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QUALITY IMPROVEMENT THROUGH THIRD-ORDER SLOPE- ROTATABLE DESIGNS OVER ALL DIRECTIONS

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ABSTRACT

Recent years have been an upsurge of interest in the role of statistical ideas and methods in improving the quality and productivity of industrial processes and products. Response Surface Methodology is, one such idea, useful for analyzing problems where several independent variables influence a dependent variable or response. The earlier study of response surface designs mainly emphasized the estimation of absolute response. Estimation of differences in response at different points in the factor space will often be of great importance. If differences at points closed together are involved, estimation of local slope i.e., the rates of change of the response surface is of interest. This problem, estimation of slopes, occurs frequently in practical situations, particularly in third order response surface. This enable the determination of the best operating conditions for the process i.e., the best combination of the levels of the controllable factors which gives the “optimum” value of the third order response function. This would also enable us to determine the best way to control the process. In this paper, we made an attempt to study the role of third order slope rotatability to improve the quality of a product. Anjaneyulu et al (1993) introduced embedding in Second–Order – Slope Rotatable Designs and constructed the same using embedding techniques similar to those of Draper (1960), Herzberg (1967). Park (1987) studied the necessary and sufficient conditions for second order slope rotatability over all directions, Anjaneyulu et al (1997) showed that these designs have the Variance-Sum Property. Anjaneyulu et al (1995) introduced TOSRD and gave a method of construction of TOSRD using Central Composite type design points. Anjaneyulu et al (2004) introduced that any Variance-Sum-Third Order Slope Rotalable Design is a Third Order Slope Rotatable Design Over All Directions introduced by Park and Lee (1995). In this paper, an attempt is made to explain the role of embedding in TOSRDOAD for Quality Improvement.

Key Words :

Quality Improvement, Response Surface Methodology, Thrid Order Slope Rotatability, Third -Order-Slope-Rotatable Designs Over All Directions, Third Order Rotatable Designs, Embedded designs.

1. INTRODUCTION

Recent years have seen an upsurge of interest in the role of statistical ideas and methods in improving the quality and productivity of industrial processes and products. Response Surface Methodology is, one such idea, useful for analyzing problems where several independent variables influence a dependent variable or response. The earlier study of response surface designs mainly emphasized the estimation of absolute response. The choice of combinations of levels of the independent variables, which will enable an experimenter to approximate a functional relationship by fitting a dth order polynomial, by the method of least squares. Emphasis was placed on judging a design on the basis of "Prediction Variance". A natural and easily attainable property was that of rotatability, which requires that the variance of a predicted value remain constant at points that are equidistant from the design center -"Stability in Prediction Variance" (See Box et al (1978)).

Data analysis procedures in general are expanding. Japan Scientists, who have successfully communicated to us the need for experimental design at the research - and -development level as a successful supplement to quality control and process control. The Japanese industrial engineer Genichi Taguchi proposed methodology for product improvement. Here focus is on " improvement" and not on "optimization" (See KACKAR (1985)). Taguchi made users aware of the need to include system or process variability as part of the response. As a result the inevitable "nonconstancy of variance is not ignored". Defining "Information" at any point as the reciprocal of the variance at that point, we must ideally then, have experimental designs which provide information contours that would be circles for two ' variables, spheres or hyper spheres for three or more variables, so that at a "Constant" distance from the centre of the design, the experimenter will gather a constant quantity of information. Such designs are termed as Rotatable Designs. These have the property that whenever the orthogonal axes of the design take any orientation (i.e., if the axes are rotated about the centre) of the confidence in a prediction made on the response at any given point will not be altered. (See Raja and Yayin (1993)).

In many applications of RSM, good estimation of the derivatives of the response function may be as important or perhaps more important than estimation of mean response. Certainly, the computation of a stationary point in second order analysts depends heavily on the partial derivatives of the estimated response surface function with respect to the design variables. The earlier study of response surface designs mainly emphasized the estimation of absolute response. Estimation of differences in response at different points in the factor space will often be of great importance. If differences at points close together are involved, estimation of local slope i.e. the rates of change of the response surface is of interest. This problem, estimation of slopes, occurs frequently in practical situations, particularly in fitting Second and Third order response surface.

