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# TOOLS AND APPROACHES USED IN LINEAR ALGEBRA AND MATRIX: A REVIEW

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**Abstract :** With the use of matrices and vectors, linear algebra helps us to begin to understand simple linear systems. The branch of mathematics dealing with the study of vectors, vector spaces, linear maps, and linear equation structures is linear algebra**.** In modern mathematics, vector spaces are a central theme; linear algebra is also commonly used in both abstract algebra and functional analysis. In analytic geometry, linear algebra also has a concrete representation and in operator theory it is generalized. In the natural sciences and social sciences, it has broad applications, since nonlinear models can often be approximated by linear ones.

**Keywords:** Linear Algebra, Matrix, Transformation, Variables, Vectors

## Introduction

Linear algebra also plays an important part in analysis, notably, in the description of higher order derivatives in vector analysis and the study of tensor products and alternating maps (Carrell 2005). Linear Map take elements from a linear space to another (or to itself), in a manner that is compatible with the addition and scalar multiplication given on the vector space. The set of all such transformations is itself a vector space. If a basis for a vector space is fixed, every linear transform 1 can be represented by a table of numbers called a matrix (Artin, 1991)

Linear Algebra is a principal branch of mathematics that is related to mathematical structures closed under the operations of addition and scalar multiplication and that includes the theory of systems of linear equations, matrices, determinants, vector spaces, and linear transformations. Linear algebra, is a mathematical discipline that deals with vectors and matrices and, more generally, with vector spaces and linear transformations (Nicosia, 2016).

A vector space of dimension n is called an n-space. Most of the useful results from 2- and 3-space can be extended to these higher dimensional spaces. Although people cannot easily visualize vectors in n-space, such vectors or n-tuples are useful in representing data. Since vectors, as n-tuples, are ordered lists of n components, it is possible to summarize and manipulate data efficiently in this framework. For example, in economics, one can create and use, say, 8-dimensional vectors or 8-tuples to represent the Gross National Product of 8 countries. One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, France, Germany, Spain, India, Japan, Australia), by using a vector (v1, v2, v3, v4, v5, v6, v7, v8) where each country's GNP is in its respective position(Tomar, et al, 2016)

Matrices are used in representing the real World data like the traits of people's population, habits, etc. Matrices can be solved physical related application and one applied in the study of electrical circuits, quantum mechanics and optics, with the help of matrices, calculation of battery power outputs, resistor conversion of electrical energy into another useful energy. These matrices play a role in calculations. Matrix mathematics simplifies linear algebra, at least in providing a more compact way to deal with groups of equations in linear algebra. Some properties of matrix mathematics are important in mathematics theory. Matrices are also used in robotics and automation in terms of base elements for the robot movements. The movements of the robots are programmed with the calculation of matrices row and column "controlling of matrices are done by calculation of matrices"( Rayate S 2018).

**LINEAR TRANSFORMATIONS**

Nicosia, 2016 conducted the study on functions of linear Algebra such as vectors as both inputs and outputs. They observed that in linear algebra vectors are objects that can be added or scalar multiplied. A linear transformation, T: U→V, is a capacity that conveys components of the vector space U (called the domain) to the vector space V (called the co domain), and which has two extra properties

T (U1 +U2) =T (U1) +T (U2) for all u1, u2 ∈U

T (αU) =αT (U) for all u∈U and all α∈C

Dummit, 2016 observed that If V and W are vector spaces, we say a map T from V to W (denoted T: V → W) is a linear transformation if, for any vectors v, v1, v2 and any scalar α, the following two properties hold:

[T1] The map respects addition of vectors: T (v1 + v2) = T (v1) + T(v2).

[T2] The map respects scalar multiplication: T (α · v) = α · v.

It is important to remember that the addition on the left side occurs within V in the statement T (v1 + v2) = T (v1) + T (v2), while the addition on the right side takes place within W. Similarly, the scalar multiplication on the left-hand side is in V in the statement T (a • v) = aT( v), while the scalar multiplication on the right-hand side is in W If V is the vector space of all differentiable functions and W is the vector space of all functions, decide if a linear transformation from V to W is the derivative map D that sends a function to its derivative.

