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A REVIEW ON MATHEMATICAL STUDY OF SURVIVAL OF AQUATIC SPECIES IN PRESENCE OF TOXICANTS/ POLLUTANTS AND NUTRIENTS

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Abstract:

In this paper authors have focused on research work done in the area of mathematical modeling and stability analysis of the survival of aquatic species in the presence of the pollutants/toxicants. The paper consists of four divisions viz Water quality, Water pollution, Plankton ecosystem affected by oxygen deficit and Effects of Pollutants in the Aquatic Environment. All these research papers include the stability analysis of the equilibrium points and from this analysis, it is shown that the impact of direct and indirect toxicity harms water, survival of biological interactive species in polluted water bodies, depleting of dissolved oxygen. And also further numerical simulations have been carried out to justify the analytic findings of water pollution and its effects on the survival of aquatic species.

Key Words: Plankton Ecosystem, Oxygen Deficit, Aquatic Environment, Nutrient Cycle, Eutrophication.

Introduction:

Water is the first basic component of the root of life. Water may be a liquid at the temperatures and weights that are most palatable for life. Water is transparent, tasteless, and odorless. Water is utilized for different sorts of purposes: for farming, industry, fishery, as well as domestic purposes. As water streams over the ground surface and gets to be accessible as water asset in streams, lakes/marshes, underground water, and coastal water, it accumulates inorganic substances from the soil, and natural substances and microorganisms produced by human exercises and characteristic environments.

Water Quality:

Debasements which exist in water incorporate not as it were the substances fundamental for supporting living animals but moreover perilous substances, which are not fair pointless for living animals, but cause wellbeing issues.

Water Quality is measuring the appropriateness of water for diverse reasons. The higher the water quality, the negligible treatment is required. Water quality is exceptionally imperative for the security of the oceanic biological system. Numerous criteria are included in evaluating the water quality such as the mineral substance, turbidity, metals. Inquire about is done on water quality in stream framework of Jajrood waterway in Tehran-Iran by S. A. Mirbagheri, M. Abaspour, and K. H. Zamani,(2009). Due to the sewage of private ranges which is dumped at the waterway and the likely defilement, the quality of water along the stream. The living creatures that are more affected are points and marine warm-blooded animals since they are at the beat of the food chain are direct revealed to top levels of harms in water and/or they feed on other points that they are contaminated. Modeling the direct and indirect effects of pollutants on the survival of fish in water bodies(Pankaj Kumar Tiwari et al.,2017).

1. Water pollution

Water pollution issues are related to wastewater release, which are broken down oxygen, suspended solids, microbes, supplements, PH and poisonous chemicals counting unstable organics, acid/base, neutrals, metals, and pesticides. The physical, chemical, and biological forms that control the predetermination of the water quality parameters. These forms are various and shifted. It is helpful to separate them into transport forms which influence all water quality parameters so also, and change forms, which are constituent-specific. Numerous of these change forms, in any case, have comparable energy so that a diverse detailing isn't required for each constituent. This is analyzed in the paper Mathematical Modeling of Disposal of Pollutant in Rivers R.V.Waghmare and S.B.Kiwne (2017).

1.1. Mathematical Modeling of the Survival of a Biological Species in Polluted Water Bodies (J.B. Shukla, A.K. Misra, and Peeyush Chandra, 2007).

In this paper, the authors have proposed a nonlinear mathematical model to study the consumption of broken down oxygen in a water body which is caused by mechanical and household release of toxins. The issue is modeled by different nonlinear forms including organic pollutants, microscopic organisms, protozoa, dissolved oxygen, and organic species entirely subordinate on it. The impact of the depleted level of dissolved oxygen on the survival of organic species in such an aquatic system has been studied.

Authors consider a food chain system in a water body consisting of organic matter pollutants, bacteria, protozoa, and a biological population whose growth rate is entirely subordinate to the concentration of dissolved oxygen. The following are modeling equations:

$$\frac{dT}{dt} = Q - \alpha_0 T - \frac{K_1 TB}{K_{12} + K_{11} T}$$

$$\frac{dB}{dt} = \frac{\lambda_1 K_1 TB}{K_{12} + K_{11} T} - \alpha_1 B - \lambda_{10} B^2 - \frac{K_2 BP}{K_{21} + K_{22} B}$$

$$\frac{dP}{dt} = \frac{\lambda_2 K_2 BP}{K_{21} + K_{22} B} - \alpha_2 P - \lambda_{20} P^2$$

$$\frac{dC}{dt} = q - \alpha_3 C - \frac{\lambda_{12} K_1 BP}{K_{12} + K_{11} T} - \frac{\lambda_{23} K_2 BP}{K_{21} + K_{22} B} - \lambda_{11}\alpha_1 B - \lambda_{22}\alpha_2 P - K_3 CF$$

$$\frac{dF}{dt} = \lambda_3 K_3 CF - \alpha_4 F - \lambda_{30} F^2$$

Where T(0) > 0; B(0) > 0,P(0) > 0, C(0) > 0, F(0) > 0.

