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CONSTRUCTION OF FUZZY CONTROL CHART FOR NUMBER OF DEFECTS USING PROCESS CAPABILITY

[1]S.Prabu and [2]Dr.C.Nanthakumar

[1]Ph.D Research Scholar, Department of Statistics, Salem Sowdeswari College, Salem – 636010.

[2]Assoicate Professor and Head, Department of Statistics, Salem Sowdeswari College, Salem – 636010.

Abstract

A control chart is a tool for representing and to monitoring a process. A control chart also detects shifts in a process and abnormal conditions in a process. If a process is monitored by control chart for number of defects (c), 'c' control charts may not be applicable due to the uncertainty of the attribute data. Many articles about fuzzy control charts, which are constructed based on information techniques with type-1 fuzzy sets, exist in literature. The fuzzy control chart based on the number of defects using process capability is constructed in this paper for the first time and the proposed control chart is applied to real world data.

Keywords: Fuzzy, Fuzzy control chart and Process capability.

1. Introduction

The control chart originated in the early 1920s, it has become a powerful tool in statistical process control (SPC) that is the most widely used in industrial processes. Shewhart (1931) control charts are designed to monitor the process of change in mean and variance; they also reflect the ability of the process. Control charts have two types: variable and attribute. Techniques of statistical process control are widely used by the manufacturing industry to detect and eliminate defects during production. Control chart technique is well-known as a key step in production process monitoring. The control chart has a major function in detecting the occurrence of assignable causes, so that the necessary correction can be made before non-conforming products are manufactured in a large amount (Rungsarit Intaramo, 2012). The control chart technique may be considered as both the graphical expression and operation of statistical hypothesis testing. It is recommended that if a control chart is employed to monitor process, some test parameters should be determined such as the sample size, the sampling interval between successive samples, and the control limits or critical regions of the chart. SPC is an efficient technique for improvement of a firm's quality and productivity. The main objective of SPC is similar to that of the control chart technique, that is, to rapidly examine the occurrence of assignable causes or process shifts.

Roland and Wang (2000) introduced fuzzy SPC theory based on the application of fuzzy logic to the SPC-zone rules. El-Shal and Morris (2000) modified SPC-zone rules to reduce false alarm and detect the real error. Zarandi *et.al.*, (2008) presented a new hybrid method based on a combination of fuzzified sensitivity criteria and fuzzy adaptive sampling rules to determine the sample size and sample interval of the control charts in order to determine the sample size and sample interval of the control charts. In fact, the problem with control charts is caused by uncertain data i.e. human, measurement devices or environmental conditions. The studies of A. Pongpullponsak, W. Suracherkiati and and R. Intaramo (2006) are important as they indicate the ambiguity data of the chart. Thus, fuzzy set theory is useful in helping to solve the problems caused by uncertain data by applying fuzzy to EV theory to develop a new chart (FEV), in order to control and improve process efficiency at its best. It was discovered by Senturk and Erginel (2009) that control charts could be used to solve the problem of uncertain data by using fuzzy theory. This research paper is summarized as the theoretical structure of fuzzy rule with control chart using process capability is given below with an illustration.

2. Methods and materials

The quality characteristic is represented as a qualitative form. Attribute control charts are used to evaluate the process for example, p (fraction of nonconforming), np (nonconforming units), c (number of nonconformities) and u (nonconformities per unit) control charts. The classical c-control chart limits proposed by W.A.Shewhart (1931) are given in the following equations.

A control chart for nonconformities with 3-sigma limits is defined as follows:

$$UCL_c = c + 3\sqrt{c}$$
$$CL_c = c$$
$$LCL_c = c - 3\sqrt{c}$$

Where UCL is the upper control limit, CL is the center line and LCL is the lower control limit of 'c' control chart.

If 'c' is not known from the population, 'c' can be estimated from the sample, like;

$$E(c) = \bar{c}$$

and

$$\bar{c} = \frac{\sum_{i=1}^{m} c_i}{m}$$

Where the expected value of 'c' equals to the mean of the nonconformities in sample.