Embedded designs may be particularly useful when the experimenter feels the necessity to introduce additional factor(s) in the known experiment without discarding the original results. Very particular in agriculture field experiments, quality improvement programme these are highly useful. With this technique repeating the same experiment for high number of factors does not arise. Hence time and cost of expenditure will be saved. In view of this practical importance many authors developed several kinds of embedded designs.

Draper (1960), Herzberg (1967) and Huda (1982) gave different methods of Embedding in Second – Order and Third Order Rotatable Designs). Draper's method has the advantage that the number of points required is usually fewer than the number required in the Hefzberg's method. These embedded designs may be particularly useful when the experimenter feels the necessity to introduce an additional factor in the experiment to after earlier study of an experiment with lower number of factors and without discarding the results.

Hader and Park (1978) introduced slope rotatability on axial directions for central composite designs.. Anjaneyulu et al (1993) constructed Second Order Slope Rotatable Designs using Draper and Herzberg type embedding techniques

Park (1987) studied the necessary and sufficient conditions for slope rotatability over all directions. Anjaneyulu et al (1997) showed that these designs have the additional property that the sum of the variances of estimate of slopes in all directions at any point is a function of the distance of the point from the design origin.

In this paper, we present some methods of construction of v –factor Third Order Slope Rotatable Design Over All Directions from known (v-1) factor TOSRDOAD using embedding techniques, in view of their practical utility in day-to-day life experiments and quality improvement techniques.

2.THIRD ORDER SLOPE ROTATABLE DESIGNS (TOSRDs) :

Anjaneyulu et al (1995) and Anjaneyulu et al (2000) introduced third order slope rotatable designs and construction of the same with CCD type design points and Doubly Balanced Incomplete Block Design points.

Let us consider a general third order response surface as

$$y(x) = b_0 + \sum_{i=1}^{v} b_i x_i + \sum_{i \neq j}^{v} \sum^{v} b_{ij} x_i x_j + \sum_{i=1}^{v} b_{ii} x_i^2$$

$$+ \sum_{i=1}^{v} b_{iii} x_i^3 + \sum_{i \neq j} \sum b_{ijj} x_i x_j^2 + \sum_{i<j<k} \sum \sum b_{ijk} x_i x_j x_k + e$$

where e's are independent random errors with same mean zero and variance, let us consider the following N design points in v-factors for fitting the above surface.

Definition of TOSRD :- A general third order response surface design is said to be a Third Order Slope Rotatable Design (TOSRD) if the variance of the estimate of first order partial derivative of y(x) with respect to each of independent variables (x_i) is only a function of the distance $\left(d^2 = \sum_{i=1}^{v} x_i^2\right)$ of the point $(x_1, x_2, \ldots\ldots, x_v)$ from the origin (centre).

(i.e)., a third order response surface design is a TOSRD if

$$V\left(\frac{\partial \hat{y}}{\partial x_i}\right) = f(d^2),\ \forall\ i = 1,2,\ldots\ldots$$

The assumptions and conditions for Third Order Slope Rotatability are :

Symmetry Assumptions :

A) All sums like $\sum x_i, \sum x_i x_j, \sum x_i^2 x_j, \sum x_j^2, \sum x_i x_j x_k, \sum x_i^2 x_j x_k$

$\sum x_i^2 x_j, \sum x_i^9 x_j^9, \sum x_i x_j x_k x_L$ ……. etc are equal to zero. That is all sums of products in which at least one of the x's is with an odd power are zero

B: (i) $\sum x_i^2 = N\lambda_2 = \text{constant}, \ \forall\ i$

(ii) $\sum x_i^4 = a\,N\lambda_4 = \text{constant}, \ \forall\ i$

(iii) $\sum x_i^6 = b\,N\,\lambda_6 = \text{constant}, \ \forall\ i$

C: (i) $\sum x_i^2 x_j^2 = N\lambda_4 = \text{constant, for } i \neq j$

(ii) $\sum x_i^2 x_j^4 = c\,N\lambda_6 = \text{constant, for } i \neq j$

(iii) $\sum x_i^2 x_j^2 x_k^2 = N\,\lambda_6 = \text{constant, for } i \neq j \neq k$

