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Even Star Decomposition of some special kind of Complete Bipartile Graph

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Abstract: In this paper, we give some necessary and sufficient condition for decomposing some special type of complete bipartite graph $K_{m,n}$ into even star subgraphs. In particular the condition for α for which $K_{2^t,\alpha}, t = 0,1,2,3,...,$ for which even star decomposition exist. The graph of the form $K_{1,\alpha}$ has even star decomposition if and only if $\alpha = n(n+1)$ for any natural number n. A Complete bipartite graph K_{2^t,α_t} admits Even star Decomposition $(S_2, S_4, \ldots, S_{2n})$ if and only if $n = k2^{t+1}$ or $n = k2^{t+1} - 1, t, k(\neq 1) \in N$.

Keywords- Graph Theory; Complete bipartite graph; Arithmetic Decomposition; Even Decomposition; Even star Decomposition.

1. Introduction

Let G = (V, E) be a simple connected graph . If $G_1, G_2, \cdots, G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, \cdots, G_n)$ is a Decomposition of G. Different types of decompositions of graphs are available in literature such as path decomposition, cycle decomposition, triangle decomposition and few papers are available in diamond decomposition . In this paper we are discussing star decomposition of some complete bipartite graph.

Star :

A complete bipartite graph of the form $K_{1,n}$ is called a star , and is denoted by S_n.

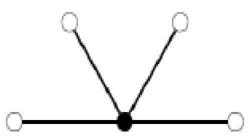
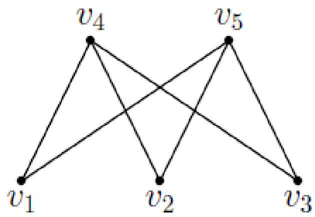


Fig 1: A Star $K_{1,4}$

Complete bipartite Graph:

A bipartite graph G which contains every edge joining V_1 and V_2 then G is a complete bipartite graph. If V_1 and V_2 have m and n vertices, we write $G = K_{(m,n)}$.Clearly $K_{m,n}$ has mn edges.



Complete Bipartile Graph $K_{2,3}$

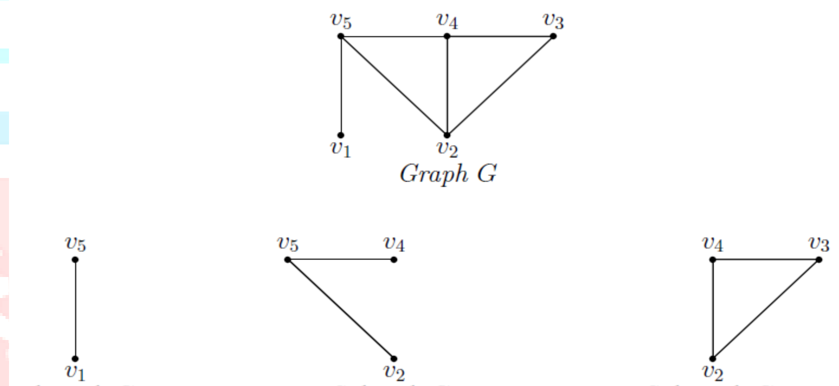
Decomposition of a Graph:

Let G = (V, E) be a simple connected graph . If G_1 , G_2 , · · · , G_n are connected edge-disjoint subgraphs of G with E(G) = E(G_1) ∪ E(G_2) ∪ ... ∪ E(G_n), then (G_1,G_2, · · · ,G_n) is a Decomposition of G.

2. **EVEN DECOMPOSITION OF A GRAPH**

Arithmetic Decomposition:

A decomposition $(G_1,G_2,G_3,....,G_n)$ of a graph G is an Arithematic Decomposition (AD) if the sequence of number of edges in the decomposed subgraphs in increasing order is an arithematic sequence.



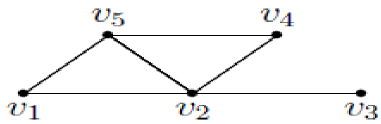
In the above G is a simple connected graph

(G_1,G_2,G_3) is a decomposition of G. $|E(G_1)|=1$,$|E(G_2)|=2$ and $|E(G_3)|=3$. Also , they are in arithmetic progression with first term 1 and common difference 1.So the decomposition

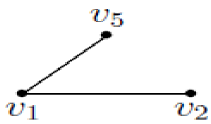
Thus (G_1,G_2,G_3) is an arithmetic decomposition.