Fuzzy set theory is very helpful for dealing with the kind of vagueness of human thought and language found in a Statistical process control. In this study, a number of nonconformities will be expressed using triangular fuzzy numbers (TFN). Let 'U' be the universe of discourse, U=[0,u].The triangular fuzzy number is defined as $\tilde{A} = (\alpha_m, \alpha_l, \alpha_r)$ also is formulated;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \le \alpha_m - \alpha_l \\ 1 + \dfrac{x - \alpha_m}{\alpha_l} & ; \alpha_m - \alpha_1 \le x \le \alpha_m \\ 1 - \dfrac{x - \alpha_m}{\alpha_r} & ; \alpha_m \le x \le \alpha_m + \alpha_r \\ 0 & ; x \ge \alpha_m + \alpha_r \end{cases}$$

Where α_m is the center (mode); α_l is left spread; α_r is right spread.

The demonstration of triangular fuzzy numbers will be as $\tilde{A} = (\alpha_m - \alpha_l; \alpha_m; \alpha_m + \alpha_r) = (a_l, a_m, a_r)$ and it is shown in Figure 1.

Fuzzy numbers (a_l, a_m, a_r) are represented as $(c_{a_{l_j}}, c_{a_{m_j}}, c_{a_{r_j}})$ for each fuzzy observation on the number of nonconformities control chart. The center line of number of nonconformities control chart $C\tilde{L}$, is mean of fuzzy samples, and it is shown as $(\bar{c}_{a_{l_j}}, \bar{c}_{a_{m_j}}, \bar{c}_{a_{r_j}})$ are the fuzzy averages of the number of nonconformities.

$$\bar{c}_{a_l} = \frac{\sum_{j=1}^{m} c_{a_{l\,j}}}{m}, \bar{c}_{a_m} = \frac{\sum_{j=1}^{m} c_{a_{m\,j}}}{m} \text{ and } \bar{c}_{a_r} = \frac{\sum_{j=1}^{m} c_{a_{r\,j}}}{m}$$

Where j=1,2,...,m.

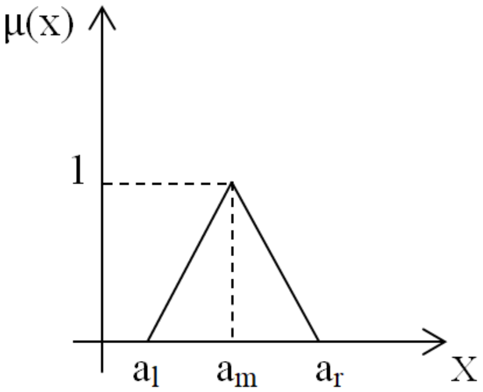

Figure 1: Representation of a sample by triangular fuzzy numbers TFN case

a. Fuzzy $\tilde{c}$ -control chart for Triangular fuzzy number

By considering the formulations of c-control limits and fuzzy numbers based on triangular membership functions, the fuzzy center line and the fuzzy upper and fuzzy lower limits of the fuzzy rule $\tilde{c}$ -control chart are given as follows:

$$\left(U\tilde{C}L_{c_{a_l}}, U\tilde{C}L_{c_{a_m}}, U\tilde{C}L_{c_{a_r}}\right) = \left(\bar{c}_{a_l} + 3\sqrt{\bar{c}_{a_l}}, \bar{c}_{a_m} + 3\sqrt{\bar{c}_{a_m}}, \bar{c}_{a_r} + 3\sqrt{\bar{c}_{a_r}}\right)$$

$$\left(\tilde{C}L_{c_{a_l}}, \tilde{C}L_{c_{a_m}}, \tilde{C}L_{c_{a_r}}\right) = \left(\bar{c}_{a_l}, \bar{c}_{a_m}, \bar{c}_{a_r}\right)$$

$$\left(L\tilde{C}L_{c_{a_l}}, L\tilde{C}L_{c_{a_m}}, L\tilde{C}L_{c_{a_r}}\right) = \left(\bar{c}_{a_l} - 3\sqrt{\bar{c}_{a_l}}, \bar{c}_{a_m} - 3\sqrt{\bar{c}_{a_m}}, \bar{c}_{a_r} - 3\sqrt{\bar{c}_{a_r}}\right)$$