Let T be the cumulative concentration of organic matters(pollutants),

B be the density of bacteria,

P be the density of protozoa,

C be the concentration of dissolved oxygen (DO),

F be the density of a biological population,

Q be the cumulative rate of discharge of organic pollutants into the water body,

T the rate of depletion of cumulative pollutants concentration caused by natural factors is assumed to be proportional to its concentration. Keeping given these considerations, this system is governed by the following differential equations.

In this research paper, it is observed non-linear forms including organic pollutants, microbes, protozoa, dissolved oxygen, and a biological species, such as fish population, entirely subordinate to nutrients. It has been assumed that oxygen is exhausted by different non-linear biochemical and biodegradation forms happening within the water body. It has appeared that as the rate of discharge of organic pollutants within the water body increments, the equilibrium concentration of dissolved oxygen diminishes due to various interactive processes beneath certain conditions. If the rate of release of organic pollutants is exceptionally high, the equilibrium concentration of dissolved oxygen gets to be insignificant and the survival of the biological population may be debilitated.

1.2. A Mathematical Approach To Study The Effect Of Pollutants/Toxicants In Aquatic Environment (Anita Chaturvedi, Kokila Ramesh, Vatsala 2017)

The acid lowers the pH levels in water bodies below what is required for the survival of aquatic life and increases the toxicity of metals. For this effect, a mathematical model has been proposed using a system of non-linear ordinary differential equations with four state variables. The dependent variables are the amount of acid and metal in water, density of favorable resources (phytoplankton), the density of fish population, and nutrient concentration under the assumption that the amount of metal present in water is less than the amount of acid present in water. Further it was also observed that, nutrients play an important role in the growth and survival of the species. The given below mathematical models:

$$\frac{dS}{dt} = S_0 - aS - gSP - \alpha(T_1 + qC_m) + kcP + kbF + k_1F^2$$
$$\frac{dP}{dt} = gSP - cP - fFP$$
$$\frac{dF}{dt} = fPF - bF - kF^2$$
$$\frac{d(T_1 + qC_m)}{dt} = Q_0 - \alpha(T_1 + qC_m) - \alpha_1(T_1 + qC_m)S$$

With the initial condition:

$$S(0) = S_{10} > 0, P(0) = P_{10} > 0, F(0) = F_{10} > 0, T(0) = T_{10} > 0$$

From the above analysis it is observed that toxicity in the nutrient pool causes an increase in the equilibrium level of resource and fish populations. The interior equilibrium exists only when the non-living equilibrium and fish extinct equilibrium are unstable. The non-living equilibriums are stable if the equilibrium level of concentration of nutrients in water is less than the fraction of the natural death rate of resource biomass to the rate of consumption of nutrients by the resource population. The fish extinct equilibrium is stable if the equilibrium level of the resource population in water is less than a fraction of the natural death rate of the fish population to the specific rate of predation of fish on the resource population. And also authors are conducted a numerical simulation to find out the feasibility of stability conditions of analytical findings.

2. Plankton ecosystem affected by oxygen deficit:

The organic pollutants and the nutrients are discharged into the water body from outside with constant rates. The system is modeled by considering the variables such as the cumulative concentration of organic pollutants, the densities of bacteria, nutrients, algae, detritus, and the concentration of DO. Mathematical modeling and analysis of the depletion of dissolved oxygen in eutrophied water bodies affected by organic pollutants (J.B.Shukla, A.K.Misra, PeeyushChandra, 2008).