Slope Rotatability Conditions :

D (i) c = 3

(ii) $\Delta_1\left[\lambda_4\left[v(5-a)-(a-3)^2\right]+\lambda_2^2[v(a-5)+4]\right]$

$$- \Delta\left[2(a-1)\lambda_4^2[3a(v+1)-9(v-1)+3a-b]\right]=0$$

Where $\Delta = \left[(a+v-1)\lambda_4 - v\lambda_2^2\right]$

$$\Delta_1 = \left[\lambda_2\lambda_6\{b(v+1)-9(v-1)\}-\lambda_4^2\{a^2(v+1)-(6a-b)(v-1)\}\right]$$

(iii) $[\lambda_2\lambda_6\{v(b-9)\}]-\lambda_4^2\{a^2v-6a(v-2)+b(v-2)-6a\}]=0$

(iv $[\lambda_2\lambda_6\{v(b-27)\}-\lambda_4^2\{a^2v-6a(v-2)+b(v-2)-18(v-1)\}]=0$

Non- Singularity Conditions :

(E) (i) $(\lambda_4/\lambda_2^2) > (v/(a+v-1))$

(ii) $\left[\lambda_2\lambda_6/\lambda_4^2\right] > \dfrac{\{a^2(v+1)-(6a-b)(v-1)\}}{[b(v+1)-9(v-1)]}$

(using c = 3)

3. VARIANCE - SUM THIRD ORDER SLOPE ROTATABLE DESIGNS :

Definition : We define a Variance – Sum Third Order Slope Rotatable Designs is one in which the sum of the variances of estimates of slopes of a third order response surface in all axial directions at any point is a function of the distance of the point from the design origin. That is, any symmetric Third Order Response Surface Design is a Variance - Sum TOSRD, if

$$\sum_{i=1}^{v} v\left(\frac{\partial \hat{y}}{\partial x_i}\right) = f(d^2), \text{ Where } d^2 = \sum_{i=1}^{v} Xi^2$$

THEOREM : The conditions for Variance-Sum Slope Rotatability for the Symmetric Third Order Response Surface Design are the following.

SYMMETRY CONDITIONS:

A: All sums of products in which at least one of the x's is with an odd power are zero.

B: (i) $\sum x_i^2 = N\lambda_2 =$ constant

(ii) $\sum x_i^4 = a\,N\lambda_4 =$ constant

(iii) $\sum x_i^6 = b\,N\lambda_6 =$ constant

C: (i) $\sum x_i^2 x_j^2 = N\lambda_4 =$ constant , for $i \neq j$

(ii) $\sum x_i^2 x_j^4 = c\,N\lambda_4 =$ constant , for $i \neq j$

(iii) $\sum x_i^2 x_j^2 x_k^2 = N\lambda_6 =$ constant , for $i \neq j \neq k$

NON- SINGULARITY CONDITIONS :

D (i) $(\lambda_4 / \lambda_2^2) > (v / (a + v - 1))$

(ii) $\left[\lambda_2 \lambda_4 / \lambda_4^2\right] > \dfrac{\{a^2 (v+1) - (6a - b)(v-1)\}}{[b(v+1) - 9(v-1)]}$

PROOF: Let D = $((x_{ij}))$, i = 1, 2, ..., N ; j = 1, 2, ...,ν be a ν factor symmetric third order response surface design satisfying conditions given below, to fit third order response surface .

Assume the third order response surface as follows

$$y(x) = b_0 + \sum_{i=1}^{v} b_i x_i + \sum_{i<j}^{v}\sum^{v} b_{ij} x_i x_j + \sum_{i=j}^{v} b_{ii} x_i^2 + \sum_{i=1}^{v} b_{iii} x_i^3$$

$$+ \sum_{i \neq j}\sum b_{ijj} x_i x_j^2 + \sum_{i<j<k}\sum\sum b_{ijk} x_i x_j x_k + e$$

Where e's are independent random errors with same mean zero and Variance σ^2.