The fuzzy control limits are defined for a fuzzy rule $\tilde{c}$ -control chart for a TFN case. The proposed standard deviation ($\tilde{\sigma}_{i.cF-C_p}$, $i = l, m, r$) for fuzzy $\tilde{c}$ -control chart with the help of process capability $C_p = \dfrac{USL_{i.cF-C_p} - LSL_{i.cF-C_p}}{6\sigma}, i = l, m, r$ using a JAVA script (Radhakrishnan and Balamurugan, 2011) is to calculate by the specified tolerance level from the relation $\dfrac{\sum_{j=1}^{m} c_{i_j}}{m}$, $i = l, m, r$ *and* $j = 1, 2, \ldots m.$

Therefore the resultant of proposed fuzzy control limits for $\tilde{c}$ using process capability is given below:

$$\left(U\tilde{C}L_{c_{a_l}-C_p}, U\tilde{C}L_{c_{a_m}-C_p}, U\tilde{C}L_{c_{a_r}-C_p}\right) = \left(\bar{c}_{a_l-C_p} + 3\tilde{\sigma}_{l.cF-C_p}, \bar{c}_{a_m-C_p} + 3\tilde{\sigma}_{m.cF-C_p}, \bar{c}_{a_r-C_p} + 3\tilde{\sigma}_{r.cF-C_p}\right)$$

$$\left(\tilde{C}L_{c_{a_l}-C_p}, \tilde{C}L_{c_{a_m}-C_p}, \tilde{C}L_{c_{a_r}-C_p}\right) = \left(\bar{c}_{a_l-C_p}, \bar{c}_{a_m-C_p}, \bar{c}_{a_r-C_p}\right)$$

$$\left(L\tilde{C}L_{c_{a_l}-C_p}, L\tilde{C}L_{c_{a_m}-C_p}, L\tilde{C}L_{c_{a_r}-C_p}\right) = \left(\bar{c}_{a_l-C_p} - 3\tilde{\sigma}_{l.cF-C_p}, \bar{c}_{a_m-C_p} - 3\tilde{\sigma}_{m.cF-C_p}, \bar{c}_{a_r-C_p} - 3\tilde{\sigma}_{r.cF-C_p}\right)$$

The α-cut control limits are also fuzzy sets which could be showed by triangular fuzzy number and the value of α-cut is determined based on the tightness of inspection, we can use a value near 1 for α.
The fuzzy $\tilde{c}$ -control limits using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}_{c_{a_l}}, U\tilde{C}L_{c_{a_m}}, U\tilde{C}L^{\alpha}_{c_{a_r}}\right) = \left(\bar{c}^{\alpha}_{a_l} + 3\sqrt{\bar{c}^{\alpha}_{a_l}}, \bar{c}_{a_m} + 3\sqrt{\bar{c}_{a_m}}, \bar{c}^{\alpha}_{a_r} + 3\sqrt{\bar{c}^{\alpha}_{a_r}}\right)$$

$$\left(\tilde{C}L^{\alpha}_{c_{a_l}}, \tilde{C}L_{c_{a_m}}, \tilde{C}L^{\alpha}_{c_{a_r}}\right) = \left(\bar{c}^{\alpha}_{a_l}, \bar{c}_{a_m}, \bar{c}^{\alpha}_{a_r}\right)$$

$$\left(L\tilde{C}L^{\alpha}_{c_{a_l}}, L\tilde{C}L_{c_{a_m}}, L\tilde{C}L^{\alpha}_{c_{a_r}}\right) = \left(\bar{c}^{\alpha}_{a_l} - 3\sqrt{\bar{c}^{\alpha}_{a_l}}, \bar{c}_{a_m} - 3\sqrt{\bar{c}_{a_m}}, \bar{c}^{\alpha}_{a_r} - 3\sqrt{\bar{c}^{\alpha}_{a_r}}\right)$$