Even Decomposition(ED):

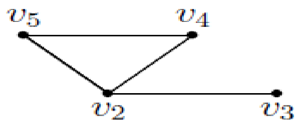
An arithmetic decomposition is said to be Even Decomposition if the first term and common difference of the sequence of number of edges in the subgraphs are 2. Since the number of edges of each sub graph of G is even, we denote the Even Decomposition as (G_2,G_4,G_{2n}).



Graph G



G_2



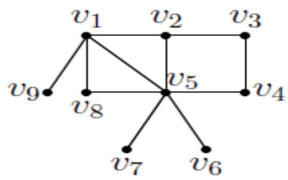
G_4

In this example, G_2 and G_4 are arithmetic decomposition of G with $a=2$ and $d=2$. So (G_2, G_4) are Even decomposition of G.

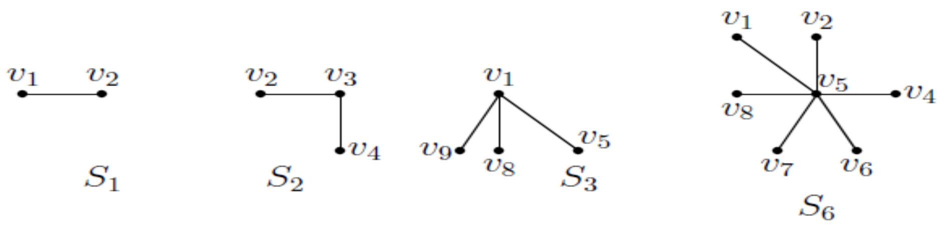
Theorem 1. Any graph G admits Even Decomposition $(G_2, G_4, G_6, \cdots, G_{2n})$, where $G_{2i} = (V_{2i}, E_{2i})$ and $|E(G_{2i})| = 2i, (i = 1, 2, 3, 4. \ldots, n)$ if and only if $q = n(n + 1)$ where n is any natural number.

Star Decomposition

A decomposition $(G_1, G_2, \ldots, G_n)$ *of* G is said to be star decomposition if each $(G_1, G_2, \ldots, G_n)$ are some stars.



G

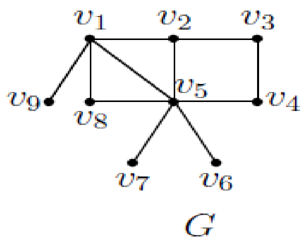


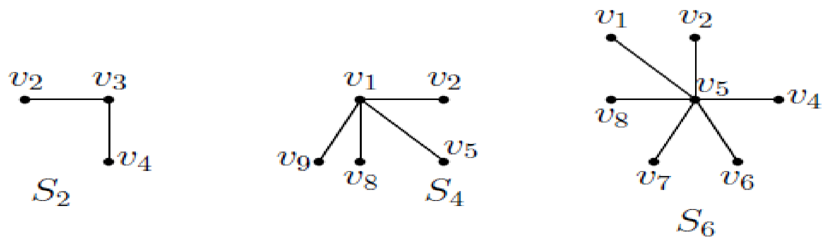
Star Decomposition (S_1, S_2, S_3, S_6) of G

Even Star decomposition(ESD)

A decomposition which is both even and star is called Even Star decomposition.

Decomposition (ESD).