Thus we have,

$$\frac{\partial \hat{y}}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii}x_i + \sum_{j\neq i} \hat{b}_{ij}x_j + 3\hat{b}_{iii}x_i^2 + \sum\sum_{j\neq i} \hat{b}_{ijjj}x_j^2$$

$$+2\sum_{j\neq i}\sum_{j<k} b_{ijj}x_i x_j + \sum\sum\sum_{i<j<k} b_{ijk}x_j x_k$$

$$V(\partial \hat{y}/\partial x_i) = V(\hat{b}_i) + 4x_1^2 V(\hat{b}_{ii}) + \sum_{j\neq i}^{v} x_j^2 V(\hat{b}_{ij}) + 9x_i^4 V(\hat{b}_{iii})$$

$$+\sum_{\substack{j\neq i\\ j=i}}^{v} x_j^4 V(\hat{b}_{ijj}) + 4\sum_{\substack{j\neq i\\ j=i}}^{v} x_i^2 x_j^2 V(\hat{b}_{iij}) + \sum_{j}^{v}\sum_{<k}^{v} x_j^2 x_k^2 V(\hat{b}_{ijk})$$

$$+6x_i^2 \mathrm{Cov}(\hat{b}_i \hat{b}_{iij}) + 2\sum_{\substack{j\neq i\\ j=1}}^{v} x_i^2 \mathrm{Cov}(\hat{b}_i \hat{b}_{ijj})$$

$$+6\sum_{\substack{j\neq 1\\ j=1}}^{v} x_i^2 x_j^2 \mathrm{Cov}(\hat{b}_{iii}\hat{b}_{ijj}) + 2\sum_{\substack{j\neq k\\ j<k}}^{v}\sum_{\neq i}^{v} x_j^2 x_k^2 \mathrm{Cov}(\hat{b}_{ijj}\hat{b}_{ikk})$$

Therefore, we have,

$$\sum_{i=1}^{v} V(\partial\hat{y}/\partial x_i) = \sum_{i=1}^{v} V(\hat{b}_i) + 4\sum_{i=1}^{v} x_i^2 V(\hat{b}_{ii}) + \sum_{i=1}^{v}\left(\sum_{\substack{j\neq i\\ j=1}}^{v} x_j^2 V(\hat{b}_{ij})\right)$$

$$+9\sum_{i=1}^{v} x_i^4 V(\hat{b}_{iii}) + \sum_{i=1}^{v}\left(\sum^{v} x_j^4 V(b_{ijj})\right)$$

$$+4\sum_{i=1}^{v}\left(\sum_{\substack{j\neq i\\ j=1}}^{v} x_i^2 x_j^2 V(\hat{b}_{iij})\right) + \sum_{i=1}^{v}\left(\sum_{j\neq k}^{v}\sum_{\substack{\neq i\\ j<k}}^{v} x_j^2 x_k^2 V(\hat{b}_{ijk})\right)$$

$$+6\sum_{i=1}^{v} x_i^2 \mathrm{Cov}(\hat{b}_i, \hat{b}_{iii}) + 2\sum_{i=1}^{v}\left(\sum_{i=1}^{v} x_j^2 \mathrm{Cov}(\hat{b}_i, \hat{b}_{ijj})\right)$$

$$+6\sum_{i=1}^{v}\left(\sum_{\substack{j\neq i\\ j=1}}^{v} x_i^2 x_j^2 \operatorname{Cov}\left(\hat{b}_{iii},\hat{b}_{ijj}\right)\right)+2\sum_{i=1}^{v}\left(\sum_{j\neq k\neq i}^{v}\sum_{j<k}^{v} x_j^2 x_k^2 \operatorname{Cov}\left(\hat{b}_{ijj},\hat{b}_{ikk}\right)\right)$$

$$=vV(\hat{b}_i)+\left[4V(\hat{b}_i)+(v-1)V\left(\hat{b}_{ij}\right)+6\operatorname{Cov}(\hat{b}_i,\hat{b}_{iii})+2(v-1)\operatorname{Cov}\left(\hat{b}_i\hat{b}_{ijj}\right)\right]\sum_{i=1}^{v} x_j^2$$