Through water runoff Nutrients are continuously flowing to the lake with a constant rate In the modeling process five variables are considered, namely concentration of nutrients, the density of algal population, density of macrophytes, density of detritus and concentration of dissolved oxygen. Mathematical Model for the depletion of dissolved oxygen in a lake due to submerged macrophytes is proposed and analyzed (A.K. Misra, 2001)

2.1. Stability Analysis of Plankton Ecosystem Model Affected by Oxygen Deficit (Yuriska Destania, Jaharuddin, Paian Sianturi, 2015)

Plankton is one of the main components of the food chain system in water. In this paper, the plankton ecosystems studied by formulating a mathematical model in nonlinear ordinary differential equations system, where the growth of plankton influenced by oxygen deficit. This model includes four variables i.e., the concentration of nutrients, density of algae, density of zooplankton, and concentration of dissolved oxygen. There are four equilibrium points, obtained. Based on the stability analysis conducted, one of the equilibrium points was stable under certain conditions. The below given the mathematical models:

$$\frac{dn}{dt} = q - \alpha n - \beta_1 na$$

$$\frac{da}{dt} = \frac{\beta_2 na}{(\alpha_1 + c_0 - c)} - v_1 a - \alpha_2 az$$

$$\frac{dz}{dt} = \frac{\alpha_3 az}{(\alpha_4 + c_0 - c)} - v_3 z$$

$$\frac{dc}{dt} = p - v_2 c + \lambda_2 a$$

with $n(0)>0, a(0)>0, Z(0)>0, C(0)>0$.

Where,

n is the concentration of nutrients,

a is the density of algae,

Z is the density of zooplankton,

C is the concentration of dissolved oxygen,

αn is the depletion rate of nutrients,

$\beta 1$ is the predation rate of nutrients by algae,

$v1$ is the natural death rate of algae,

$\alpha 2$ the predation rate of algae by zooplankton,

$v3$ is the natural death rate of zooplankton,

p is the concentration of dissolved oxygen enters the system from a variety of sources,

$\lambda\ 1a$ is algae give the supply of oxygen through photosynthesis process,

$v2$ is the natural depletion rate of oxygen,

$C0$ is the saturation value of dissolved oxygen,

$C0-C$ is the oxygen deficit,

$\beta 2$ and $\alpha 3$ are positive constants,

$\alpha 1$ and $\alpha 4$ are saturation constants.

It is observed that the nonlinear mathematical model for plankton ecosystems has been modified and analyzed. The model exhibits four equilibrium points. Stability analysis shows that, one of the equilibrium points will be stable under certain conditions. Numerical simulations give the result, that this ecosystem will reach a stable condition. Numerical simulations also show the dynamics that occur until the ecosystem reaches a stable condition.

2.2. Modeling the effect of the depleting dissolved oxygen on the existence of interacting planktonic population(S. Khare, S. Kumar, and C. Singh, Elixir Appl. Math, 55 (2013))

The mathematical model is proposed to study the effect of the depleting dissolved oxygen on the existence of an interacting planktonic population. The model includes four state variables viz., nutrient concentration, density of algae, and density of the zooplankton population and concentration of dissolved oxygen. All the feasible equilibrium of the system is obtained and the conditions for the existence of the interior equilibrium are determined. The mathematical models are given below:

$$\frac{dn}{dt} = q - \alpha n - \beta_1 na$$

$$\frac{da}{dt} = \frac{\beta_2 na}{(\alpha_1 + c_0 - c)} - v_1 a - \alpha_2 az$$

$$\frac{dP}{dt} = \frac{\alpha_3 aP}{(\alpha_4 + c_0 - c)} - v_3 P$$

$$\frac{dc}{dt} = q_0 - v_2 c$$

With the initial conditions $n(0) = n10 > 0$, $a(0) = a10 > 0$, $C(0) = C10 > 0$, $P(0) = P10 > 0$.

Where,

n be the cumulative concentration of various nutrients,

a be the density of algae,

P be the density of the zooplankton population,

C be the concentration of dissolved oxygen. We

q is the cumulative rate of discharge of nutrients into the aquatic system

α n is a constant which is depleted with rate due to natural factors.

na is the concentration of nutrient

is the growth rate of algae is proportional

n1, ν3 is the natural depletion rate of algae and zooplankton are respectively.

α2 is the rate of predation of algae by zooplankton.

q0 assumed to be a constant

ν2 is the natural depletion rate of concentration C.

is further assumed that the growth rate of zooplankton is proportional

are half-saturation constants,

C 0 is DO saturation value

C 0 - C is an oxygen deficit.

In this paper authors are proposed and analyzed the mathematical model of the algal bloom in the aquatic system. The model exhibits three non-zero equilibrium E1, E2 and E3.

