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CONSTRUCTION OF FOUR LEVEL VARIANCE-SUM THIRD ORDER SLOPE ROTATABLE DESIGNS

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Abstract

Designs which are used for the study of response surfaces are called response surface designs response surface methodology usually adopts sequential procedure our objective here is to rapidly and efficiently lead the experimenter to the general vicinity of the optimum. In this paper it is proposed to construct a response surface design known as Variance-Sum Third Order Slope Rotatable Design.

Key Words: Second Order Rotatable Design, Second Order Slope Rotatable Design, Third Order Slope Rotatable Design, Variance-Sum - Third Order Slope Rotatable Design.

1. Introduction:-

The fitting of the Response Polynomial can be complex and costly if done haphazardly thus the process requires expert knowledge on design and analysis of experiments. To cut on costs, an experimenter has to make a choice of the experimental design prior to experimentation. Rotatability is a natural and desirable property, which requires that the variance of a predicted response at a point remains constant at all such points that are equidistant from the design centre. Bose and Draper(1990) point out that the technique of fitting a Response Polynomial is one widely used to aid in the statistical analysis of experimental work in which the response of a product depends in some unknown fashion, on one or more controllable variables. A Particular selection of settings or factor levels at which observations are to be taken is called a design. Designs are usually selected to satisfy some desirable criteria chosen by the experimenter. These criteria include the rotatability criterion and the criterion of minimizing the mean squire error of estimation over a given region in the factor space. They considered a problem arising in the design of experiments for empirically investigating the relationship between a dependent and several independent variables assuming that the form of the functional relationship is unknown but that within the region of interest. Draper constructed Third order rotatable designs in three dimensions by combining pairs of II order rotatable designs. second and Third order rotatable designs in three dimensions but did not give the optimality criteria for the designs. There is a need to give hypothetical examples to all the existing designs to make them ready for the experimenters to apply in the production processes. In this paper an attempt is made to construct Variance-Sum Third Order Slope Rotatable Design.

VARIANCE-SUM THIRD ORDER SLOPE ROTATABLE DESIGNS

Anjaneyulu *et al.*, (1997) established that SOSRD (OAD) has the additional interesting property that the sum of the variances of estimates of slopes in all axial directions at any point is a function of the distance of the point from the design origin. We define a Variance - Sum Third Order Slope Rotatable Design is one in which the sum of the variances of estimates of slopes of a third order Response Polynomial in all axial directions at any point is a function of the distance of the point from the design origin. That is, any symmetric THIRD Order Response Polynomial Design is a Variance Sum TOSRD, if

$$\sum_{i=1}^{v} V\left(\frac{\partial \hat{y}}{\partial x_i}\right) = f\left(d^2\right), \quad \text{Where } d^2 = \sum_{i=1}^{v} x_i^2$$

Anjaneyulu *et al.*, (1994) introduced a new method of Construction Of THIRD Order Slope Rotatable Design Through Doubly Balanced Incomplete Block Design. Anjaneyulu *et al.*, (1997) established that SOSRD (OAD) has the additional interesting property that the sum of the variances of estimates of slopes in all axial directions at any point is a function of the distance of the point from the design origin. Anjaneyulu *et al.*, (2000) introduced THIRD Order Slope Rotatable Designs. Anjaneyulu *et al.*, (2002)

introduced Variance-Sum THIRD Order Slope Rotatable designs.Anjaneyulu et al., (2006) constructed Sequential THIRD Order Slope Rotatable Designs Overall All Directions.Anjaneyulu *et al.*, (2006) introduced and constructed Embedded Type THIRD Order Slope Rotatable Designs Over All Directions.

Anjaneyulu *et al.*, (2006) constructed THIRD Order Slope Rotatable Designs Over All Directions Using Embedding Techniques. Anjaneyulu *et al.*, (2006) introduced Embedding In THIRD Order Slope Rotatable Designs Over All Directions. Anjaneyulu *et al.*, (2008) studied Construction Of Variance-Sum THIRD Order Slope Rotatable Designs. Anjaneyulu *et al.*, (2009) introduced and constructed Variance-Sum Group-Divisible THIRD Order Slope Rotatable Designs.

Guravaiah et al (2021 a, b, c) constructed some New Variance – Sum Second and Third Order Slope Rotatable Designs.

THEOREM: The conditions for Variance-Sum Slope Rotatability for the Symmetric THIRD Order Response Polynomial Design are the following :

SYMMETRY CONDITIONS:

A: All sums of products in which at least one of the x's is with an odd power are zero.