$$+\left[9V(\hat{b}_{iii})+(v-1)V\left(\hat{b}_{ijj}\right)\right]\left(\sum_{i=1}^{v} x_i^2\right)^2+\left[V\left(\hat{b}_{iij}\right)(4-2(v-1))+\frac{(v-i)(v-(i+1))}{2}V\left(\hat{b}_{ijk}\right)\right.$$

$$\left.+6\operatorname{Cov}\left(\hat{b}_{iii},\hat{b}_{ijj}\right)+2v(v-1)(v-2)\operatorname{Cov}\left(\hat{b}_{ijj},\hat{b}_{ikk}\right)-18V(\hat{b}_{iii})\right]\sum_{i\neq}^{v}\sum_{\substack{j\\ i,j=1}}^{v} x_i^2 x_j^2$$

$$=vV(\hat{b}_i)+\left[4V(\hat{b}_{ii})+(v-1)V\left(\hat{b}_{ij}\right)+6\operatorname{Cov}(\hat{b}_i,\hat{b}_{iii})+2(v-1)\operatorname{Cov}\left(\hat{b}_i,\hat{b}_{ijj}\right)\right]d^2$$

$$+\left[9V(\hat{b}_{iii})+(v-1)V\left(\hat{b}_{ijj}\right)\right]\left[d^2\right]^2+\left[V\left(\hat{b}_{iij}\right)(4-2(v-1))\right.$$

$$\left.+\frac{(v-i)(v-(i+1))}{2}V\left(\hat{b}_{ijk}\right)+6\operatorname{Cov}\left(\hat{b}_{iii},\hat{b}_{ijj}\right)+2v(v-1)(v-2)\operatorname{Cov}\left(\hat{b}_{ijj},\hat{b}_{ikk}\right)-18V(\hat{b}_{iii})\right]d^2$$

$= f(d^2)$

Thus, the sum of the variances of the slopes in axial directions at any point is a function of the distance of the point from the design origin in any symmetric Third Order response surface design. Hence we call these designs as Variance - sums TOSRD on all axial directions. Thus the conditions of Variance - sum TOSRD are

A: All sums of products in which at least one of the x's is with an odd power are zero.

B: (i) $\sum x_i^2 = N\lambda_2 =$ constant for all i = 1,2,…, v

(ii) $\sum x_i^4 = a\,N\lambda_4 =$ constant for all i = 1,2,…, v

(iii) $\sum x_i^6 = b\,N\lambda_6 =$ constant for all i = 1,2,…, v

C: (i) $\sum x_i^2 x_j^2 = N\lambda_4 =$ constant , for $i \neq j$ for all i = 1,…., v

j = 1,….v,

ii) $\sum x_i^2 x_j^4 = c N\lambda_4 =$ constant , for $i \neq j$ for all i = 1,…., v

j = 1,…., v

(iii) $\sum x_i^2 x_j^2 x_k^2 = N\lambda_6 =$ constant , for $i \neq j \neq k$ for all i = 1,…., v

j = 1,…., v

k = 1,…… v

NON- SINGULARITY CONDITIONS :

D (i) $(\lambda_4 / \lambda_2^2) > (v/(a+v-1))$

(ii) $$[\lambda_2 \lambda_4 / \lambda_4^2] > \frac{\{a^2(v+1)-(6a-b)(v-1)\}}{\{b(v+1)-9(v-1)\ \}} \quad \ldots \quad (3.1)$$

Any symmetric third order response surface designs is a TOSRD (OAD). Hence every TOSRD (OAD) has the additional property that the trace of the dispensions matrix of estimates of slopes in all directions has a type of Variance-Sum Third Order Slope Rotatbility property. Hence we may also all these designs as Variance-Sums TOSRDS. Hence every TOSRD(OAD) is Variance-Sum TOSRD on axial directions and conversely.

4.Construction of Embedded TOSRDOAD :-

If the experimenter feels to improve the quality of a product by adding one or more desirable factor (s) to the already existing TOSRDOAD experimental set-up keeping the earlier results as it is, he can feel free to choose any factor or any number of factors as per the customer's specifications. If it is an agriculture experiment or it can be a food experiment, the desired factors can be added to make the considered (v-1) or (v-1) TORDOARD as v dimensional TOSRDOAD, which satisfied the moment conditions given in (3.1). Unlike in other methods of embedding, here, in the construction of embedded TOSRD(OAD) no extra selective or restrictive point sets needed except the required and suitable factor (s). The total number of design pints in the embedded TOSRDOAD are considerably less when compared embedded TORD or embedded TOSRD over axial direction. The embedding procedure is also quite simple. The procedure is as follows.

