m = 4.0895

σ_m^2 = 15.2361

Table showing ranking of companies based on excess return to beta

Company	R_i	$R_i - R_f$	Q_i	$\frac{(R_i - R_f)}{Q_i}$	Rank
Hero Motocorp	1.7095	-6.5605	0.9818	-6.6818	11
Bajaj Auto	1.4917	-6.7783	1.1351	-5.9711	10
Axis Bank Ltd	0.7778	-7.4922	2.3182	-3.2318	6
ICICI Bank Ltd	0.1348	-8.1352	3.3359	-2.4386	4
ACC	0.8463	-7.4237	1.4972	-4.958	9
Ambuja	1.273	-6.997	1.0339	-6.7674	12
Hero Motocorp	1.7095	-6.5605	0.9818	-6.6818	11

Where,

$R_i - R_f$ = Excess Return

$\frac{(R_i - R_f)}{\beta_i}$ = Excess return to beta ratio

STEP 2:- The companies are rearranged here according to their ranks with decreasing excess return to beta ratios.

Table No.: Table showing rearranging of companies according to excess return to beta

Company	R_i	$R_i - R_f$	Q_i	$\frac{(R_i - R_f)}{Q_i}$	Rank
ICICI	0.1348	-8.1352	3.3359	-2.4386	4
Axis	0.7778	-7.4922	2.3182	-3.2318	6
ACC	0.8463	-7.4237	1.4972	-4.958	9
Bajaj Auto	1.4917	-6.7783	1.1351	-5.9711	10
Hero Motcorp	1.7095	-6.5605	0.9818	-6.6818	11
Ambuja	1.2730	-6.997	1.0339	-6.7674	12

STEP 3:- C Values for each company is calculated using the following formula

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^N \frac{(R_i - R_f)\beta_i}{\sigma^2}}{1 + \sigma_m^2 \sum_{i=1}^N \frac{\beta_i^2}{\sigma_{ei}^2}}$$

$$\sigma_m^2 = 15.2361$$

Table No.:

Table showing calculation of cut-off

Company	$(R_i - R_f) * Q_i$	$\frac{(R_i - R_f) * Q_i}{2}$	$\sum (R_i - R_f) * Q_i$	σ_{ei}^2	$\sum \sigma_{ei}^2$	Cut-off
1.ICICI	-0.2213	-0.4149	-6.3214	0.0908	0.1764	-1.7142c*
2.Axis	-0.0701	-0.5452	-8.3067	0.0271	0.3213	
3.ACC	-0.2745	-1.0855	-16.5387	-0.0554	0.3337	
4.Bajaj Auto	-0.239	-1.3245	-20.1802	0.0402	0.3739	
5.Hero Motocorp	-0.1454	-1.4699	-22.3955	0.0217	0.3956	
6.Ambuja	-0.2118	-1.6817	-25.6225	0.0323	0.4279	

STEP 4:- The C_i values go's on increasing up to a certain point and then starts decreasing in the table shown. The highest point is called cut off point , the securities which are above C^* point are chosen to the portfolio. i.e., the companies to be included in the portfolio are: Table No.:

Table showing selected companies for investment

FINDINGS:

1. The Single index model is able to provide complete comparative study on selected lot of stocks which are yielding varied returns.
2. By calculating single cut-off rate one can easily and accurately arrive at optimal portfolio of stocks to be selected for investment.
3. Single index considers and provides various weightage aspects which will provide clear picture and possibilities of investment for investors.
4. According to excess return beta ratio company Ashok Leyland ranks first followed by ,ICICI.
5. Z_i Helps to determine the proportion of funds deployed in each security, which ensures optimal return for investors.
6. Among the 15 companies Ashok Leyland is giving highest returns with 2.95%

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