IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Physical picture of mensuration

Ram Swaroop Meghwal[1*]

Harish Kumar Sublania[2]

Dept. of Physics, Govt . College, Kota (Raj.)- India,

Abstract-

Under plane geometry area of circle is evaluated. The ellipse is mapped to get a circle. The cone and sphere are mapped from curved to plane geometry and areas are calculated. Volume pyramid is decided and it is used to calculated volumes of cone and sphere. The entire computation is based on elementary mathematics rather than calculus. Such method is not available elsewhere.

Key words

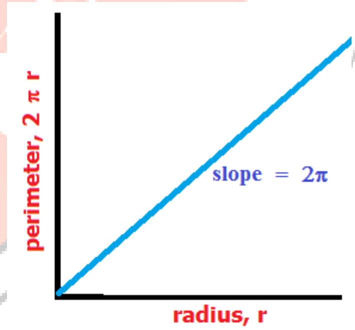
Mensuration, area and circumference of circle and ellipse, cone , sphere, volume of cone, sphere.

1. Introduction

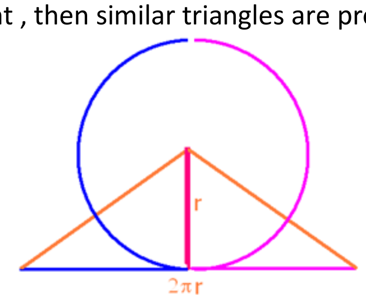
The Great Mathematician Aryabhatta invented that perimeter of a circle and its diameter has a constant value, we call it ‘π’ now. Geometrical shapes and bodies occupy a major space in physical and mathematical problems. At elementary level subject is delivered as to remember the formulas. At intermediate level calculus is adopted to derive them. Here I will present analytical consideration at very elementary level so that there should no more need to memorize or forget such interesting things.

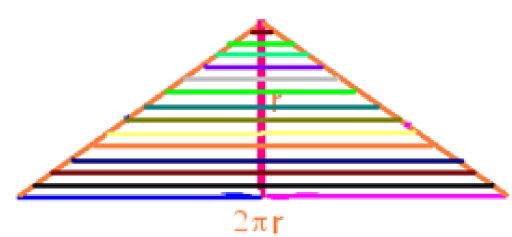
2. Area of circle

A circular area[1] can be regarded as composed of infinite number of circular paths ranging from centre to the perimeter. Now take a set A of radii and set B of semi-perimeter and plot on graph, we will obtain a straight line.



Hence if we take linear perimeters as base and radius as height , then similar triangles are produced.





Hence we can calculate the area very easily as follows

$$area\,of\ circle = area\,of\ triangle = \frac{1}{2}\times height \times base = \frac{1}{2}\times r\times 2\pi r = \pi r^2$$ ...(1)

3. Area and perimeter of an ellipse

Here an interesting approach is developed to do the job. First it is useful to transform the ellipse into the circle.

The equation of ellipse is given by

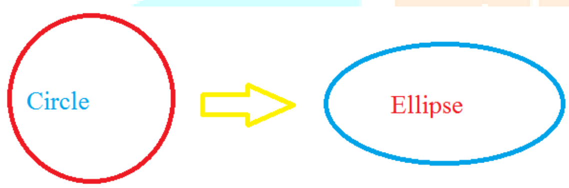
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ ...(2)

The following substitution will transform the ellipse into the circle.

$$\frac{x}{a}=\frac{x'}{r},\frac{y}{b}=\frac{y'}{r}$$ ...(3)

$$x'^2+y'^2=r^2$$ ...(4)

The physical aspect is if we squeeze a circle then we have an ellipse.



This mechanism can be incorporated into below mathematical ansatz.

$$a=r+\in\ ,\quad b=r-\in$$ ...(5)

It is to be noted that their sum (a+b) and product (ab) yield new dimensions of thinking. The sum is equal to the diameter of the circle irrespective of measure of $\in$, while the product equals square of the radius if $\in$ is infinitesimally small .

