



HEPTAPARTITIONED NEUTROSOPHIC SET

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Abstract: The focus of this paper is to propose a new notion of Heptapartitioned Neutrosophic Set and study some basic properties.

Keywords: neutrosophic set, heptapartitioned neutrosophic set

1 INTRODUCTION

The fuzzy set was introduced by Zadeh [19] in 1965. The concept of Neutrosophic set was introduced by F. Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.

Smarandache in proposed neutrosophic sets [14]. In neutrosophic sets, the indeterminacy membership function walks along independently of the truth membership or of the falsity membership. Neutrosophic theory has been widely explored by researchers for application purpose in handling real life situations involving uncertainty. Although the hesitation margin of neutrosophic theory is independent of the truth or falsity membership, looks more general than intuitionistic fuzzy sets yet. Recently, in Atanassov et al. [3] studied the relations between inconsistent intuitionistic fuzzy sets, picture fuzzy sets, neutrosophic sets and intuitionistic fuzzy sets; however, it remains in doubt that whether the indeterminacy associated to a particular element occurs due to the belongingness of the element or the non-belongingness. This has been pointed out by Chatterjee et al. [4] while introducing a more general structure of neutrosophic set viz. quadripartitioned single valued neutrosophic set (QSVNS). The idea of QSVNS is actually stretched from Smarandache's four numerical-valued neutrosophic logic and Belnap's four valued logic, where the indeterminacy is divided into two parts, namely, "unknown" i.e., neither true nor false and "contradiction" i.e., both true and false. In the context of neutrosophic study however, the QSVNS looks quite logical. Also, in their study, Chatterjee [4] et al. analyzed a real-life example for a better understanding of a QSVNS environment and showed that such situations occur very naturally.

In 2018 Smarandache [17] generalized the Soft Set to the Hyper Soft Set by transforming the classical uni-argument function F into a multi-argument function.

In 2020, Rama Malik [11] introduced Pentapartitioned sets and discussed some of its properties.

In 1995 F. Smarandache [13] introduced Seven Symbol-Valued Neutrosophic Logic as I refined (split) as U , C , G , but T also is refined as T_A = absolute truth and T_R = relative truth, and F is refined as F_A = absolute falsity and F_R = relative falsity. Where: U = neither (T_A or T_R) nor (F_A or F_R) (i.e., undefined); $C = (T_A \text{ or } T_R) \wedge (F_A \text{ or } F_R)$ (i.e., Contradiction), which involves the Extenics; $G = (T_A \text{ or } T_R) \vee (F_A \text{ or } F_R)$ (i.e., Ignorance). All are symbols. But if T_A , T_R , F_A , F_R , U , C , G are subsets of $[0, 1]$, then we get a Seven Numerical valued numerical logic. By using this we can define Heptapartitioned Neutrosophic set and discuss some of its properties.

2 PRELIMINARIES

2.1 Definition [14]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership.

Here, $T_A(x)$ and $F_A(x)$ are dependent neutrosophic components and $I_A(x)$ is an independent component.

2.2 Definition: [4]

Let X be a universe. A Quadripartitioned neutrosophic set A with independent neutrosophic components on X is an object of the form

$$A = \{ \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle : x \in X \}$$

and $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

2.3 Definition [11]

Let P be a non-empty set. A PNS A over P characterizes each element p in P by a truth-membership function T_A , a contradiction membership function C_A , an ignorance membership function G_A , unknown membership function U_A and a falsity membership function F_A , such that for each $p \in P$,

$$0 \leq T_A + C_A + U_A + G_A + F_A \leq 5$$

3 HEPTAPARTITIONED NEUTROSOPHIC SET

3.1 Definition

Let R be a non-empty Universe. A Heptapartitioned neutrosophic set (HNS) A over R characterizes each element r in R by an absolute truth-membership function T_A , a relative truth membership function M_A , a contradiction membership function C_A , an ignorance membership function I_A , an unknown membership function U_A , an absolute falsity membership function F_A and a relative falsity membership function K_A such that for each