3. Survival of Interacting Species in Aquatic Environment

Three species interactions in a food chain, with the assumption that the interactions in an aquatic Habitat are analyzed by J. N. Ndam, J. P. Challam, and T. G. Kassem(2011). The results indicate the possibility of a stable coexistence of the three interacting species in the form of stable oscillations under the reflecting boundary conditions. Habitat segregation also occurs under these conditions.

Modeling the direct and indirect effects of copper on phytoplankton–zooplankton interactions Loïc Prosnier, Michel Loreau, Florence D. Hulot(2015). In this paper, the author has analyzed the effects of pollution at the community level is difficult because of the complex impacts of ecosystem dynamics and properties. To predict the effects of copper on a plant-herbivore interaction in a freshwater ecosystem. We include two types of direct effects of copper on Scenedesmus and Daphnia that results from hormesis: a deficiency effect at low concentration and a toxic effect at high concentration.

Study a nonlinear mathematical model of fishery management to understand the dynamics of a fishery resource system in an aquatic environment that consists of two zones; one is a free fishing zone and another is reserve zone where fishing is strictly prohibited. Mathematical Modeling Applied to Sustainable Management of Marine Resources (Md. Haider Ali Biswasa, Md Rajib Hossain, Mitun Kumar Mondala, 2016)

3.1. The fate of dissolved oxygen and survival of the fish population in the aquatic ecosystem with nutrient loading: a model (O. P. Misra, Divya Chaturvedi, 2016)

In this paper, a nonlinear mathematical model is proposed and analyzed to study the depletion of dissolved oxygen and survival or extinction of the fish population in a nutrient-enriched aquatic ecosystem. It is assumed in the model that there is an external constant input of nutrients (phosphorus and nitrogen) in the water body on account of anthropogenic activities. Stability analysis of the equilibria of the model is carried out and from the analysis it is shown that the fish population will survive at a very low equilibrium level due to reduced concentration of dissolved oxygen and excessive presence of algal biomass on account of nutrient loading. It considers an aquatic ecosystem in which nutrients (phosphorus and nitrogen) are continuously discharged due to anthropogenic activities such as runoff from agriculture and development, pollution from septic systems and sewers, sewage sludge spreading, etc.

Let F and C denote the density of fish population and concentration of dissolved oxygen in water bodies respectively.

P denotes the concentration of nutrients (phosphorus and nitrogen) and

N represents the algal biomass. Keeping in view the above the considerations the mathematical model describing the system is given by the following set of differential equations:

$$\frac{dF}{dt} = R(c)F - \frac{r_0 F^2}{K(N)}$$
$$\frac{dC}{dt} = -d_{B0} + d_{B1}(N_0 - N) + K_2(N)(C_s - C)$$
$$\frac{dP}{dt} = 1 - rP - \frac{d_1 PN}{b+P} + \beta_1 aN$$
$$\frac{dN}{dt} = \frac{d_1 PN}{b+P} - aN - gN^2$$

With the initial condition as

In this present analysis, we assume the following forms for function

Where,

R(C) denotes the growth rate of the fish population which depends upon dissolved oxygen and is assumed to be an increasing function of dissolved oxygen.

DO as dissolved oxygen

K(N) denotes the carrying capacity of fish population

K 2(N) be Reaeration coefficient also depends upon algal biomass and decreases as algal biomass increases.

is the intrinsic growth rate of fish,

is the control parameter for the growth of fish population depending upon the level of dissolved oxygen, is the threshold level of concentration of dissolved oxygen,

is natural carrying capacity,

is a reduced rate in carrying capacity due to algal biomass,

is natural depletion rate of dissolved oxygen,

is the threshold level of algal biomass

N is more than the threshold level

concentration of dissolved oxygen

is the growth rate coefficient of dissolved oxygen depending upon the level of algal biomass, is natural reaeration rate,

be saturated concentration of dissolved oxygen,

I is the input rate of nutrients (phosphorus, nitrogen),

r is the depletion rate of nutrients (phosphorus, nitrogen),

is nutrients (phosphorus, nitrogen) recycling coefficient.

a is depletion rate of algal biomass,

g is the depletion rate of algal biomass due to crowding.

represents the growth of algal biomass due to nutrients present in the water body,

is the maximum specific growth rate of algae population,

b is half-saturation constant.

Here, all the parameters are taken to be positive constants.