Consider (v-1) or (v-l) dimensional TOSRDOAD. Add the vth factor or l factors to the considered design, in such way that which satisfies the moment conditions given in equation (3.1). Therefore v – dimensional design is a TOSRDOAD. Suppose the considered design is TORD in lower dimension but the experimenter requires TOSRDOAD of higher dimensions without discarding the earlier model and results, then apply embedding techniques proposed by various authors, which suits to the model, the embedded higher order TORD, itself is the desired TOSRDOAD of higher dimensions.

Similarly the considered design is TOSRD over axial directions. Here also the experimenter feels not to loose the earlier model of the experiment and not to discard the results, but feels to have a TOSRDOAD, then it can be achieved by variance-sum property introduced by Anjaneyulu et al (2004). Embedding in SOSRDOAD is always less restrictive and unconditional. In this, always, the total number of design pints are less.

5. CONDITIONS OF THIRD ORDER SLOPE ROTATABLE DESIGNS OVER ALL DIRECTIONS :

The conditions for slope rotatability over all directions for the third order polynomial. model with four independent variables are the following. (Park and Lee (1995).)

1. $4V_{ii} + 6C_{i,iii} + \sum_{j \neq i} (V_{ij} + 2C_{j,iij})$ equals for all i = 1,2,3,4.

2. $9V_{iii} + \sum_{j \neq 1} V_{iij}$ equals for all i = 1,2,3,4,

$4V_{iij} + 4V_{jji} + \sum_{k \neq i,j} V_{ijk} + 2C_{iik,jjk} + 6C_{jjj,iij} + 6C_{iii,jji}$ equals for all i < j and the value of the second equation is two times of that of the first equation.

3. All coefficients of the terms, except for constant,

$x_1^2, x_2^2, x_3^2, x_4^2, x_1^4, x_2^4, x_3^4, x_4^4, x_1^2x_2^2, x_1^2, x_3^2, x_1^2x_4^2, x_2^2x_3^2, x_2^2x_4^2, x_3^2x_4^2$, equal zero. ...(4.1)

6. SYMMETRY CONDITIONS FOR VARIANCE-SUM THIRD ORDER SLOPE ROTATABLE DESIGNS:

Anjaneyulu et al (2004) have derived the following conditions for Variance-sum Third Order Slope Rotatable Designs:

A) All sums like $\sum x_i, \sum x_i x_j, \sum x_i^2 x_j, \sum x_j^2, \sum x_i x_j x_k, \sum x_i^2 x_j x_k$

$\sum x_i^2 x_j, \sum x_i^9 x_j^9, \sum x_i x_j x_k x_L$ ……. etc are equal to zero. That is all sums of products in which at least one of the x's is with an odd power are zero.

B: (i) $\sum x_i^2 = N\lambda_2 =$ constant, $\forall$ i

(ii) $\sum x_i^4 = a\,N\lambda_4 =$ constant, $\forall$ i

(iii) $\sum x_i^6 = b\,N\,\lambda_6 =$ constant, $\forall$ i

C: (i) $\sum x_i^2 x_j^2 = N\lambda_4 =$ constant, for $i \neq j$

(ii) $\sum x_i^2\; x_j^4 = c\,N\lambda_6 =$ constant, for $i \neq j$

(iii) $\sum x_i^2 x_j^2 x_k^2 = N\,\lambda_6 =$ constant, for $i \neq j \neq k$ (5.1)

7.Construction of Third Order Slope Rotatable Designs Over All Directions using Herzberg Method :

The technique of fitting a response surface is one has been extensively used as quality improvement techniques or industrial experimentation to aid in the statistical analysis of experimental work in which the " Yield = Quality " of a product depends, on some controllable variables. To carry out such analysis, experiments must be performed at predetermined levels of the controllable factors. For this choose suitable response surface. Let us consider a TOSRDOAD with N points in v dimensions.

$$(x_{1u},\ x_{2u},\ \ldots\ldots, x_{vu}),\quad u = 1,2,\ldots\ldots, N.$$

Where in the u^{th} experiment, factor t is at level x_{iu} .