$r \in R, T_A, M_A, C_A, U_A, I_A, K_A, F_A \in [0,1]$ and

$$A = [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)) : r \in R]$$

$$0 \leq T_A(r) + M_A(r) + C_A(r) + U_A(r) + I_A(r) + K_A(r) + F_A(r) \leq 7$$

3.2 Definition

A Heptapartitioned neutrosophic set A is said to absolute Heptapartitioned neutrosophic set Δ if and only if its absolute truth-membership, a relative truth membership, a contradiction membership, an ignorance membership, an unknown membership, an absolute falsity membership and a relative falsity membership are defined as follows,

$$T_A(r) = 1, M_A(r) = 1, C_A(r) = 1, U_A(r) = 0, I_A(r) = 0, K_A(r) = 0$$

$$\text{and } F_A(r) = 0$$

3.3 Definition

A Heptapartitioned neutrosophic set A is said to relative Heptapartitioned neutrosophic set \emptyset if and only if its absolute truth-membership, a relative truth membership a contradiction membership, an ignorance

membership, an unknown membership, an absolute falsity membership and a relative falsity membership are defined as follows,

$$T_A(r) = 0, M_A(r) = 0, C_A(r) = 0, U_A(r) = 1, I_A(r) = 1, K_A(r) = 1 \\ \text{and } F_A(r) = 1$$

3.4 Definition

For any two Heptapartitioned neutrosophic sets A and B over R, A is said to be contained in B iff $T_A(r) \leq T_B(r), M_A(r) \leq M_B(r), C_A(r) \leq C_B(r),$

$$U_A(r) \geq U_B(r), I_A(r) \geq I_B(r), K_A(r) \geq K_B(r) \text{ and } F_A(r) \geq F_B(r)$$

3.5 Definition

The complement of Heptapartitioned neutrosophic sets A over the universe R is denoted by A^C and is defined as

$$A^C = [r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): \forall r \in R]$$

3.6 Definition

The union of any two Heptapartitioned neutrosophic sets A and B over R is denoted by $A \cup B$ and is defined as

$$A \cup B = [r, (\max(T_A(r), T_B(r)), \max(M_A(r), M_B(r)), \max(C_A(r), C_B(r)), \min(U_A(r), U_B(r)), \min(I_A(r), I_B(r)), \min(K_A(r), K_B(r)) \text{ and } \min(F_A(r), F_B(r))): r \in R]$$

3.7 Definition

The intersection of any two Heptapartitioned neutrosophic sets A and B over R is denoted by $A \cap B$ and is defined as

$$A \cap B = [r, (\min(T_A(r), T_B(r)), \min(M_A(r), M_B(r)), \min(C_A(r), C_B(r)), \max(U_A(r), U_B(r)), \max(I_A(r), I_B(r)), \max(K_A(r), K_B(r)) \text{ and } \max(F_A(r), F_B(r))): r \in R]$$

3.8 Example

Consider two HNSs over R, given as

$$A = [0.5, 0.6, 0.9, 0.4, 0.2, 0.7, 0.3]/r_1 + [0.1, 0.6, 0.8, 0.4, 0.5, 0.9, 0.4]/r_2 + [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7]/r_3$$

$$B = [0.9, 0.8, 0.2, 0.7, 0.1, 0.6, 0.5]/r_1 + [0.6, 0.2, 0.4, 0.5, 0.9, 0.4, 0.8]/r_2 + [0.9, 0.7, 0.4, 0.5, 0.6, 0.7, 0.8]/r_3$$

$$A^C = [0.3, 0.7, 0.2, 0.6, 0.9, 0.6, 0.5]/r_1 + [0.4, 0.9, 0.5, 0.6, 0.8, 0.6, 0.1]/r_2 + [0.7, 0.6, 0.5, 0.6, 0.3, 0.2, 0.1]/r_3$$

$$A \cup B = [0.9, 0.8, 0.2, 0.4, 0.1, 0.6, 0.3]/r_1 + [0.6, 0.6, 0.8, 0.4, 0.5, 0.4, 0.4]/r_2 + [0.9, 0.7, 0.4, 0.4, 0.5, 0.6, 0.7]/r_3$$

$$A \cap B = [0.5, 0.6, 0.2, 0.7, 0.2, 0.7, 0.5]/r_1 + [0.1, 0.2, 0.4, 0.5, 0.9, 0.9, 0.8]/r_2 + [0.1, 0.2, 0.3, 0.5, 0.6, 0.7, 0.8]/r_3$$