Where

$c^{\alpha}_{a_l} = c_{a_l} + \alpha\left(c_{a_m} - c_{a_l}\right)$ and $c^{\alpha}_{a_r} = c_{a_r} + \alpha\left(c_{a_r} - c_{a_m}\right)$

The proposed standard deviation ($\tilde{\sigma}^{\alpha}_{a_l.cF-C_p}$ and $\tilde{\sigma}^{\alpha}_{a_r.cF-C_p}$) for fuzzy $\tilde{c}$-control chart with the help of process capability $C_p = \dfrac{USL^{\alpha}_{i.cF-C_p} - LSL^{\alpha}_{i.cF-C_p}}{6\sigma} r = a_l$ and a_r, using α-cut method is to calculate by the specified tolerance level from the relation

$$\frac{\sum_{j=1}^{m} c_{a_{l\,j}}}{m} + \alpha\left(\frac{\sum_{j=1}^{m} c_{a_{m\,j}}}{m} - \frac{\sum_{j=1}^{m} c_{a_{l\,j}}}{m}\right) \text{ for } \tilde{\sigma}^{\alpha}_{a_l.cF-Cp}$$

$$\text{and } \frac{\sum_{j=1}^{m} c_{a_{r\,j}}}{m} + \alpha\left(\frac{\sum_{j=1}^{m} c_{a_{r\,j}}}{m} - \frac{\sum_{j=1}^{m} c_{a_{m\,j}}}{m}\right) \text{ for } \tilde{\sigma}^{\alpha}_{a_r.cF-Cp},\ j = 1,2,\ldots m.$$

The proposed fuzzy $\tilde{c}$-control limits with process capability using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}{}_{c_{a_l}-C_p}, U\tilde{C}L_{c_{a_m}-C_p}, U\tilde{C}L^{\alpha}{}_{c_{a_r}-C_p}\right) = \left(\bar{c}^{\alpha}_{a_l-C_p} + 3\tilde{\sigma}^{\alpha}_{l.cF-C_p}, \bar{c}_{a_m-C_p} + 3\tilde{\sigma}_{m.cF-C_p}, \bar{c}^{\alpha}_{a_r-C_p} + 3\tilde{\sigma}^{\alpha}{}_{r.cF-C_p}\right)$$

$$\left(\tilde{C}L^{\alpha}{}_{c_{a_l}-C_p}, \tilde{C}L_{c_{a_m}-C_p}, \tilde{C}L^{\alpha}{}_{c_{a_r}-C_p}\right) = \left(\bar{c}^{\alpha}_{a_l-C_p}, \bar{c}_{a_m-C_p}, \bar{c}^{\alpha}_{a_r-C_p}\right)$$

$$\left(L\tilde{C}L^{\alpha}{}_{c_{a_l}-C_p}, L\tilde{C}L_{c_{a_m}-C_p}, L\tilde{C}L^{\alpha}{}_{c_{a_r}-C_p}\right) = \left(\bar{c}^{\alpha}_{a_l-C_p} - 3\tilde{\sigma}^{\alpha}_{l.cF-C_p}, \bar{c}_{a_m-C_p} - 3\tilde{\sigma}_{m.cF-C_p}, \bar{c}^{\alpha}_{a_r-C_p} - 3\tilde{\sigma}^{\alpha}{}_{r.cF-C_p}\right)$$

In this research article, the α-level fuzzy midrange transformation technique issued for the construction of fuzzy attribute control charts with the help of process capability based on fuzzy trapezoidal number.

The control limits of α-level fuzzy midrange for α-cut fuzzy $\tilde{c}$-control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{c.mid} = \left(\frac{\bar{c}^{\alpha}_{a_l} + \bar{c}^{\alpha}_{a_r}}{2}\right) + \left[3\sqrt{\left(\frac{\bar{c}^{\alpha}_{a_l} + \bar{c}^{\alpha}_{a_r}}{2}\right)}\right]$$

$$C\tilde{L}^{\alpha}_{c.mid} = \left(\frac{\bar{c}^{\alpha}_{a_l} + \bar{c}^{\alpha}_{a_r}}{2}\right)$$

$$L\tilde{C}L^{\alpha}{}_{c.mid} = \left(\frac{\bar{c}^{\alpha}_{a_l} + \bar{c}^{\alpha}_{a_r}}{2}\right) - \left[3\sqrt{\left(\frac{\bar{c}^{\alpha}_{a_l} + \bar{c}^{\alpha}_{a_r}}{2}\right)}\right]$$