[T1]: We have $D(f_1 + f_2) = (f_1 + f_2)' = f'_1 + f'_2 = D(f_1) + D(f_2)$.

[T2]: Also, $D(\alpha \cdot f) = (\alpha f)' = \alpha \cdot f' = \alpha \cdot D(f)$.

since both parts of the definition are satisfied, the derivative is a linear transformation

More generally, on the vector space of n-times differentiable functions, the map T which sends a function y to the function $y^n + P_n(x)\, y^{n-1} + \cdot \cdot \cdot + P_2(x)\, y' + P_1(x)\, y$ is a linear transformation, for any functions $P_n(x), \cdot \cdot \cdot, P_1(x)$

**Properties of Linear Transformation**

Gilbert, 2009 gives the four properties of linear Transformation. Let V and W be two vector spaces. Suppose T: V to W is a linear transformation .Then

1. T(0) = 0
2. T (−v) = −T (v) for all v ∈ V.
3. T(u – v) = T (u) − T (v) for all u, v ∈ V
4. If v = c1v1 + c2v2 + · · · + cnvn then T (v) = T(c1v1 + c2v2 + · · · + cnvn) = c1T (v1)+c2T (v2)+· · ·+cnT (vn).

**LINEAR EQUATION**

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. Linear equations can have one or more variables. Linear equations occur abundantly in most subareas of mathematics and especially in applied mathematics A particular case of matrix multiplication is tightly linked to linear equations: if x designates a column vector (i.e. n×1- matrix) of n variables x1, x2, ..., xn, and A is an m-by-n matrix, then the matrix equation Ax = b, where b is some m×1-column vector, is equivalent to the system of linear equations $A_{1,1}x_1 + A_{1,2}x_2 + \ldots + A_{1},nxn = b_1$

$$A_{2,1}x_1 + A_{2,2}x_2 + \ldots\ldots + A_{m,nxn} = b_2$$

$$A_{m,\,1x1} + A_{m,\,2x2} + \ldots + A_{m,nxn} = b_m\ .$$

This way, matrices can be used to compactly write and deal with multiple linear equations, i.e. systems of Linear equations do not include exponents. This article considers the case of a single equation for which one searches the real solutions. All its content applies for complex solutions and, more generally for linear equations with coefficients and solutions in any field. (Baker and Andrew, 2003).

Saraswati et al, 2016 conducted the new approaches of Linear equation with one variable is a beginning algebraic topic taught in the 7th grade. Cai, et al., (2005) clarified that "algebra has been characterized as an important 'gatekeeper'in mathematics". Besides, in Al Khwarizmi's book, it is stated that "a motivation for studying algebra was the solution of equations" (Krantz, 2006). It demonstrates that to promote learning of other topics in mathematics, linear equations with one variable are really necessary. The learning process used in Indonesia, however, does not help students to understand the notion of solving linear equations with one variable. In teaching linear equation with one variable, most learning processes are only familiar with the formal strategy((Jupri, 2015).

Solving linear equation is particularly important concepts in algebra and on that causes confusions for students (Marauder, 2012). Then, Marauder (2012) clarified that there are three primary subtopics where students found difficulties when solving equations such as symbolic understanding. The meaning of the equal sign a reliance on procedural knowledge without conceptual Understanding. Moreover, Drivers Panhuizen (2014) stated that one of common mistakes on understanding the concept of linear equation is applying arithmetic operation. For instance, to find the value of x in the equation $3x = 5$, it has to be 5 divided by 3. However, the students commonly come with $x= 5 - 3$. Students struggle to balance conceptual and procedural knowledge to learn the subject of solving linear equations with one variable. (Magruder, 2012). Linear equations are often difficult for students in transition from a concrete mathematics to an abstract concept. So, a learning that bridges the students' thinking from abstract to real is needed (cooper 2005).