3.9 Theorem

Consider any two HNSs defined over R and HNSs satisfy the following properties under the abovementioned set theoretic operations

❖ Commutative law

a) $A \cup B = B \cup A$

b) $A \cap B = B \cap A$

❖ Associative law

c) $(A \cup B) \cup C = A \cup (B \cup C)$

d) $(A \cap B) \cap C = A \cap (B \cap C)$

❖ Distributive law

e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$f) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

❖ Absorption law

$$g) A \cup (A \cap C) = A$$

$$f) A \cap (A \cup C) = A$$

❖ Involution law

$$i) (A^c)^c = A$$

❖ Law of contradiction

$$j) A \cap A^c = \emptyset$$

❖ De Morgan's law

$$k) (A \cup B)^c = A^c \cap B^c$$

$$l) (A \cap B)^c = A^c \cup B^c$$

Proof:

(a) Let $A = [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]$ and

$$B = [r, (T_B(r), M_B(r), C_B(r), U_B(r), I_B(r), K_B(r), F_B(r)): r \in R]$$

$$A \cup B = [r, (\max(T_A(r), T_B(r)), \max(M_A(r), M_B(r)), \max(C_A(r), C_B(r)), \min(U_A(r), U_B(r)), \min(I_A(r), I_B(r)), \min(K_A(r), K_B(r)) \text{ and } \min(F_A(r), F_B(r)): r \in R]$$

Throughout this paper, we denote $T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r)$ and $F_A(r)$ by $T_A, M_A, C_A, U_A, I_A, K_A$ and F_A .

$$A \cup B = [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B)): r \in R]$$

Let $x_i \in A \cup B$

$$\Rightarrow x_i \in [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B)): r \in R]$$

$$\Rightarrow x_i \in [r, (\max(T_B, T_A), \max(M_B, M_A), \max(C_B, C_A), \min(U_B, U_A), \min(I_B, I_A), \min(K_B, K_A) \text{ and } \min(F_B, F_A)): r \in R]$$

$$\Rightarrow x_i \in B \cup A$$

$$\Rightarrow A \cup B \subset B \cup A \quad \text{----- (1)}$$

Let $y_i \in B \cup A$

$$\Rightarrow y_i \in [r, (\max(T_B, T_A), \max(M_B, M_A), \max(C_B, C_A), \min(U_B, U_A), \min(I_B, I_A), \min(K_B, K_A) \text{ and } \min(F_B, F_A)): r \in R]$$

$$\Rightarrow y_i \in [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B)): r \in R]$$

$$\Rightarrow y_i \in A \cup B$$

$$\Rightarrow B \cup A \subset A \cup B \quad \text{----- (2)}$$

Therefore, from (1) and (2) we obtain

$$A \cup B = B \cup A$$

(b) Similarly, we can prove $A \cap B = B \cap A$

(c) Let $x_i \in A \cup (B \cup C)$

$\Rightarrow x_i \in A \cup [r, (\max(T_B, T_C), \max(M_B, M_C), \max(C_B, C_C), \min(U_B, U_C), \min(I_B, I_C), \min(K_B, K_C) \text{ and } \min(F_B, F_C))): r \in R]$

$\Rightarrow x_i \in [r, (\max(T_A, T_B, T_C), \max(M_A, M_B, M_C), \max(C_A, C_B, C_C), \min(U_A, U_B, U_C), \min(I_A, I_B, I_C), \min(K_A, K_B, K_C) \text{ and } \min(F_A, F_B, F_C))): r \in R]$

$\Rightarrow x_i \in [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B))): r \in R] \cup C$

$\Rightarrow A \cup (B \cup C) \subset (A \cup B) \cup C$ ----- (3)

Let $y_i \in (A \cup B) \cup C$

$\Rightarrow y_i \in [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B))): r \in R] \cup C$

$\Rightarrow y_i \in [r, (\max(T_A, T_B, T_C), \max(M_A, M_B, M_C), \max(C_A, C_B, C_C), \min(U_A, U_B, U_C), \min(I_A, I_B, I_C), \min(K_A, K_B, K_C) \text{ and } \min(F_A, F_B, F_C))): r \in R]$

