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Application of Ellipsoid Method in Convex Optimization Problem

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Abstract:

The feasibility problem is the problem of finding a point in S. The convex optimization problem is the problem of minimizing a convex function over S. We present a new algorithm for the feasibility problem. The notion of a volumetric center of a polytope and a related ellipsoid of maximum volume inscribable in the polytope are central to the algorithm. Our algorithm has a significantly better global convergence rate and time complexity than the ellipsoid algorithm. The algorithm for the feasibility problem easily adapts to the convex optimization problem.

Keywords: *Ellipsoid method, convex optimization problem, linear programming.*

Introduction:

In mathematical optimization, the ellipsoid method is an iterative method for minimizing convex functions. When specialized to solving feasible linear_optimization problems with rational data, the ellipsoid method is an algorithm which finds an optimal solution in a finite number of steps.[1-3]

The ellipsoid method generates a sequence of ellipsoids whose volume uniformly decreases at every step, thus enclosing a minimizer of a convex function.[4]

History:

The ellipsoid method has a long history. As an iterative method, a preliminary version was introduced by Naum Z. Shor. In 1972, an approximation algorithm for real convex minimization was studied by Arkadi Nemirovskiand David B. Yudin (Judin). As an algorithm for solving linear programming problems with rational data, the ellipsoid algorithm was studied by Leonid Khachiyan: Khachiyan's achievement was to prove the polynomial-time solvability of linear programs.[5-7]

Convex optimization problem

The general form of an optimization problem (also referred to as a mathematical programming problem or minimization problem) is to find some $x^* \in \chi$

such that $f(x^*) = \min\{ f(x): x \in \chi \}$ for some $\chi \subset \mathbf{R}^n$ feasible set and objective f unction $f(\mathrm{x}) = \mathbf{R}^n \to \mathbf{R}$. The optimization problem is called a convex optimization problem if χ is a convex set and $f(\mathrm{x})$ is a convex function defined on $\mathbf{R}^n$.

Alternatively, an optimization problem of the form

Minimize $f(\mathrm{x})$

Subject to : $g_i \leq 0$, $i = 1, 2, 3, ..., m$ is called convex if the functions $f, g_1, ...g_m : \mathbf{R}^n \to \mathbf{R}$ are all convex functions.

A convex minimization problem consists of a convex_function $f_0(\mathrm{x}) = \mathbf{R}^n \to \mathbf{R}$ to be minimized over the variable *x*, convex inequality constraints of the form $f_i(\mathrm{x}) \leq 0$, where the functions $f_i(\mathrm{x})$ are convex, and linear equality constraints of the form $h_i(\mathrm{x}) = 0$. We are also given an initial ellipsoid $\varepsilon^0 \subseteq \mathbf{R}^n$ defined as $\varepsilon^0 = \{\mathrm{z} \in \mathbf{R}^n : (\mathrm{z} - \mathrm{x}_0)^T \mathrm{P}^{-1}{}_{(0)} (\mathrm{z} - \mathrm{x}_0) \leq 1\}$ containing a minimizes x^*, where $P \succ 0$ and x_0 is the center of ε. Finally, we require the existence of a cutting-plane oracle for the function *f*. One example of a cutting-plane is given by a subgradient *g* of *f*.

At the *k*-th iteration of the algorithm, we have a point $\varepsilon^{(k)} = \{\mathrm{x} \in \mathbf{R}^n : (\mathrm{x} - \mathrm{x}^{(k)})^T \mathrm{P}_{(k)}{}^{-1} (\mathrm{x} - \mathrm{x}^{(k)} \leq 1)\}$ at the center of an ellipsoid.