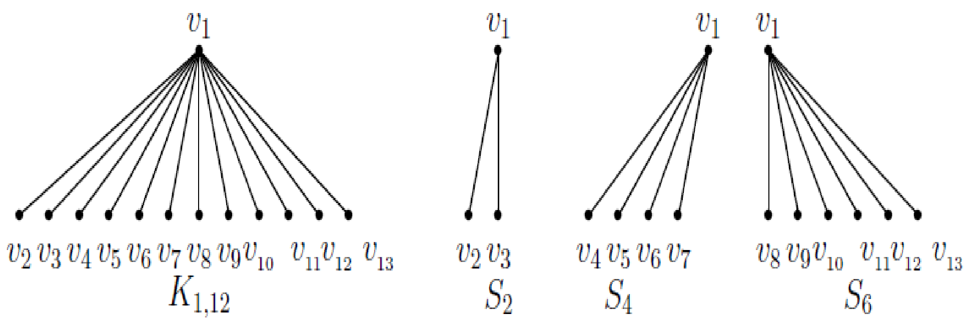


Even Star Decomposition (S_2, S_4, S_6) of G

3. EVEN STAR DECOMPOSITION OF COMPLETE BIPARTITE GRAPH

Number of edges in $K_{1,1}$ is 1. This number 1 cannot be expressed as $n(n+1)$ *for* any natural number n. So it is not an ED, and hence not an ESD. In the similar manner $K_{1,3}$, $K_{1,4}$, $K_{1,5}$, $K_{1,7}$ etc has no ESD.

Number of edges in $K_{1,2}$ is 2. This number 2 can be expressed as $1(1+1)$. So it is an ED, and hence $K_{1,2}$ have an decomposition $G_2 = S_2$. In the similar manner $K_{1,6}$, $K_{1,12}$ etc has ESD.



Theorem 2:

The graph $K_{1,\alpha}$ has an ESD $(S_2, S_4, \ldots, S_{2n})$ if and only if α = n(n + 1) , n ϵ N.

Proof.

Assume that $K_{1,\alpha}$ has an ESD $(S_2, S_4, \ldots, S_{2n})$. Then $|E(K_{1,\alpha})| = \sum_{i=1}^{n} |E(S_{2i})|$

$$\Rightarrow \alpha = \sum_{i=1}^{n} 2i \quad \Rightarrow \alpha = n(n+1)$$

Conversely assume that $\alpha = n(n+1)$. We have to prove that $K_{1,\alpha} = K_{1,n(n+1)}$ has an ESD $(S_2, S_4, \ldots, S_{2n})$. Let us prove his by mathematical induction. For $n = 1, \alpha = 1*(1+1) = 2$ clearly $K_{1,2}$ has ESD S_2. Assume that the result is true for n = m. That is $K_{1,m(m+1)}$ has ESD $(S_1, S_2, \ldots, S_{2m})$. We have to show that it is true for $n = m+1$. that is to prove that $K_{1,(m+1)(m+1+1)}$ has ESD $(S_1, S_2, \ldots, S_{2(m+1)})$. We have, $(m+1)(m+2) = m(m+1) + 2(m+1)$.

Also, $K_{1,(m+1)(m+1+1)} = K_{1(m+1)(m+2)} = K_{1(m+1)(m+2)} = K_{1,m(m+1)(m+1+1)}$

$K_{1,m(m+1)(m+1+1)}$ can be decomposed in to $(K_{1,m(m+1),}K_{1,2(m+1)})$.
By hypothesis $K_{1,m(m+1)}$ has ESD $(S_1,S_2,...,S_{2m})$.
$\Rightarrow (K_{1,m(m+1),}K_{2(m+1)})$ can be decomposition into $(S_1,S_2,...,S_{2m},K_{1,2(m+1)})$.
$\Rightarrow (K_{1,m(m+1),}K_{2(m+)})$ can be decomposition into $(S_1,S_2,...,S_{2m},K_{1,2(m+1)})$.
$\Rightarrow (K_{1,m(m+1),}K_{2(m+1)})$ has the ESD $(S_1,S_2,...,S_{2m},S_{2(m+1)})$.
Hence the proof.

Theorem 3:
A complete bipartile graph $K_{2,\alpha}$ admits Even star Decomposition$(S_2,S_4,\dots,S_{2(2^2k-1)})$ if and only if $\alpha = 2k(2^2k-1)$, where $n = 2^2k-1$, $(K \neq 1)\epsilon N$.