It is concluded here that if the nutrients are excess in amount than the required level then the survival of the fish population is threatened. These numerical results suggest the role of nutrient loading on the fate of dissolved oxygen and consequently on the growth dynamics of algal biomass and fish population.

3.2. Modeling Effect of Eutrophication On The Survival Of Fish Population Incorporating Nutrient Recycling (Anita Chaturvedi, O P Misra, 2010)

It is known that the aquatic bodies undergo eutrophication process due to the nutrient enrichment caused by waste disposal from industries and discharge of chemicals from the agricultural fields consequently, increasing the concentration of phytoplankton and algae thereby decreasing the dissolved oxygen and transparency of the aquatic environment. Due to oxygen deficit and loss of transparency, the growth of much aquatic population such as fish are adversely affected. In this paper mathematical models are proposed to study the survival or extinction of fish populations incorporating the impact of direct and indirect recycling of the nutrients under the adverse effects of eutrophication.

Linear habitat at $0 \le Z \le a$

$$\frac{\partial H}{\partial t} = \beta P - \alpha_L H$$
$$\frac{\partial F}{\partial t} = r(D)F - \frac{r_0 F^2}{k(N)} + D_0 \frac{\partial^2 F}{\partial Z^2}$$
$$\frac{\partial C}{\partial t} = -k_1 CP - d_B(N) + k_2(m)(C_s - C) + D_1 \frac{\partial^2 P}{\partial Z^2}$$
$$\frac{\partial P}{\partial t} = I - rP - \frac{d_1 PN}{b+P} + \alpha_1 m_d D_N + D_2 \frac{\partial^2 P}{\partial Z^2}$$
$$\frac{\partial N}{\partial t} = \frac{d_1 PN}{b+P} - aN - gN^2 + D_3 \frac{\partial^2 N}{\partial Z^2}$$
$$\frac{\partial D_N}{\partial t} = I_n + \alpha a N - m_d D_N + D_3 \frac{\partial^2 N}{\partial Z^2}$$

With the initial conditions which are given as follows

$$H(Z,0) = f_1(Z) \ge 0, F(Z,0) = f_2(Z) \ge 0, C(Z,0) = f_3(Z) \ge 0$$
$$P(Z,0) = f_4(Z) \ge 0, H(Z,0) = f_5(Z) \ge 0, D_N(Z,0) = f_6(z) = 0$$

The model is associated with the following boundary conditions=0, $H = H^*, C = C^*, F = F^*, P = P^*, N = N^*, D_N = D_N^*$

$H^*, F^*, C^8, P^*, N^*, D_N^*$ are the equilibrium values(or steady-state)

For the analysis of the model given, we assume the following forms for the functions r(D), K(N), dB(N) and k2(m).

D is assumed to be D=Cs-C,

Mathematical Model without Diffusion

$$\frac{dH}{dt} = \beta P - \alpha_n H$$
$$\frac{dF}{dt} = r(D)F = \frac{r_0 F^2}{k(N)}$$
$$\frac{dC}{dt} = -k_1 CP - d_B(N) + k_2(m)(C_s - C)$$
$$\frac{dP}{dt} = I - rP - \frac{d_1 PN}{b+P} + \alpha_1 m_d D_N$$
$$\frac{dN}{dt} = \frac{d_1 PN}{b+P} - aN - gN^2$$
$$\frac{dD_N}{dt} = I_n + \alpha a N - m_d D_N$$

It is observed that both the populations of fish and phytoplankton will coexist but the equilibrium level of fish population decreases if the density of the phytoplankton population and oxygen deficit simultaneously increases. It is noted from the analysis that if the dispersal (diffusion) is allowed in the system then the Modelling Effects of Eutrophication on the Survival of Fish. Equilibrium point E1 is stable which shows that the fish population and phytoplankton both die out. Whereas, in the case of equilibrium point E2 which is stable if dispersal (diffusion) is introduced in the system, the fish population again dies out but phytoplankton exists.

4. Effects of Pollutants in the Aquatic Environment

A nonlinear mathematical model is proposed and analyzed to see the indirect effect of air pollutants on the prey-predator type fish population in a closed population (lake). it is shown that as the pollutant concentration in the environment increases, the concentration of the acidic chemicals in the lake increases, and consequently the equilibrium level of the fish population decreases (Mini Ghosh, Peeyush Chandra, and Prawal Sinha(2002)).