An algebraic equation is a linear equation in which each term is either a constant or the product of a constant and a single variable (the first power of). One or more variables may have linear equations. In most sub-areas of mathematics, and particularly in applied mathematics, linear equations exist abundantly. While, when modeling many phenomena, they emerge very naturally,(Pathik 2021)

**System of Linear Equation**

David, (2005) conducted the study of linear equation if you have a particular solution $x^p$ a linear equation and add a sum of multiples of homogeneous solutions to it you obtain another particular solution.

Consider a system of equations with exactly one solution

$2x-y=1$

$3x+2y=12$

If we solve the first equation for y in terms of x, we get the equation $y=2x-1$

Now substitute this equation for y into the second equation gives

$3x+2(2x-1) =12$

$3x+4x-2=12$

$7x=14$

$X=2$

Finally, we can obtain the following by substituting this value of x into the expression for y

$2(2)-y=1$

$4-y=1$

$Y=3$

**NOTE:-** the result can be checked by substituting the values $x = 2$ and $y = 3$ into the equations.

$2(2)-3=1$

$3(2) +2(3) =12$

by this verification, he concluded that point (2, 3) lies on both lines.

**Matrices**

A matrix is a two dimensional array of numbers or expressions arranged in a set of rows and columns. An $m \times n$ matrix A has m rows and n columns .A matrix having either a single row ($m = 1$) or a single column ($n = 1$) is defined to be a vector because it is often used to define the coordinates of a point in a multi-dimensional space. (Rowell 2002).

**Elementary Matrix Arithmetic**

Rowell, 2002 observed that the operation of addition of two matrices is only defined when both matrices have the same dimensions. If A and B are both ($m \times n$), then the sum $C=A+B$ is also ($m \times n$) and is defined to have each element the sum of the corresponding elements of A and B. Matrix addition is both associative, that is $A + (B + C) = (A + B) + C$ and commutative $A + B = B + A$. The subtraction of two matrices is similarly defined; if A and B have the same dimensions, then the difference $C = A – B$.

Klopper et al (2007) conducted the study on Matrices contain verbal information, quotes, summarized text, extracts from notes, memos, standardized responses and, in general, data integrated around a point or research theme that makes sense. They explained aspects of research, and allowed the researcher to get a quick overview of data related to a certain point. In this very sense they serve a similar purpose to that of tables employed in quantitative research. Because matrix are powerful interpretive tools, they are currently being used in a wide variety of disciplines such as physics, engineering, economics, statistics, mathematics, logic, cryptography, linguistics, communication science, health science and information science.

**Matrix Multiplication**

Two matrices may be multiplied together only if they meet conditions on their dimensions that allow them to conform. Let A have dimensions m × n, and B be n × p, that is A has the same number as columns as the number of rows in B, then the product C = AB, Matrix multiplication is associative, that is A (BC) = (AB) C, but is not commutative in general AB not equal to BA. It is interesting to note in passing that unlike the scalar case, the fact that AB = 0 does not imply that either A = 0 or that B = 0. (Rowell, 2002)

Williams, 2003 gives the new operation regarding Matrix multiplication is one of the most basic mathematical operations outside of everyday arithmetic. Many other essential matrix operations can be efficiently reduced to it, such as Gaussian elimination, LUP decomposition, the determinant or the inverse of a matrix [1]. Matrix multiplication is also used as a subroutine in many computational problems that, on the face of it, have nothing to do with matrices.

Pandya et al (2013) conducted the new method of matrix multiplication. The product of two matrices is one of the most basic operations in mathematics and computer science. Here, They focus on mainly three methods of matrices multiplication. 1) Conventional Method, 2) Stassen's Method and 3) Coppersmith and Wino grad. The only multiplication appears in the innermost loop. Since this loop will iterate a total of n3 times, we will perform exactly $n_3$ multiplication operations.

**Conclusion**

From the above discussion, it can be said that if there are small matrices, then nearly equal credit is given by all the methods. For any processor used, it should be noted that this is valid. If the matrix size is increased, then much less time is seen. The researcher suggests that teachers should use algebra tiles supported with co-operation to help students'understanding of solving linear equation with one variable**.**

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