$\Rightarrow y_i \in A \cup [r, (\max(T_B, T_C), \max(M_B, M_C), \max(C_B, C_C), \min(U_B, U_C), \min(I_B, I_C), \min(K_B, K_C) \text{ and } \min(F_B, F_C))): r \in R]$

$\Rightarrow y_i \in A \cup (B \cup C)$

$(A \cup B) \cup C \subset A \cup (B \cup C)$ ----- (4)

Therefore, from (3) and (4) we obtain

$A \cup (B \cup C) = (A \cup B) \cup C$

(d) Similarly, we can prove that

$A \cap (B \cap C) = (A \cap B) \cap C$

(e) Let $x_i \in A \cup (B \cap C)$

$\Rightarrow x_i \in A \cup [r, (\min(T_B, T_C), \min(M_B, M_C), \min(C_B, C_C), \max(U_B, U_C), \max(I_B, I_C), \max(K_B, K_C) \text{ and } \max(F_B, F_C))): r \in R]$

$\Rightarrow x_i \in [r, (\max(T_A, (\min(T_B, T_C))), \max(M_A, \min(M_B, M_C)), \max(C_A, \min(C_B, C_C)), \min(U_A, \max(U_B, U_C)), \min(I_A, \max(I_B, I_C)), \min(K_A, \max(K_B, K_C)) \text{ and } \min(F_A, \max(F_B, F_C))): r \in R]$

$\Rightarrow x_i \in [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B))): r \in R] \cap [r, (\max(T_A, T_C), \max(M_A, M_C), \max(C_A, C_C), \min(U_A, U_C), \min(I_A, I_C), \min(K_A, K_C) \text{ and } \min(F_A, F_C))): r \in R]$

$\Rightarrow x_i \in (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ -----(5)

Assume that $y_i \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y_i \in [r, (\max(T_A, T_B), \max(M_A, M_B), \max(C_A, C_B), \min(U_A, U_B), \min(I_A, I_B), \min(K_A, K_B) \text{ and } \min(F_A, F_B))): r \in R] \cap [r, (\max(T_A, T_C), \max(M_A, M_C), \max(C_A, C_C), \min(U_A, U_C), \min(I_A, I_C), \min(K_A, K_C) \text{ and } \min(F_A, F_C))): r \in R]$

$\Rightarrow y_i \in [r, (\max(T_A, (\min(T_B, T_C))), \max(M_A, \min(M_B, M_C)), \max(C_A, \min(C_B, C_C)), \min(U_A, \max(U_B, U_C)), \min(I_A, \max(I_B, I_C)), \min(K_A, \max(K_B, K_C)) \text{ and } \min(F_A, \max(F_B, F_C))): r \in R]$

$\Rightarrow y_i \in A \cup [r, (\min(T_B, T_C), \min(M_B, M_C), \min(C_B, C_C), \max(U_B, U_C), \max(I_B, I_C), \max(K_B, K_C) \text{ and } \max(F_B, F_C)): r \in R]$

$\Rightarrow y_i \in A \cup (B \cap C)$

$(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ -----(6)

From (5) and (6), we conclude that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(f) similarly, we can prove

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) Let $x_i \in A \cup (A \cap C)$

$\Rightarrow x_i \in A \cup [r, (\min(T_A, T_C), \min(M_A, M_C), \min(C_A, C_C), \max(U_A, U_C), \max(I_A, I_C), \max(K_A, K_C) \text{ and } \max(F_A, F_C)): r \in R]$

$\Rightarrow x_i \in [r, (\max(T_A, (\min(T_A, T_C))), \max(M_A, \min(M_A, M_C)), \max(C_A, \min(C_A, C_C)), \min(U_A, \max(U_A, U_C)), \min(I_A, \max(I_A, I_C)), \min(K_A, \max(K_A, K_C)) \text{ and } \min(F_A, \max(F_A, F_C)): r \in R]$

$\Rightarrow x_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]$

$\Rightarrow x_i \in A$

$\Rightarrow A \cup (A \cap C) \subset A$ -----(7)