A Mathematical Model For Toxic Waste In A River And Its Remediation By Freshening Dr. D.V. Ramalinga Reddy A.Sreenivasa Chari (2013) present a simple mathematical model for river pollution and examine the effect of freshening on the humiliation of toxin The model consists of a pair of coupled feedback diffusion advection equations for the toxin and dissolved oxygen concentrations, respectively. The critical value is not reached in The Krishna River because of the finite length over which pollution is discharged so that the river reaches the sea before this environmental catastrophe can occur.

4.1.Combined Effects of Acid and Metal on the Survival of Resource-Based Population Incorporating Nutrient Recycling: A Mathematical Model (Asha Bharathi A., Anita Chaturvedi, Radha Gupta and Kokila Ramesh(2015))

The aquatic environment is getting polluted by many different types of toxic metal which are discharged from the industries and agricultural fields. The chief pollutants which are produced by the industries are heavy metals and radioactive substances. The models are given below:

$$\frac{dT_1}{dt} = H_0 - \alpha_1 T_1 - \beta_1 T_1 S$$

$$\frac{dC_w}{dt} = Q_0 - \alpha_2 C_w - \beta_2 C_w S$$

$$\frac{dB}{dt} = gSB - cB - fBN$$

$$\frac{dN}{dt} = fBn - bN$$

$$\frac{dS}{dt} = S_0 - aS - gSB - \beta_1 T_1 S - \beta_2 s C_w + KcB + KbN$$

Where, $T_1(0) = T_{10} > 0, C_w(0) = C_{10} > 0, B(0) = B_{10} > 0, N(0) = N_{10} > 0, S(0) = S_{10} > 0$

Where,

T1 is Concentration of acid in water,

Cw is Concentration of metal in water,

B is the density of a favorable resource.

N is the density of fish population,

S is the concentration of nutrients,

H0 is the total input rate of acid,

Q0 is the total input rate of metal,

α1 and α2 natural washout rates of acid and metal respectively,

β1 and β2 are Depletion rate of nutrients due to acid and metal respectively in water,

g is the rate of consumption of nutrients by the resource population,

f is a specific rate of predation of fish on resource population,

b, c is the natural death rates resources and fish population,

S0 is constant nutrient input,

α is the nutrient leaching rate.

From the above analysis it is concluded that toxicity in the nutrient pool causes an increase in the equilibrium level of resource and fish populations. The interior equilibrium exists only when the non-living equilibrium and fish extinct equilibrium are unstable. the results are interpreted analytically and graphically for all possible cases using time series, phase space, and phase plan graph.

4.2. A Mathematical Approach To Study The Effect Of Pollutants/Toxicants In Aquatic Environment (Anita Chaturvedi, Kokila Ramesh, Vatsala 2017)

The acid lowers the pH levels in water bodies below what is required for the survival of aquatic life and increases the toxicity of metals. For this effect, a mathematical model has been proposed using a system of non-linear ordinary differential equations with four state variables. The dependent variables are the amount of acid and metal in water, density of favorable resources (phytoplankton), the density of fish population, and nutrient concentration under the assumption that the amount of metal present in water is less than the amount of acid present in water. Further it was also observed that, nutrients play an important role in the growth and survival of the species. The mathematical model is given below:

$$\frac{dS}{dt} = S_0 - aS - gSP - \alpha(T_1 + qC_m) + kcP + kbF + k_1F^2$$

$$\frac{dP}{dt} = gSP - cP - fFP$$

$$\frac{dF}{dt} = fPF - bF - kF^2$$

$$\frac{d(T_1 + qC_m)}{dt} = Q_0 - \alpha(T_1 + qC_m) - \alpha_1(T_1 + qC_m)S$$

With the initial condition:

$$S(0) = S_{10} > 0, P(0) = P_{10} > 0, F(0) = F_{10} > 0, T(0) = T_{10} > 0$$

Where,

Q0 is the total input rate of metal,

α1 and α2 natural washout rates of acid and metal respectively,

$\beta 1$ and $\beta 2$ are Depletion rate of nutrients due to acid and metal respectively in water,

g is the rate of consumption of nutrients by the resource population,

f is a specific rate of predation of fish on resource population,

b, c is the natural death rates resources and fish population,

S0 is constant nutrient input,

α is the nutrient leaching rate.

