IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

To Study The Skin-Friction And The Volume Flow For The Flow Of Dusty Visco- Elastic Fluid (Kuvshiniski type) Between Two Parallel Plates.

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Abstract:

The flow of dusty visco-elastic fluid between two parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its plane is considered in view of its growing importance in various technical problems.

In the present paper, consider the unsteady laminar flow of visco-elastic fluid containing uniformly small solid particles between two infinitely extended parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane. The analytical expressions for velocity fields of fluid and dust particles are obtained which are in elegant forms. The skin friction at the lower plate wall and the total volume flow in between the plates are obtained and discuss the effects of various parameters on skin-friction and volume flow between the parallel walls.

Key Words: Visco-elastic fluid, laminar flow, elastic element, dusty fluid, skin-friction, harmonic oscillation, volume flow.

Introduction:

In recent years, the study of non-Newtonian fluids has drawn special attention under a wide range of geometrical, dynamical, and rheological conditions. A few examples are the flow of nuclear fuel slurries, the flow of liquid metals and alloys such as the flow of gallium at ordinary temperatures ($30\ ^{\circ}c$), the flow of plasma, the flow of mercury amalgams, handling of biological fluids, the flow of blood, a Bingham fluid with some thixotropic behaviours, coating of paper, petroleum production, molten paper pulp, emulsion, paints, lubrication with heavy oils and greases, etc., as important raw materials and chemical products in a large variety of industrial processes. The subject of Rheology is of great technological importance as in many branches of industry, the problem arises of designing apparatus to transport or to process substances which cannot be governed by the classical stress-strain velocity relations. Visco-Elastic fluids are particular cases of non-Newtonian fluids that exhibit appreciable elastic behaviour and stress-strain velocity relations and are time-dependent. Many common fluids such as oils, certain paints, polymer solutions, some organic liquids, and many new materials of industrial importance exhibit both viscous and elastic properties. Though the above fluids, called visco-elastic fluids, are also being studied extensively.

Saffman has expressed model equations describing the influence of dust particles on the motion of fluids. Several authors using equations of Saffman have investigated several dusty gas flow problems in different situations. Kapur and Sukla investigated the problem of two immiscible viscous liquids between two fixed parallel plates under a certain pressure gradient. The flow of visco-elastic Maxwell liquid down an inclined plane was investigated by Bagchi. The unsteady flow of two immiscible visco-elastic conducting liquids between two inclined parallel plates has been studied by Lahiri and Ganguly. Mandal et al have considered unsteady flow of dusty visco-elastic (kuvshiniski type) liquid between two oscillating plates. Johari et al have studied the MHD flow of a dusty Visco-elastic (kuvshiniski type) liquid past in an inclined plane. Recently Singh et al have studied the MHD flow of a dusty visco-elastic fluid past on an inclined plane. Most recently the unsteady flow of visco-elastic liquid (Kuvshinisky type) between two parallel plates has been discussed by Dr.K.C.Nandy in the year 2020.

Mathematical Formulation Of The Problem And Its Solution:

We suppose that the dusty visco-elastic fluid fills the region between two horizontal infinite parallel flat plates at a distance h apart. The lower plate is kept at rest and the upper one begins to perform harmonic oscillations with a frequency ω in its own plane. The present analysis takes a co-ordinate system such that the x-axis coincides with the lower fixed plate and the z-axis is perpendicular to it.

The dust particles are assumed to be spherical in shape and uniform in size and the number density of dust particles is taken as constant throughout the flow and it be ρ_0. Since the plates are infinite, the velocity will depend on z and time t mainly. For the constitutive equation we adopt Kuvshiniski type fluid, given by

$$P_{ij} = -p\delta_{ij} + p'_{ij}$$

$$\left(1+\lambda_0 \frac{D}{Dt}\right) p'_{ij} = 2\mu e_{ij}$$

$$\frac{D}{Dt} p'_{ij} = \frac{\partial p'_{ij}}{\partial t} + u_m \frac{\partial p'_{ij}}{\partial x_m} \quad \text{.........................(1)}$$

$$2e_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

Where P_{ij} is stress tensor and p'_{ij} the deviatoric stress tensor, $\frac{D}{Dt}$ is the convective time derivative following a fluid element and u_i is the velocity of the fluid particle. Here λ_0 and μ denote the elastic coefficient and viscosity of the fluid. P.G.Saffman and using equation (1) we get the equation of motion of dusty visco-elastic fluid (dropping dashes)