Assume that $y_i \in A$

$\Rightarrow y_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]$

$\Rightarrow y_i \in [r, (\max(T_A, (\min(T_A, T_C))), \max(M_A, \min(M_A, M_C)), \max(C_A, \min(C_A, C_C)), \min(U_A, \max(U_A, U_C)), \min(I_A, \max(I_A, I_C)), \min(K_A, \max(K_A, K_C)) \text{ and } \min(F_A, \max(F_A, F_C)): r \in R]$

$\Rightarrow y_i \in A \cup [r, (\min(T_A, T_C), \min(M_A, M_C), \min(C_A, C_C), \max(U_A, U_C), \max(I_A, I_C), \max(K_A, K_C) \text{ and } \max(F_A, F_C)): r \in R]$

$\Rightarrow y_i \in A \cup (A \cap C)$

$\Rightarrow A \subset A \cup (A \cap C)$ -----(8)

From (7) and (8) we conclude that

$A \cup (A \cap C) = A$

(h) similarly, we can prove that

$A \cap (A \cup C) = A$

(i) $A = [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]$

$A^C = [r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R]$

Let $x_i \in (A^C)^C$

$\Rightarrow x_i \in [r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R]^C$

$\Rightarrow x_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]$

$\Rightarrow x_i \in A$

$\Rightarrow (A^C)^C \subset A$ -----(9)

Assume that $y_i \in A$

$\Rightarrow y_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]$

$\Rightarrow y_i \in [r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R]^C$

$$\Rightarrow y_i \in (A^c)^c$$

$$\Rightarrow A \subset (A^c)^c \quad \text{-----(10)}$$

From (9) and (10), we get

$$(A^c)^c = A$$

(j) Let $x_i \in A \cap A^c$

$$\Rightarrow x_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R] \cap [(r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R]$$

$$\Rightarrow x_i \in [r, (\min(T_A, F_A), \min(M_A, K_A), \min(C_A, I_A), \max(U_A, 1 - U_A), \max(I_A, C_A), \max(K_A, M_A) \text{ and } \max(F_A, T_A)): r \in R]$$

$$\Rightarrow x_i \in \phi$$

$$\Rightarrow A \cap A^c \subset \phi \quad \text{-----(11)}$$

Assume that $y_i \in \phi$

$$\Rightarrow y_i \in [r, (\min(T_A, F_A), \min(M_A, K_A), \min(C_A, I_A), \max(U_A, 1 - U_A), \max(I_A, C_A), \max(K_A, M_A), \max(F_A, T_A)): r \in R]$$

$$\Rightarrow y_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R] \cap [(r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R]$$

$$\Rightarrow y_i \in A \cap A^c$$

$$\Rightarrow \phi \subset A \cap A^c \quad \text{-----(12)}$$

From (11) and (12), we get

$$A \cap A^c = \phi$$

(k) Let $x_i \in (A \cup B)^c$

$$\Rightarrow x_i \in [r, (\max(T_A(r), T_B(r)), \max(M_A(r), M_B(r)), \max(C_A(r), C_B(r)), \min(U_A(r), U_B(r)), \min(I_A(r), I_B(r)), \min(K_A(r), K_B(r)), \min(F_A(r), F_B(r)): r \in R]^c$$

$$\Rightarrow x_i \in [r, (\min(F_A(r), F_B(r)), \min(K_A(r), K_B(r)), \min(I_A(r), I_B(r)), \max(1 - U_A(r), 1 - U_B(r)), \max(C_A(r), C_B(r)), \max(M_A(r), M_B(r)), (\max(T_A(r), T_B(r)): r \in R]$$

$$\Rightarrow x_i \in [(r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R] \cap [(r, (F_B(r), K_B(r), I_B(r), 1 - U_B(r), C_B(r), M_B(r), T_B(r)): r \in R]$$

$$\Rightarrow x_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]^c \cap$$

$$[r, (T_B(r), M_B(r), C_B(r), U_B(r), I_B(r), K_B(r), F_B(r)): r \in R]^c$$

$$\Rightarrow x_i \in A^c \cap B^c$$

$$\Rightarrow (A \cup B)^c \subset A^c \cap B^c \quad \text{-----(13)}$$

Again, Assume that $y_i \in A^c \cap B^c$

$$\Rightarrow y_i \in [r, (T_A(r), M_A(r), C_A(r), U_A(r), I_A(r), K_A(r), F_A(r)): r \in R]^c \cap$$