The definition of α-level fuzzy midrange of sample j for fuzzy $\tilde{c}$-control chart is

$$S^{\alpha}_{j:c.mid} = \frac{\left(c_{a_{l\,j}} + c_{a_{r\,j}}\right) + \alpha\left[\left(c_{a_{m\,j}} - c_{a_{l\,j}}\right) - \left(c_{a_{r\,j}} - c_{a_{m\,j}}\right)\right]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ;\ L\tilde{C}L^{\alpha}{}_{\tilde{c}.mid} \le S^{\alpha}_{j:\tilde{c}.mid} \le U\tilde{C}L^{\alpha}{}_{\tilde{c}.mid} \\ \text{Out-of-control} & ;\ \text{Otherwise} \end{cases}$$

The proposed standard deviation ($\tilde{\sigma}^{\alpha}_{Mid.cF-C_p}$) for α-level fuzzy midrange at $\tilde{c}$-control chart with the help of process capability $C_p = \dfrac{USL^{\alpha}_{Mid.cF-C_p} - LSL^{\alpha}_{Mid.cF-C_p}}{6\sigma}$ using α-cut method is to calculate by the specified tolerance level from the relation

$$\left\{\frac{\sum_{j=1}^{m} c_{a_{l\,j}}}{m}+\alpha\left(\frac{\sum_{j=1}^{m} c_{a_{m\,j}}}{m}-\frac{\sum_{j=1}^{m} c_{a_{l\,j}}}{m}\right)\right\}+\left\{\frac{\sum_{j=1}^{m} c_{a_{r\,j}}}{m}+\alpha\left(\frac{\sum_{j=1}^{m} c_{a_{r\,j}}}{m}-\frac{\sum_{j=1}^{m} c_{m_j}}{m}\right)\right\} \text{ for } \tilde{\sigma}^{\alpha}_{Mid.cF-Cp},\ ,\ j=1,2,\ldots m.$$

The proposed control limits using process capability of α-level fuzzy midrange for α-cut fuzzy $\tilde{c}$ - control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{c.mid:C_p}=\left(\frac{c^{\alpha}_{a_l}+c^{\alpha}_{a_r}}{2}\right)+\left[3\tilde{\sigma}^{\alpha}_{Mid.cF-C_p}\right]$$

$$C\tilde{L}^{\alpha}_{c.mid:C_p}=\left(\frac{c^{\alpha}_{a_l}+c^{\alpha}_{a_r}}{2}\right)$$

$$L\tilde{C}L^{\alpha}{}_{c.mid:C_p}=\left(\frac{c^{\alpha}_{a_l}+c^{\alpha}_{a_r}}{2}\right)-\left[3\tilde{\sigma}^{\alpha}_{Mid.cF-C_p}\right]$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control}=\begin{cases}\text{in control} & ;\ L\tilde{C}L^{\alpha}{}_{c.mid:C_p}\le S^{\alpha}_{j:c.mid}\le U\tilde{C}L^{\alpha}{}_{c.mid:C_p}\\ \text{Out-of-control} & ;\ \text{Otherwise}\end{cases}$$

b. Illustration

The example provided by Montgomery (2008, Page No. Page No. 277) is considered here. The Table 1 presents the number of defects observed in 26 successive samples of 100 printed Circuit boards.