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = R\frac{\partial^2 u}{\partial z^2} + \frac{f}{\tau}\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u) \quad \text{................ ....(2)}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u-v) \quad \text{............. ....(3)}$$

Here, $u' = \frac{u}{h\omega}, \quad v' = \frac{v}{h\omega}, \quad t' = \omega t, \quad z' = \frac{z}{h}, \quad \alpha = \lambda_0\omega, \quad R = \frac{\gamma}{h^2\omega}$

Where, v is the dusty velocity of the particle.

f = Mass concentration = $\frac{mB_0}{\rho}$

τ = Relaxation time = $\frac{m\,\omega}{K}$

The relevant initial and boundary conditions in non-dimensional form are

$t \le 0$

$u = \frac{\partial u}{\partial t} = 0$ for all z ....(4)

$t > 0$

$u = a\,Sint$ at $z = 1$

$u = 0$ at $z = 0$ ....(5)

Taking $u = \underline{u(z)}\, e^{-bt}$, $v = \underline{v(z)}\, e^{-bt}$ $(b > 0)$ ………………. (6)

Equations (2) and (3) takes the form

$$R\frac{d^2\underline{u}}{dz^2} + b(1 - b\alpha)\underline{u} + \frac{f}{\tau}(1 - b\alpha)(\underline{v} - \underline{u}) = 0 \quad \text{……………………..(7)}$$

And $\underline{v} = \frac{1}{1-b\tau}\underline{u}$ ………………………………..(8)

Boundary conditions are $(t > 0)$

$\underline{u} = a\, e^{bt}$ Sint at $z = 1$ ……...……………………...(9)

$\underline{u} = 0$ at $z = 0$

Substituting equation (8) into equation (7) we get

$$\frac{d^2\underline{u}}{dz^2} - M^2\underline{u} = 0 \quad \text{………………………………(10)}$$

Where, $M^2 = \frac{(b\alpha-1)bF}{R(1-b\tau)}$ and $F = 1 - b\tau + bf$

Thus the solution of the equation (10) using boundary conditions (9), the velocity profile of dusty fluids

$$u = \frac{aSint\ SinhMz}{SinhM} \quad (t > 0)$$ (Which is same with Dr.K.C.Nandy's result) ….(14)

and the velocity profile of dust particles

$$v = \frac{a\,Sint}{(1-b\tau)}\frac{SinhMz}{SinhM}, \quad t > 0$$ (Which is same with Dr.K.C.Nandy's result) …….(15)

The dimensionless shearing stress τ_p at the lower plate due to the dusty visco-elastic fluid is

$$\tau_p = \left[\left(1 - \alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial z}\right]_{z=0}$$

$$\tau_p = \frac{a\left(\sqrt{k^2+1}\right)M}{SinhM} Sin(t - \theta_k), \quad \text{where } tan\theta_k = k \ (k = 1,2,3,\dots) \text{ .(16)}$$

The volume flow of dusty Visco-elastic fluid discharged per unit breadth of the plate is given by

$$\phi = 2\int_0^1 u\, dz = \frac{2\,a\,Sint}{M\,SinhM}(CoshM - 1) \text{……………………….(17)}$$

Results And Discussion:

The effects of elastic elements in the fluid, the mass concentration, and relaxation time of dust particles on the velocity profiles are obtained. The skin-friction and the total volume flow between the walls are obtained. Finally, for some representative values of τ and f, skin-friction at the lower plate is calculated numerically for different values of elastic parameters.

In figures (1), (2), and (3) the skin-friction of dusty visco elastic fluid between the walls is plotted for a=1, R=1, b=2, $\tau = 1$ and different values of f(mass concentration),α(slip parameter) and t(time). Figures (1), (2), and (3) show that the skin-friction(τ_p) fluctuates sinusoidally with time (t) for fixed value of α and f.