$$[r, (T_B(r), M_B(r), C_B(r), U_B(r), I_B(r), K_B(r), F_B(r)): r \in R]^c$$

$$\Rightarrow y_i \in [(r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R] \cap [(r, (F_B(r), K_B(r), I_B(r), 1 - U_B(r), C_B(r), M_B(r), T_B(r)): r \in R]$$

$$\Rightarrow y_i \in [r, (\min(F_A(r), F_B(r)), \min(K_A(r), K_B(r)),$$

$$\min(I_A(r), I_B(r)), \max(1 - U_A(r), 1 - U_B(r)), \max(C_A(r), C_B(r)), \max(M_A(r), M_B(r)), (\max(T_A(r), T_B(r))): r \in R]$$

$$\Rightarrow y_i \in [r, (\max(T_A(r), T_B(r)), \max(M_A(r), M_B(r)), \max(C_A(r), C_B(r)), \min(U_A(r), U_B(r)), \min(I_A(r), I_B(r)), \min(K_A(r), K_B(r)), \min(F_A(r), F_B(r))): r \in R]^C$$

$$\Rightarrow y_i \in (A \cup B)^C$$

$$\Rightarrow A^C \cap B^C \subset (A \cup B)^C \quad \text{-----(14)}$$

From (13) and (14), we conclude that

$$(A \cup B)^C = A^C \cap B^C$$

$$(i) \text{ Let } x_i \in (A \cap B)^C$$

$$\Rightarrow x_i \in [r, (\min(T_A, T_C), \min(M_A, M_C), \min(C_A, C_C), \max(U_A, U_C), \max(I_A, I_C), \max(K_A, K_C) \text{ and } \max(F_A, F_C)): r \in R]^C$$

$$\Rightarrow x_i \in [r, (\max(F_A(r), F_B(r)), \max(K_A(r), K_B(r)), \max(I_A(r), I_B(r)), \min(1 - U_A(r), 1 - U_B(r)), \min(C_A(r), C_B(r)), \min(M_A(r), M_B(r)), (\min(T_A(r), T_B(r))): r \in R]$$

$$\Rightarrow x_i \in [(r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R] \cup [(r, (F_B(r), K_B(r), I_B(r), 1 - U_B(r), C_B(r), M_B(r), T_B(r)): r \in R]$$

$$\Rightarrow x_i \in A^C \cup B^C$$

$$\Rightarrow (A \cap B)^C \subset A^C \cup B^C \quad \text{-----(15)}$$

Again, Assume that $y_i \in A^C \cup B^C$

$$\Rightarrow x_i \in [(r, (F_A(r), K_A(r), I_A(r), 1 - U_A(r), C_A(r), M_A(r), T_A(r)): r \in R] \cup [(r, (F_B(r), K_B(r), I_B(r), 1 - U_B(r), C_B(r), M_B(r), T_B(r)): r \in R]$$

$$\Rightarrow x_i \in [r, (\max(F_A(r), F_B(r)), \max(K_A(r), K_B(r)), \max(I_A(r), I_B(r)), \min(1 - U_A(r), 1 - U_B(r)), \min(C_A(r), C_B(r)), \min(M_A(r), M_B(r)), (\min(T_A(r), T_B(r))): r \in R]$$

$$\Rightarrow x_i \in [r, (\min(T_A, T_C), \min(M_A, M_C), \min(C_A, C_C), \max(U_A, U_C), \max(I_A, I_C), \max(K_A, K_C) \text{ and } \max(F_A, F_C)): r \in R]^C$$

$$\Rightarrow A^C \cup B^C \subset (A \cap B)^C \quad \text{-----(16)}$$

From (15) and (16), we conclude that

$$(A \cap B)^C = A^C \cup B^C$$

Hence the proof.

4 Conclusion

In this article we have develop Heptapartitioned neutrosophic set. The Heptapartitioned neutrosophic set is an extension of Pentapartitioned neutrosophic set. The concept of complement law, inclusion law, union law, commutative law, etc. have been defined on Heptapartitioned neutrosophic set. Future works may comprise of the study of different types of operators on heptapartitioned neutrosophic sets dealing with actual problems and implementing them in decision making problems.

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