Table 1: Data on the Number of defects in samples of 100 printed Circuit boards

Sample Number	Number of defects	Sample Number	Number of defects	Sample Number	Number of defects
1	21	11	20	21	30
2	24	12	24	22	24
3	16	13	16	23	16
4	12	14	19	24	19
5	15	15	10	25	17
6	5	16	17	26	15
7	28	17	13		
8	20	18	22		
9	31	19	18		
10	25	20	39		

Table 2: Triangular fuzzy numbers for fuzzy $\tilde{c}$ -control chart

Sample	a_l	a_m	a_r
1	15	21	24
2	23	24	30
3	12	16	17
4	10	12	14
5	11	15	17
6	4	5	10
7	26	28	31
8	15	20	22
9	27	31	32
10	22	25	28
11	14	20	23
12	18	24	27
13	14	16	21
14	15	19	20
15	8	10	16
16	10	17	20
17	8	13	15
18	20	22	27
19	14	18	19
20	33	39	42
21	26	30	31
22	18	24	22
23	10	16	19
24	20	19	22
25	20	17	18
26	13	15	20

$\bar{c}_{a_l} = 16.38, \bar{c}_{a_m} = 19.85$ and $\bar{c}_{a_r} = 22.58$

The fuzzy center line and the fuzzy upper and fuzzy lower limits of the fuzzy rule $\tilde{c}$ -control chart are given as follows:

$$\left(U\tilde{C}L_{c_{a_l}}, U\tilde{C}L_{c_{a_m}}, U\tilde{C}L_{c_{a_r}}\right) = \left(16.38+3\sqrt{16.38}, 19.85+3\sqrt{19.85}, 22.58+3\sqrt{22.58}\right)$$

$$= \left(28.5, 33.2, 36.8\right)$$

$$\left(\tilde{C}L_{c_{a_l}}, \tilde{C}L_{c_{a_m}}, \tilde{C}L_{c_{a_r}}\right) = \left(16.38, 19.85, 22.58\right)$$

$$\left(L\tilde{C}L_{c_{a_l}}, L\tilde{C}L_{c_{a_m}}, L\tilde{C}L_{c_{a_r}}\right) = \left(16.38-3\sqrt{16.38}, 19.85-3\sqrt{19.85}, 22.58-3\sqrt{22.58}\right)$$

$$= \left(4.2, 6.5, 8.3\right)$$

The resultant of proposed fuzzy control limits for $\tilde{c}$ using process capability is given below:

$$\left(U\tilde{C}L_{c_{a_l}-C_p}, U\tilde{C}L_{c_{a_m}-C_p}, U\tilde{C}L_{c_{a_r}-C_p}\right) = \left[16.38+(3\times 0.312), 19.85+(3\times 0.334), 22.58+(3\times 0.277)\right]$$
$$=(17.3, 20.8, 23.4)$$
$$\left(\tilde{C}L_{c_{a_l}-C_p}, \tilde{C}L_{c_{a_m}-C_p}, \tilde{C}L_{c_{a_r}-C_p}\right) = (16.38, 19.85, 22.58)$$
The fuzzy
$$\left(L\tilde{C}L_{c_{a_l}-C_p}, L\tilde{C}L_{c_{a_m}-C_p}, L\tilde{C}L_{c_{a_r}-C_p}\right) = \left[16.38-(3\times 0.312), 19.85-(3\times 0.334), 22.58-(3\times 0.277)\right]$$
$$=(15.4, 18.8, 21.7)$$

$\tilde{c}$ -control limits using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}{}_{c_{a_l}}, U\tilde{C}L_{c_{a_m}}, U\tilde{C}L^{\alpha}{}_{c_{a_r}}\right) = \left(18.63+3\sqrt{18.63}, 19.85+3\sqrt{19.85}, 24.35+3\sqrt{24.35}\right)$$
$$=(31.6, 33.2, 39.2)$$
$$\left(\tilde{C}L^{\alpha}{}_{c_{a_l}}, \tilde{C}L_{c_{a_m}}, \tilde{C}L^{\alpha}{}_{c_{a_r}}\right) = (18.63, 19.85, 24.35)$$
$$\left(L\tilde{C}L^{\alpha}{}_{c_{a_l}}, L\tilde{C}L_{c_{a_m}}, L\tilde{C}L^{\alpha}{}_{c_{a_r}}\right) = \left(18.63-3\sqrt{18.63}, 19.85-3\sqrt{19.85}, 24.35-3\sqrt{24.35}\right)$$
$$=(5.7, 6.5, 9.5)$$