Figure-1 for $f = 0.01$

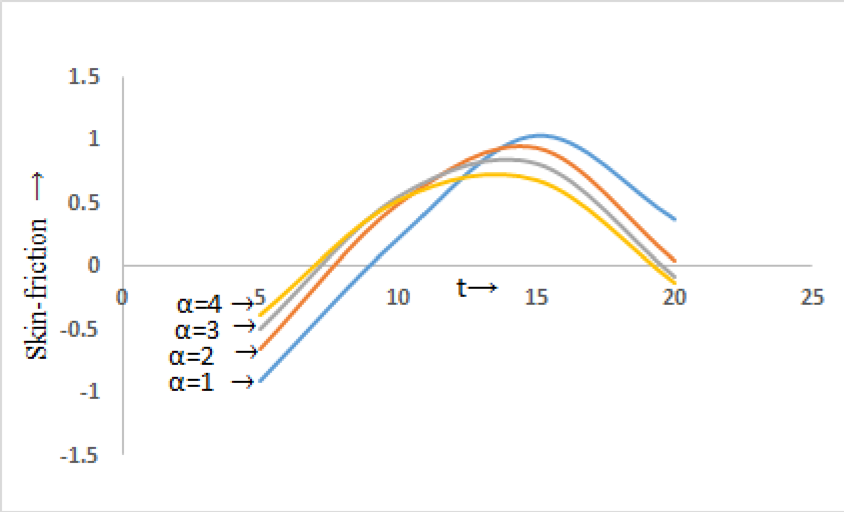


Figure- 2 for $f = 0.02$

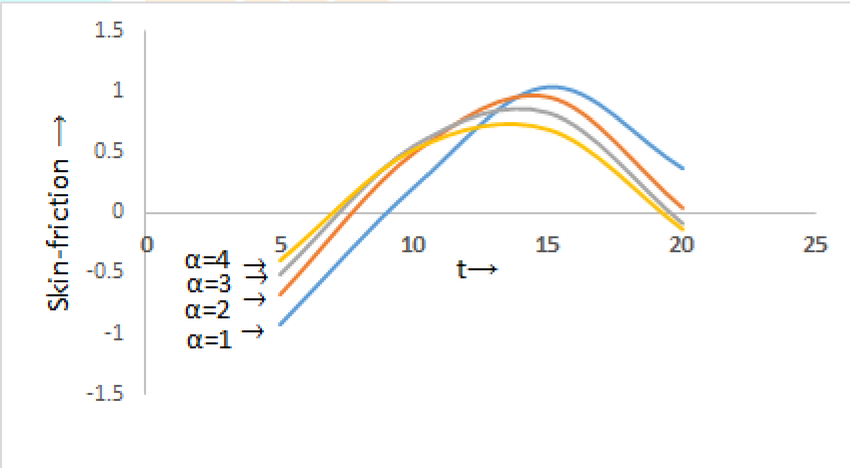
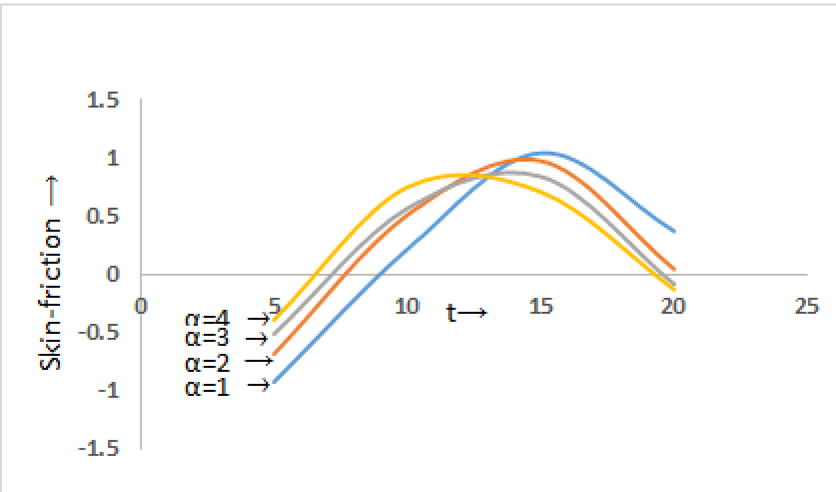


Figure-3 for $f = 0.03$



In Tables (1), (2) and (3) ,the total volume flow of dusty visco-elastic fluid discharged per unit breadth is studied for a=1,R=1,b=2, $\tau = 1$ and different values of f(mass concentration), α(slip parameter) and t(time).It is observed from tables(1), (2), and (3) that the magnitude of volume flow$|\phi|$ decreases with increaseα.