Proof:
Assume that $K_{2,\alpha}$ admits Even star Decomposition$(S_2,S_4,\dots,S_{2(2^2k-1)})$.
Then $|E(K_{2,\alpha})| = \sum_{i=1}^{n}|E(S_{2i})|$
$\Rightarrow 2\alpha = \sum_{i=1}^{n}2i$
$\Rightarrow 2\alpha = n(n+1)$
$\Rightarrow 2\alpha = 2^2k(2^2k-1)$
$\Rightarrow \alpha = 2k(2^2k-1)$.
Conversely assume that $\alpha = 2k(2^2k-1)$. Let $\{u_1,u_2,v_1,v_2,\dots,v_\alpha\}$ be the vertex set of $K_{2,\alpha}$. For every α , $K_{2,\alpha\_}$ can be decomposed into $K_{2,\alpha}^{u_1}$ with vertex set $\{u_1,v_1,v_2,\dots,v_\alpha\}$ and $K_{2,\alpha}^{u_2}$ with vertex set $\{u_2,v_1,v_2,\dots,v_\alpha\}$.Since number of edges in $K_{2,\alpha}$ can be expressed in the form $n(n+1)$, it has an even decomposition $(G_2,G_4,\dots,G_{2(n)})$. This even decomposition can be Partitioned into two sets $U = \{G_2,G_4,\dots,G_{2(K-1)},G_{2(n-(k-1))},\dots,G_{2n}\}$ and $V = \{G_{2k},G_{2(k+1)},\dots,G_{2k+1}\}$. Using these two sets U and V, we can decompose $K_{2,\alpha}^{u_1}$ and $K_{2,\alpha}^{u_2}$, Whose union gives the decomposition of $K_{2,\alpha}$. $K_{2,\alpha}^{u_1}$ can be decomposed as $\{S_2,S_4,\dots,S_{2(K-1)},S_{2(n-(k-1))},\dots,S_{2n}\}$ and $K_{2,\alpha}^{u_2}$ can be decomposed as $\{S_{2k},S_{2(k+1)},\dots,S_{2k+1}\}$, whose union gives the ESD of $K_{2,\alpha}$.Hence the proof.

Theorem 4:
A complete bipartile graph $K_{2,\alpha}$ admits Even star Decomposition$(S_2,S_4,\dots,S_{2(2^2k)})$ if and only if $\alpha = 2k(2^2k+1)$, where $n = 2^2k$, $(K \neq 1)\epsilon N$.

proof.
Assume that $K_{2,\alpha}$ admits Even star Decomposition$(S_2,S_4,\dots,S_{2(2^2k)})$.
Then $|E(K_{2,\alpha})| = \sum_{i=1}^{n}|E(S_{2i})|$
$\Rightarrow 2\alpha = \sum_{i=1}^{n}2i$
$\Rightarrow 2\alpha = n(n+1)$
$\Rightarrow 2\alpha = 2^2k(2^2k+1)$
$\Rightarrow \alpha = 2k(2^2k+1)$.
Conversely assume that $\alpha = 2k(2^2k+1)$. Let $\{u_1,u_2,v_1,v_2,\dots,v_\alpha\}$ be the vertex set of $K_{2,\alpha}$. For every α , $K_{2,\alpha}$ can be decomposed into $K_{2,\alpha}^{u_1}$ with vertex set $\{u_1,v_1,v_2,\dots,v_\alpha\}$ and $K_{2,\alpha}^{u_2}$ with vertex set $\{u_2,v_1,v_2,\dots,v_\alpha\}$.Since number of edges in $K_{2,\alpha}$ can be expressed in the form $n(n+1)$, it has an even decomposition $(G_2,G_4,\dots,G_{2(n)})$. This even decomposition can be Partitioned into two sets $U = \{G_2,G_4,\dots,G_{2(K)},G_{2(n-(k-1))},\dots,G_{2n}\}$ and $V = \{G_{2(k+1)},G_{2(k+2)},\dots,G_{2(n-k)}\}$. Using these two sets U and V, we can decompose $K_{2,\alpha}^{u_1}$ and $K_{2,\alpha}^{u_2}$, Whose union gives the decomposition of $K_{2,\alpha}$. $K_{2,\alpha}^{u_1}$ can be decomposed as $\{S_2,S_4,\dots,S_{2(k1)},S_{2(n-(k-1))},\dots,S_{2n}\}$ and $K_{2,\alpha}^{u_2}$ can be decomposed as $\{S_{2(k+1)},S_{2(k+2)},\dots,S_{2(n-k)}\}$, whose union gives the ESD of $K_{2,\alpha}$.Hence the proof.