From the above analysis it has been observed that toxicity in the nutrient pool causes an increase in the equilibrium level of resource and fish populations. The interior equilibrium exists only when the non-living equilibrium and fish extinct equilibrium are unstable. The non-living equilibriums are stable if the equilibrium level of concentration of nutrients in water is less than the fraction of the natural death rate of resource biomass to the rate of consumption of nutrients by the resource population. The fish extinct equilibrium is stable if the equilibrium level of the resource population in water is less than a fraction of the natural death rate of the fish population to the specific rate of predation of fish on the resource population.

4.3.Modeling and Analysis of the Algal Bloom in a Lake caused by Discharge of Nutrients.
(J.B. Shukla, A.K. Misra, Peeyush Chandra, 2008)

In this paper, it is analyzed a nonlinear model for the algal bloom in a lake caused by excessive flow of nutrients from domestic drainage and water runoff from agricultural fields, is proposed and analyzed. This model considers interactions of cumulative concentration of nutrients, the density of algal population, density of detritus, and concentration of dissolved oxygen in the lake. It is assumed that detritus, which is formed due to the death of algae, supplements the cumulative concentration of the nutrients in the water body, using dissolved oxygen in the process. It is shown that the equilibrium level of the algal population is highly dependent on the cumulative rate of discharge of inputs of nutrients in the lake and as this cumulative discharge increases the equilibrium level of algal population increases leading to eutrophication. The following assumptions are made in this modeling process:

(i) Nutrients like nitrate and phosphorus are supplied to the water body from domestic drainage as well as from water runoff from agricultural fields with a cumulative rate, which is assumed to be a constant.

(ii) The algal population density is wholly dependent on the cumulative concentration of nutrients.

(iii) The detritus in the water body is formed by the death of algae (phytoplankton), which is transformed into nutrients by the remineralization process using dissolved oxygen.

(iv) The level of dissolved oxygen in the water body increases by air, water interaction at the water surface with a constant rate as well as by photosynthesis/respiration by algae. Its concentration is depleted by its utilization by detritus information of nutrients, the amount of which is assumed to be proportional to the concentration of dissolved oxygen.

$$\frac{dn}{dt} = q - \alpha_0 n - \frac{\beta_1 na}{\beta_{12} + \beta_{11} n} + \prod \delta S$$

$$\frac{da}{dt} = \frac{\theta_1 \beta_1 na}{\beta_{12} + \beta_{11} n} - \alpha_1 a - \beta_{10} a^2$$

$$\frac{dS}{dt} = \prod_1 \alpha_1 a + \prod_2 \beta_{10} a^2 - \delta S$$

$$\frac{dc}{dt} = q_c - \alpha_2 c + \lambda_2 c + \lambda_{11} a - \delta_1 S$$

n be the cumulative concentration of various nutrients,

a be the cumulative density of various algal populations,

S be the density of detritus

C be the concentration of dissolved oxygen (DO)

q be t the cumulative rate of discharge of nutrients into the aquatic system from outside of the water body

a0n is depleted with rate due to natural factors.

is the growth rate of nutrients by detritus is dS

ai's are depletion rate coefficients, and 1 are constants of proportionality and are positive.

The positive constant 10 is a coefficient corresponding to crowding of the algal population.

The proportionality constants are such that 0 < < 1.

The modeling analysis shows that as the supply of nutrients in the water body increases, the cumulative density of algal population increases, and algal bloom occurs causing eutrophication. It has also been shown that the density of detritus increases correspondingly leading to a decrease in the concentration of dissolved oxygen (as the net production of oxygen, formed due to photosynthesis by floating algae does not affect the concentration of dissolved oxygen), threatening the survival of the fish population.

This Paper focuses on water pollution and the impact of water pollution is considered from 2000 to up to the date. The quality of water is decreasing and increasing over utilization. It has been concluded that oxygen is depleted by various biochemical and biodegradation processes occurring in the water body. Plankton is the main component of the food chain, the impact of water pollution on plankton. From this water pollution survival of aquatic species are threatened due to the toxicants and organic pollutants. It is concluded here that if the nutrients are excess in amount than the required level then the survival of the fish population is threatened.

We have conquered nature to pollute it but still we have failed to understand the nature policy even less than 10%. Water pollution can disrupt life on our planet to a great extent. We should think of even those generations which have still to appear on this earth. We must become familiar with our local water resources and learn about ways for disposing of harmful household wastes so they don't end up in sewage treatment plants that can't handle them or landfills not designed to receive hazardous materials.

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