Consider the following point sets in four dimensions

$S(\pm p, \pm p,\ 0,\ 0)$ ....12 Points

$S(\pm e, \pm e,\ 0,\ 0)$ ....12 Points

$S(\pm d,\ 0,\ 0,\ 0)$ ....8 Points

$S(\pm q, \pm q, \pm q, \pm q)$ ....16 Points

The point sets together will form a TOSRDOAD if the following conditions are satisfied

A: All sums of products in which at least one of the x's is with an odd power are zero.

B: (i) $\sum x_i^2 = N\lambda_2$ = constant
(ii) $\sum x_i^4 = a\, N\lambda_4$ = constant
(iii) $\sum x_i^6 = b\, N\, \lambda_6$ = constant

C: (i) $\sum x_i^2 x_j^2 = N\lambda_4$ = constant , for $i \neq j$
(ii) $\sum x_i^2 x_j^4 = c\, N\lambda_4$ = constant , for $i \neq j$
(iii) $\sum x_i^2 x_j^2 x_k^2 = N\, \lambda_6$ = constant , for $i \neq j \neq k$

D: (i) $\sum x_i^4 x_j^2 = a \sum x_i^2 x_j^2$ = constant , for $i \neq j$

(ii) $\sum x_i^6 = b \sum x_i^2 x_j^2 x_k^2$ = constant, for $i \neq j \neq k$

(iii) $\sum x_i^2 x_j^4 = c \sum x_i^2 x_j^2 x_k^2$ = constant , for $i \neq j \neq k$ ... (6..1)

In this we have 48 points.

$$\sum x_i^{\,2} = 12p^2 + 12e^2 + 2d^2 + 16q^2$$

$$\sum x_i^{\,4} = 12p^4 + 12e^4 + 2d^4 + 16q^4$$

$$\sum x_i^{\,6} = 12p^6 + 12e^6 + 2d^6 + 16q^6$$

$$\sum x_i^{\,2} x_j^{\,2} = 4p^4 + 4e^4 + 16q^4$$

$$\sum x_i^{\,4} x_j^{\,2} = 4p^6 + 4e^6 + 16q^6$$

$$\sum x_i^{\,2} x_j^{\,2} x_K^{\,2} = 16q^6$$

We have

$$(i) \sum x_i^{\,6} = b \sum x_i^{\,2} x_j^{\,2} x_k^{\,2}$$

$$12p^6 + 12e^6 + 2d^6 + 16q^6 = b16q^6$$

$$12p^6 + 12e^6 + 2d^6 = (b-1)16q^6 \qquad ....(1)$$

$$\therefore q^6 = \frac{12p^6 + 12e^6 + 2d^6}{16(b-1)}$$

$$(ii) \sum x_i^{\,4} = a \sum x_i^{\,2} x_j^{\,2}$$

$$12p^4 + 12e^4 + 2d^4 + 16q^4 = a\left[4p^4 + 4e^4 + 16q^4\right]$$

$$d^4 = 2q^4 - 4qp^4 - 4e^4$$

$$and \quad p^4 = \frac{2q^4 - d^4 + 4e^4}{4a}$$

$$\sum x_i^{\,4} x_j^{\,2} = c\left[\sum x_i^{\,2} x_j^{\,2} x_k^{\,2}\right]$$

$$4p^6 + 4e^6 + 16q^6 = c\left[16q^6\right]$$

$$p^6 + e^6 = 4q^6(c-1) \qquad ....(2)$$

Solving (1) and (2) we get,

$$d^6 = (b-1)16q^6 - 48(c-1)q^6$$

For any values of p,q,d and e satisfying the above conditions, the considered design is a Third Order Slope Rotatable Design Over All Directions. Herzberg and Draper constructed third order rotatable designs in four dimensions compared to their designs the proposed design is a better design, because the number of design points required in the design are less than the number of design points required for their method.

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