Theorem 5:
A Complete bipartile graph K_{2^t,α_t} admits Even star Decomposition $(S_2,S_4,...,S_{2n})$if and only if $n = k2^{t+1}$ or $n = k2^{t+1}-1, t, k(\neq 1) \in N$.
Proof:

Assume that K_{2^t,α_t} admits Even star Decomposition $(S_2, S_4, \ldots, S_{2n})$.

$\Rightarrow 2^t\alpha_t = \sum_{i=1}^{n} |E(S_{2i})|$

$\Rightarrow 2^t\alpha_t = \sum_{i=1}^{n} 2i$

$\Rightarrow 2^t\alpha_t = n(n+1)$

$\Rightarrow \alpha_t = \frac{n(n+1)}{2^t}$. , where $\alpha_t \in 2N$.

$\Rightarrow \frac{n(n+1)}{2^t} = 2k$, where $k \in N$

$\Rightarrow n\{n+1) = 2^{t+1}k$.

$\Rightarrow n = 2^{t+1}k$ or $n+1 = 2^{t+1}k$.

$\Rightarrow n = 2^{t+1}k$ or $n = 2^{t+1}k - 1$.

Conversely assume that $n = 2^{t+1}k$ or $n = 2^{t+1}k - 1$.

Case 1: $= 2^{t+1}k$.

When $n = 2^{t+1}k$, $\alpha_t = \frac{2^{t+1}k\,(2^{t+1}k+1)}{2^t} = 2k\,(k2^{t+1}+1)$,we have to prove K_{2^t,α_t} admits Even star Decomposition. Applying induction on 't' the result is true when t=1.Suppose the result is true when t=m. that is K_{2^m,α_m} admit ESD $(S_2, S_4, \ldots, S_{2.k2^{m+1}})$.We have to show that the result is true for $t=m+1$, that is to prove $K_{2^{m+1},\alpha_{m+1}}$ admit ESD $(S_2, S_4, \ldots, S_{2.k2^{m+1}}, S_{2(k2^{m+1}+1)}, \ldots, S_{2(k2^{m+2})})$.

We have ,

$|E(K_{2^{m+1},\alpha_{m+1}})| = 2^{m+1}\alpha_{m+1} = k2^{m+2}(k2^{m+2}+1)$ … … … … … … … … … … … … … (1)

Also , $|E(K_{2^m,\alpha_m})| = 2^m\alpha_m = k2^m(k2^{m+1}+1)$ … … … … … … … … … … … … … … … … … … (2)

There fore , $|E(K_{2^{m+1},\alpha_{m+1}})| - |E(K_{2^m,\alpha_m})| = 3K^2 2^{2m+2} + k2^{m+1}$

Now $|E(S_{k2^{g+2}+2})| + \ldots + |E(S_{k2^{g+3}})| = 3K^2 2^{2m+2} + k2^{m+1}$……………………………(3)

Therefore , From (2) and (3) $|E(K_{2^{m+1},\alpha_{m+1}})| = |E(K_{2^m,\alpha_m})| + |E(S_{k2^{m+2}+2})| + \ldots + |E(S_{k2^{m+3}})|$

Therefore result is true for t=m+1

Hence , K_{2^t,α_t} admits Even star Decomposition $(S_2, S_4, \ldots, S_{2n})$ Where n= $k2^{t+1}, t, k(\neq 1) \in \mathrm{N}$.

Case 2: $n = 2^{t+1}k$-1.

The proof is similar to *case 1*.

Hence the proof.

4. Result and Discussion

In this study, we introduce a concept known as even star decomposition of graphs and determined this parameter for few es, especially for some complete bipartile graph.. The graph $K_{1,\alpha}$ has an ESD $(S_2, S_4, \ldots, S_{2n})$ if and only if $\alpha = n(n+1), n \in \mathrm{N}$. A Complete bipartite graph K_{2^t,α_t} admits Even star Decomposition $(S_2, S_4, \ldots, S_{2n})$ if and only if $n = k2^{t+1}$ or $n = k2^{t+1} - 1, t, k(\neq 1) \in N$.

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