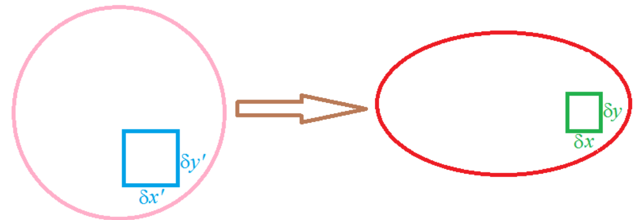
$$a+b=2r\quad ,\quad a-b=2\in$$

$$r^2=(a-\in)(b+\in)=ab+\in^2$$

Thus formula of circumference C=π(2r) is transformed to

$$C=\pi(a+b)$$ ...(6)

We also get the area of the ellipse as $A_{circle}=\pi r^2 \rightarrow A_{ellipse}=\pi(ab)$, but the condition on $\in$ is pursuing the picture. To overcome it we follow a different route.



$$\frac{\delta x}{a}=\frac{\delta x'}{r},\frac{\delta y}{b}=\frac{\delta y'}{r}\Rightarrow\frac{\delta A}{\delta A'}=\frac{\delta x\delta y}{\delta x'\delta y'}=\frac{ab}{r^2}$$

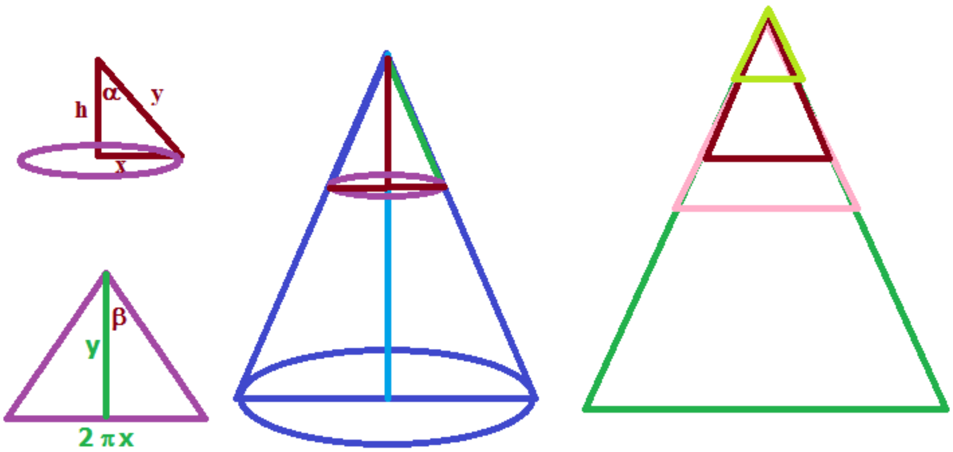
$$\frac{\delta x}{a}=\frac{\delta x'}{r},\frac{\delta y}{b}=\frac{\delta y'}{r}\Rightarrow\frac{\delta A}{\delta A'}=\frac{\delta x\delta y}{\delta x'\delta y'}=\frac{ab}{r^2}$$

$$A=\sum\delta x\delta y=\frac{ab}{r^2}\sum\delta x'\delta y'=\frac{ab}{r^2}.\pi r^2=\pi ab$$ ...(7)

Thus we succeed to overcome the restriction.

4. Area of cone

Now we consider a right circular cone[2] and set A belonging to the points forming slant height 'y' on curved surface of the cone and set B belonging to the perimeter of radius 'x' at height 'h'. if we plot set B against set A, we get straight line of slope '2πsinα', where 'α' is the semi vertex angle. Now we map set B onto set C on the plane surface of length '2πx'. The semivertex angle of the triangle of the plane is β=arctan(πsinα). In this way we can map all the points of curved surface of cone onto plane triangle of height equal to slant height of the cone and base equal to perimeter of the base.