The proposed fuzzy $\tilde{c}$ -control limits with process capability using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}{}_{c_{a_l}-C_p}, U\tilde{C}L_{c_{a_m}-C_p}, U\tilde{C}L^{\alpha}{}_{c_{a_r}-C_p}\right) = \left[18.63+(3\times 0.327), 19.85+(3\times 0.334), 24.35+(3\times 0.249)\right]$$
$$=(19.6, 20.8, 25.1)$$
$$\left(\tilde{C}L^{\alpha}{}_{c_{a_l}-C_p}, \tilde{C}L_{c_{a_m}-C_p}, \tilde{C}L^{\alpha}{}_{c_{a_r}-C_p}\right) = (18.63, 19.85, 24.35)$$
The
$$\left(L\tilde{C}L^{\alpha}{}_{c_{a_l}-C_p}, L\tilde{C}L_{c_{a_m}-C_p}, L\tilde{C}L^{\alpha}{}_{c_{a_r}-C_p}\right) = \left[18.63-(3\times 0.327), 19.85-(3\times 0.334), 24.35-(3\times 0.249)\right]$$
$$=(17.7, 18.8, 23.6)$$

control limits of α-level fuzzy midrange for α-cut fuzzy $\tilde{c}$ -control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{c.mid} = \left(\frac{18.63+24.35}{2}\right) + \left[3\sqrt{\left(\frac{18.63+24.35}{2}\right)}\right] = 35.4$$
$$C\tilde{L}^{\alpha}_{c.mid} = \left(\frac{18.63+24.35}{2}\right) = 21.5$$
$$L\tilde{C}L^{\alpha}{}_{c.mid} = \left(\frac{18.63+24.35}{2}\right) - \left[3\sqrt{\left(\frac{18.63+24.35}{2}\right)}\right] = 7.6$$

The proposed control limits using process capability of α-level fuzzy midrange for α-cut fuzzy $\tilde{c}$ -control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{c.mid:C_p} = \left(\frac{18.63+24.35}{2}\right) + [3\times 0.288] = 22.4$$
$$C\tilde{L}^{\alpha}_{c.mid:C_p} = \left(\frac{18.63+24.35}{2}\right) = 21.5$$
$$L\tilde{C}L^{\alpha}{}_{c.mid:C_p} = \left(\frac{18.63+24.35}{2}\right) - [3\times 0.288] = 20.6$$

Table 3: Fuzzy rule $\tilde{c}$ -control chart limits and process condition c~

Sample	$S^{\alpha}_{j:c.mid}$	Process condition	
		$\tilde{c}.mid$	$c.mid : C_p$
1	20	in control	Out-of- control
2	25	in control	Out-of- control
3	15	in control	Out-of- control
4	12	in control	Out-of- control
5	15	in control	Out-of- control
6	6	Out-of- control	Out-of- control
7	28	in control	Out-of- control
8	19	in control	Out-of- control
9	30	in control	Out-of- control
10	25	in control	Out-of- control
11	19	in control	Out-of- control
12	23	in control	Out-of- control
13	17	in control	Out-of- control
14	18	in control	Out-of- control
15	11	in control	Out-of- control
16	16	in control	Out-of- control
17	12	in control	Out-of- control
18	23	in control	Out-of- control
19	17	in control	Out-of- control
20	38	Out-of- control	Out-of- control
21	29	in control	Out-of- control
22	23	in control	Out-of- control
23	15	in control	Out-of- control
24	20	in control	Out-of- control
25	18	in control	Out-of- control
26	16	in control	Out-of- control

3. Conclusion

If the process is monitored by Shewhart's control charts, than these traditional control charts assume that the data has crisp values. Thus fuzzy control charts are inevitable tools for monitoring the process. The fuzzy c control chart based on process capability is constructed for the first time in this paper and is applied to real world data. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system. Furthermore, in the case of non-normality, it is recommended to use proposed fuzzy $\tilde{c}$ -control chart as an alternative to Shewhart control chart.

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