Table- 1

a=1,R=1,b=2,$\tau = 1$,$f = 0.01$,t=5

α	1	2	3	4
$\|\phi\|$	0.827918	0.663068	0.561400	0.493325

a=1,R=1,b=2,$\tau = 1$,$f = 0.01$,t=10

α	1	2	3	4
$\|\phi\|$	0.615835	0.376174	0.318495	0.279875

a=1,R=1,b=2,$\tau = 1$,$f = 0.01$,t=15

α	1	2	3	4
$\|\phi\|$	0.561446	0.449654	0.380709	0.334545

a=1,R=1,b=2,$\tau = 1$,$f = 0.01$,t=20

α	1	2	3	4
$\|\phi\|$	0.788220	0.631274	0.534482	0.469671

Table-2

a=1,R=1,b=2,$\tau = 1$,$f = 0.02$,t=5

α	1	2	3	4
$\|\phi\|$	0.829482	0.666176	0.565586	0.496609

a=1,R=1,b=2,$\tau = 1$,$f = 0.02$,t=10

α	1	2	3	4
$\|\phi\|$	0.470586	0.377938	0.320870	0.281738

a=1,R=1,b=2,$\tau = 1$,$f = 0.02$,t=15

α	1	2	3	4
$\|\phi\|$	0.562508	0.451763	0.383548	0.336772

a=1,R=1,b=2,$\tau = 1$,$f = 0.02$,t=20

α	1	2	3	4
$\|\phi\|$	0.789710	0.634234	0.538467	0.472797

Table-3

a=1,R=1,b=2,$\tau = 1, f = 0.03$,t=5

α	1	2	3	4
$\|\phi\|$	0.832598	0.670859	0.569810	0.501038

a=1,R=1,b=2,$\tau = 1, f = 0.03$,t=10

α	1	2	3	4
$\|\phi\|$	0.472353	0.380595	0.323267	0.284251

a=1,R=1,b=2,$\tau = 1, f = 0.03$,t=15

α	1	2	3	4
$\|\phi\|$	0.564620	0.454938	0.386412	0.339775

a=1,R=1,b=2,$\tau = 1, f = 0.03$,t=20

α	1	2	3	4
$\|\phi\|$	0.792676	0.638693	0.542488	0.477014

In tables (4), (5), and (6), skin friction of dusty visco-elastic fluid between the walls is studied for a=1, R=1, b=2, $\tau = 1$, and different values of f(mass concentration), α(slip parameter) and t(time). It is observed from the tables that magnitude of skin-friction(τ_p) increases with increasing f for different values of $\alpha$and t.

Table-4

$\boldsymbol{t = 5\ \alpha = 1}$

$\boldsymbol{f}$	0.01	0.02	0.03
τ_p	0.913522	0.917315	0.924875

$\boldsymbol{t = 5\ \alpha = 2}$

$\boldsymbol{f}$	0.01	0.02	0.03
τ_p	0.662103	0.670114	0.682240

$\boldsymbol{t = 5\ \alpha = 3}$

$\boldsymbol{f}$	0.01	0.02	0.03
τ_p	0.496265	0.507236	0.518390

$\boldsymbol{t = 5\ \alpha = 4}$

$\boldsymbol{f}$	0.01	0.02	0.03
τ_p	0.383262	0.391746	0.403378

Table-5

$t = 10\ \alpha = 1$

f	0.01	0.02	0.03
τ_p	0.216914	0.217815	0.219610

$t = 10\ \alpha = 2$

f	0.01	0.02	0.03
τ_p	0.491994	0.497947	0.506959

$t = 10\ \alpha = 3$

f	0.01	0.02	0.03
τ_p	0.541036	0.552996	0.565158

$t = 10\ \alpha = 4$

f	0.01	0.02	0.03
τ_p	0.514829	0.526227	0.541763

Table-6

$t = 15\ \alpha = 1$

f	0.01	0.02	0.03
τ_p	1.036584	1.040887	1.049464

$t = 15\ \alpha = 2$

f	0.01	0.02	0.03
τ_p	0.941224	0.952612	0.969850

$t = 15\ \alpha = 3$

f	0.01	0.02	0.03
τ_p	0.803209	0.820965	0.839018

$t = 15\ \alpha = 4$

f	0.01	0.02	0.03
τ_p	0.675338	0.690288	0.710667

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