We query the cutting-plane oracle to obtain a vector $\mathrm{g}^{(k+1)} \in \mathbf{R}^n$ such that $\mathrm{g}^{(k+1)T} (\mathrm{x}^* - \mathrm{x}^{(k)}) \leq 0$

We therefore conclude that $x^* \in \varepsilon^{(k)} \bigcap \{\mathrm{z} : \mathrm{g}^{(k+1)T} (\mathrm{z} - \mathrm{x}^{(k)}) \leq 0\}$

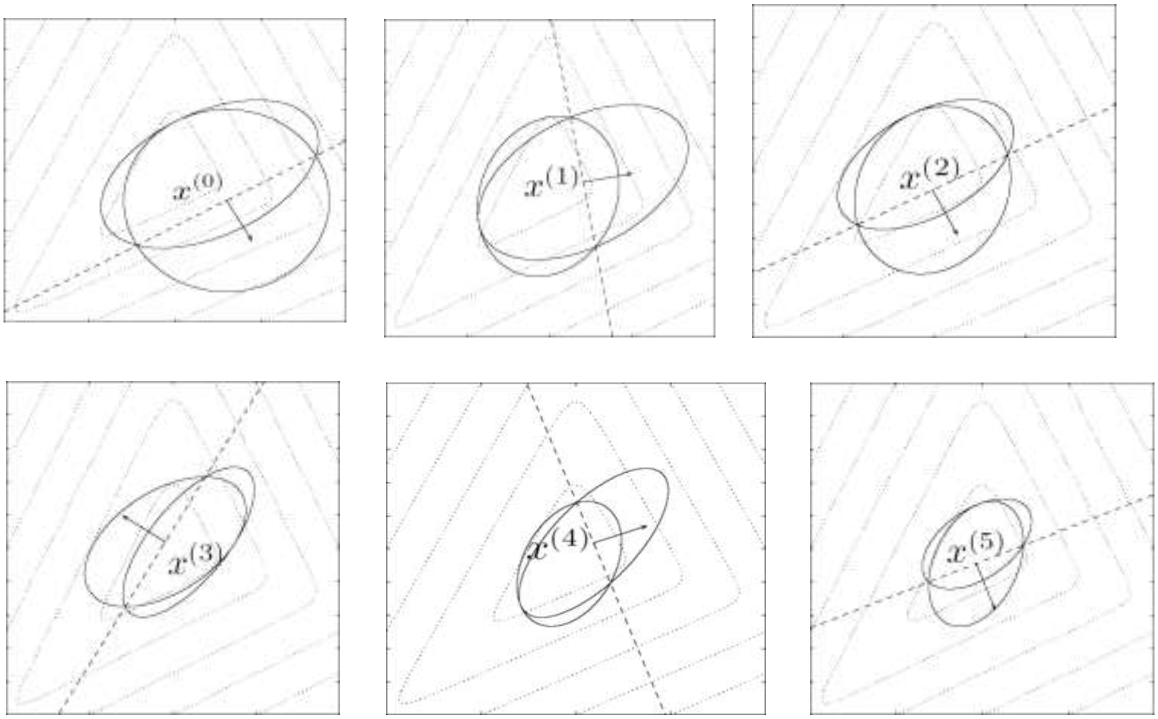
We set $\in^{(k+1)}$ to be the ellipsoid of minimal volume containing the half-ellipsoid described above and compute $x^{(k+1)}$. [8-14]

The update is given by

$$\text{Where } x^{(k+1)} = x^{(k)} - \frac{1}{n+1} P_{(k)} \overset{\square (k+1)}{g}$$

The stopping criterion is given by the property that $\sqrt{g^{(k)T} P_{(k)} g^{(k)}} \leq \in$

Sample sequence of iteration for k=0,1,2,3,4



Application to Linear Programming:

Inequality-constrained minimization of a function that is zero everywhere corresponds to the problem of simply identifying any feasible point. It turns out that any linear programming problem can be reduced to a linear feasibility problem (e.g. minimize the zero function subject to some linear inequality and equality constraints). One way to do this is by combining the primal and dual linear programs together into one program, and adding the additional (linear) constraint that the value of the primal solution is no worse than the value of the dual solution. Another way is to treat the objective of the linear program as an additional constraint, and use binary search to find the optimum value.

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