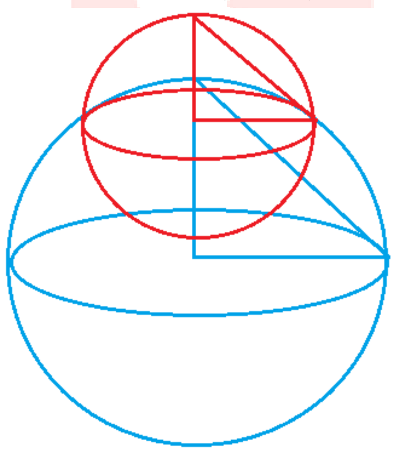


Hence the curved surface area

$$C = \tfrac{1}{2}.l.2\pi r = \pi rl \qquad ...(8)$$

5. Area of sphere

For the great circle one thing is obviously observable that each point of the great circle is at a distance '√2. R', from the vertex of great sphere. Similarly, the same type of construction is possible for sphere of radius 'r', having its own great circle at a distance '√2. R ',from its own vertex. Now we take single point for all vertices and map onto plane surface. '√2. R',



We consider a hemisphere and define set A as points on curved surface. Now we map these points on plane figure of set B in the following manner.

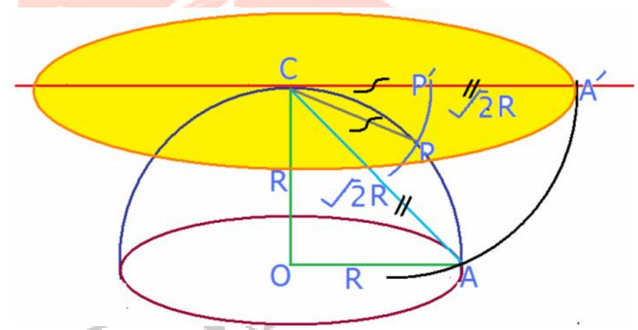
Using Pythagoras theorem it is easy to find that the di stance

$$CA = \sqrt{2}\,.R$$

Any chord cuts the circle or sphere at most at two po ints only.

The concentric spheres or circles of distinct radii neve r intersect.

Drawing the arcs of radius CA=CA', CP=CP',.....we can map all the points of curved surface to the correspon ding points on the circle of radius CA and centre at C. Moreover, this mapping has one -to-one correspondence uniquely.



Thus, all the area of curved surface of hemisphere con verts to area of circle with radius CA= √2 .R.

∴area of curved surface of hemisphere

$A = \pi(\sqrt{2}.R)^2 = 2\pi R^2$.

Area of Complete Sphere[3] $= 4\pi R^2$ ...(9)

6. Volume of pyramid[4]-[6]

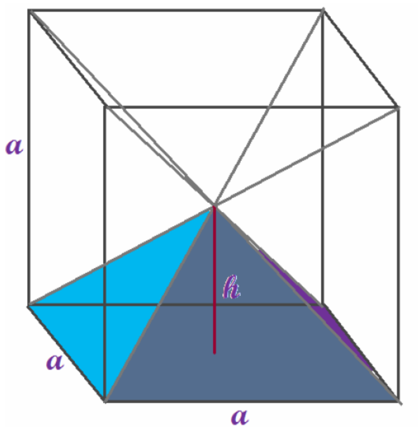
Body diagonals with faces make six pyramids within the cube.. As a general ansatz we assume that the volume must possess area of base and height as factors in the expression.

∴ volume of pyramid =k(area of base)(height)

⇒ volume of six pyramids $=6ka^2h$= volume of cube

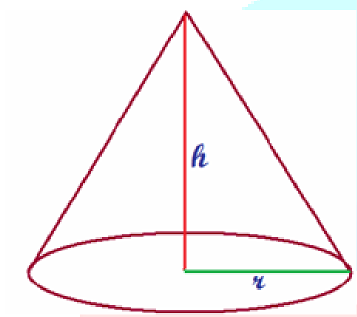
$\Rightarrow 6ka^2 a/2 = a^3 \Rightarrow k = 1/3$

$\therefore$ volume of pyramid =(1/3)(area of base)(height)



7. **Volume of cone**

Now using this idea we find volume of the cone.



$$V = \frac{1}{3}\pi r^2 h$$

...(10)

8. **Volume of sphere**

Now construct cones of infinitesimal base and height equal to radius within the sphere. Sum of the base areas will be equal to area of the sphere and height is the same.

Therefore volume will be V =(1/3)(area of base)(height)

$$V = \frac{1}{3}\left(4\pi r^2\right)r = \frac{4}{3}\pi r^3$$

...(11)

Conclusion:

The transformation method adopted here is extremely useful to simplify the problem.

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