B: (i) $\sum x_i^2 = N\lambda_2 = \text{constant}$

(ii) $\sum x_i^4 = a N\lambda_4 =$ constant

(Third) $\sum x_i^6 = b N\lambda_6 =$ constant

C: (i) $\sum x_i^2 x_j^2 = N\lambda_4 =$ constant , for $i \neq j$

(ii) $\sum x_i^2 x_j^4 = c N\lambda_4 =$ constant , for $i \neq j$

(Third) $\sum x_i^2 x_j^2 x_k^2 = N\lambda_6 =$ constant , for $i \neq j \neq k$

NON - SINGULARITY CONDITIONS:

D. (i) $[\lambda_4/\lambda_2^2] > [v (a + v - 1)]$

(ii) $$\left[\lambda_2\lambda_6/\lambda_4^2\right] > \frac{\left\{a^2(v+1)-(6a-b)(v-1)\right\}}{\left[b(v+1)-9(v-1)\right]}$$

2. CONSTRUCTION OF FOUR LEVEL VARIANCE-SUM THIRD ORDER SLOPE ROTATABLE DESIGNS

The four level Third order slope rotatable design is constructed by combining a pair of Third order rotatable designs in four levels, suitably chosen to satisfy the non-singularity conditions.

The Considered Design points are

[S (±α, ±α, ±α, ±α,) + S (±β, ±β, ±β, ±β) + S (±ϒ, 0, 0, 0) + S (±ϒ$_1$, 0, 0, 0)]

α α α α

α α α -α

α α -α α

α α -α -α

α -α α α

α -α α -α

α -α -α α

α -α -α -α

-α α α α

-α α α -α

-α α -α α

-α α -α -α

-α -α α α

-α -α α -α

-α -α -α α

-α -α -α -α

β β β β

β β β -β

β β -β β

β β -β -β

β -β β β

β -β β -β

β -β -β β

β -β -β -β

-β β β β

-β β β -β

-β β -β β

-β β -β -β

-β -β β β

-β -β β -β

-β -β -β β

$-\beta$	$-\beta$	$-\beta$	$-\beta$
Υ	0	0	0
$-\Upsilon$	0	0	0
0	Υ	0	0
0	$-\Upsilon$	0	0
0	0	Υ	0
0	0	$-\Upsilon$	0
0	0	0	Υ
0	0	0	$-\Upsilon$
Υ	Υ	0	Υ
Υ_1	0	0	0
$-\Upsilon_1$	0	0	0
0	Υ_1	0	0
0	$-\Upsilon_1$	0	0
0	0	Υ_1	0

0	0	$-\Upsilon_1$	0
0	0	0	Υ_1
0	0	0	$-\Upsilon_1$
Υ_1	0	0	0
$-\Upsilon_1$	0	0	0
0	Υ_1	0	0
0	$-\Upsilon_1$	0	0
0	0	Υ_1	0
0	0	$-\Upsilon_1$	0
0	0	0	Υ_1
0	0	0	$-\Upsilon_1$

The considered design points gives us

1. $\sum x_i^2 = 16\alpha^2 + 16\beta^2 + 8\gamma^2 + 8\gamma_1^2 = N\lambda_2$..........(2.1)
2. $\sum x_i^4 = 16\alpha^2 + 16\beta^4 + 8\gamma^4 + 8\gamma_1^4 = aN\lambda_4$.........(2.2)
3. $\sum x_i^6 = 16\alpha^6 + 16\beta^6 + 8\gamma^6 + 8\gamma_1^6 = bN\lambda_6$.........(2.3)
4. $\sum x_i^2 x_j^2 = 16\alpha^4 + 16\beta^4 = N\lambda_4$..........(2.4)
5. $\sum x_i^2 x_j^4 = 16\alpha^6 + 16\beta^6 = 3N\lambda_6$.........(2.5)
6. $\sum x_i^2 x_j^2 x_k^2 = 16\alpha^6 = N\lambda_6$..........(2.6)

Where α=1,

β=1.99912, γ=1.4142, γ_1=0.7471

on solve the equations it gives

$N = 48$
$\lambda_2 = 2.090509$
$\lambda_4 = 5.657286$
$\lambda_6 = 0.33334$

Solving the equation (2.2) and equation (2.4) gives values of ‘a’ and ‘b’

a = 1.1267538

b = 68.912866

c = 16.48260

$\text{var}(\hat{b}_0) = (0.422955)\sigma^2$
$\text{var}(\hat{b}_i) = (0.009965)\sigma^2$
$\text{var}(\hat{b}_{ij}) = (0.003683)\sigma^2$
$\text{var}(\hat{b}_{ii}) = (0.0016973)\sigma^2$

Where ‘α’ is arbitrary and has a positive value

The considered design points forms a Variance-Sum Third Order Slope Rotatable arrangement of order three for the values of the constants given in (2.1). Substituting (2.2) in non-singularity conditions gives the values of λ_2, λ_4 and λ_6 which finally satisfies the non-singularity conditions. Hence the considered design points forms a Variance-Sum Third Order Slope Rotatable Design in four dimensions.

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