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CONSTRUCTION OF FUZZY CONTROL CHART FOR FRACTION DEFECTIVES USING PROCESS CAPABILITY

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Abstract

In traditional control charts, all data should be exactly known, whereas there are many quality characteristics that cannot be expressed in numerical scale, such as characteristics for appearance, softness, and colour. Fuzzy sets theory is powerful mathematical approach to analyse uncertainty, ambiguous and incomplete that can linguistically define data in these situations (Sogandi *et.al.*, 2014). Fuzzy control charts have been extended by converting the fuzzy sets associated with linguistic or uncertain values into scalars regarded as representative values. In this paper, we develop a new fuzzy control chart for fraction defectives with an example.

Keywords: Fuzzy, Fuzzy control chart and Process capability.

1. Introduction

Statistical process control (SPC) is a major tool in many manufacturing environment for implementing quality improvement programs. This process includes observation, evaluation, diagnosis, decision and implementation. Control charts are widely applied in the specified tools. Despite the first control charts proposed during 1920s by Shewhart, they still have an extensive application especially in manufacturing processes in industrial applications.

Control charts were designed to monitor a process and detect shifts in mean and variance of quality characteristics to assure that the processes are performing in an acceptable manner. Two main types of control charts include variable and attribute control charts. The first is used to monitor measurable characteristics on numerical scales (Sogandi et al. 2014). Quality characteristics cannot be easily represented in numerical form monitored by second. In contrast to variable control charts, attribute control charts could monitor more than one quality characteristic simultaneously and need less cost and time for inspections. However the observation of these control charts accompany with ambiguous and vague. In classical control charts for fraction defectives, products are clearly categorized as conformed and non-conformed. In many situations, binary classification may not be appropriate since they have several intermediate levels and the necessity to apply mathematical powerful tool in order to increase the performance of control charts. Hence, recently fuzzy control charts have been extended to analyse uncertainty, ambiguous and incomplete or linguistically defined data. Fuzzy sets convert associated linguistic or uncertain values into scalars regarded as representative values.

Wang and Raz (1990) illustrate two approaches for constructing variable control charts based on linguistic data. Afterwards, Raz and Wang (1995) assigned fuzzy sets to each linguistic term in order to create and design control charts for linguistic data. Gulbay and Kahraman (2004) constructed α-level fuzzy control charts for attributes data to represent the ambiguous of the data and strange of the inspection. In real-applications, there are many cases with uncertainty, ambiguous and incomplete or linguistically defined data. Obviously, mentioned data effect on the performance of attribute control chart. Hence, it is necessary to use a new approach that increases flexibility in range of observation whereas improve the performance of attribute control chart in detection of assignable cause. We consider a new method for control charts to transform fuzzy sets into scalars based on α-level fuzzy midrange. This research paper is summarized as the theoretical structure of fuzzy rule with control chart using process capability is given below with an illustration.

2. Methods and materials

In statistical quality control, we apply control chart for fraction defectives to monitor fraction rejected units of products. It shows the number of nonconforming items, exist in entire process. In the traditional approach the formulation of upper bound and lower bound of p-control charts were based on crisp data and calculated by given following equations:

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL_p = \bar{p}$$

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Where UCL is the upper control limit, CL is the center line and LCL is the lower control limit of 'p' control chart.

If 'p' is not known from the population, 'p' can be estimated from the sample, like;

$$E(p) = \bar{p}$$

and

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{m}$$

where the value of 'D' equals to the number of defects in sample.

Fuzzy set theory is very helpful for dealing with the kind of vagueness of human thought and language found in a Statistical process control. In this study, a number of nonconformities will be express educing triangular fuzzy numbers (TFN). Let 'U' be the universe of discourse, U=[0,u].The triangular fuzzy number is defined as $\tilde{A} = (\alpha_m, \alpha_l, \alpha_r)$ also is formulated;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \le \alpha_m - \alpha_l \\ 1 + \dfrac{x - \alpha_m}{\alpha_l} & ; \alpha_m - \alpha_1 \le x \le \alpha_m \\ 1 - \dfrac{x - \alpha_m}{\alpha_r} & ; \alpha_m \le x \le \alpha_m + \alpha_r \\ 0 & ; x \ge \alpha_m + \alpha_r \end{cases}$$

Where α_m is the center (mode); α_l is left spread; α_r is right spread.

The demonstration of triangular fuzzy numbers will be as $\tilde{A} = (\alpha_m - \alpha_l; \alpha_m; \alpha_m + \alpha_r) = (a_l, a_m, a_r)$ and it is shown in Figure 1.

Fuzzy numbers (a_l, a_m, a_r) are represented as $\left(\bar{p}_{a_{l_j}}, \bar{p}_{a_{m_j}}, \bar{p}_{a_{r_j}}\right)$ for each fuzzy observation on the number of nonconformities control chart. The center line for the control chart $C\tilde{L}$ is as follows:

$$\bar{p}_{a_l} = \frac{\sum_{j=1}^{m} D_{a_{l\,j}}}{m}, \bar{p}_{a_m} = \frac{\sum_{j=1}^{m} D_{a_{m\,j}}}{m} \text{ and } \bar{p}_{a_r} = \frac{\sum_{j=1}^{m} D_{a_{r\,j}}}{m}$$

Where j=1,2,...,m.

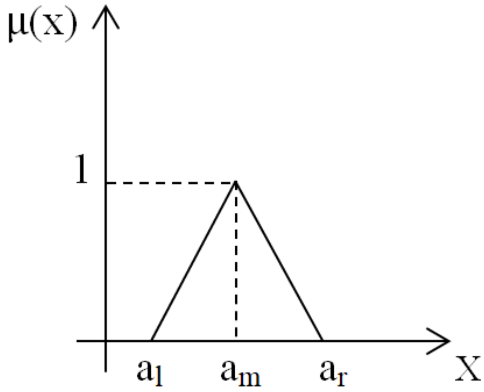

Figure 1:Representation of a sample by triangular fuzzy numbers TFN case

a. Fuzzy p-control chart for Triangular fuzzy number

By considering the formulations of $\tilde{p}$-control limits and fuzzy numbers based on triangular membership functions, the fuzzy center line and the fuzzy upper and fuzzy lower limits of the fuzzy rule $\tilde{p}$-control chart are given as follows:

$$\left(U\tilde{C}L_{p_{a_l}}, U\tilde{C}L_{p_{a_m}}, U\tilde{C}L_{p_{a_r}}\right) = \left(\bar{p}_{a_l} + 3\sqrt{\frac{\bar{p}_{a_l}(1-\bar{p}_{a_l})}{n}}, \bar{p}_{a_m} + 3\sqrt{\frac{\bar{p}_{a_m}(1-\bar{p}_{a_m})}{n}}, \bar{p}_{a_r} + 3\sqrt{\frac{\bar{p}_{a_r}(1-\bar{p}_{a_r})}{n}}\right)$$

$$\left(\tilde{C}L_{p_{a_l}}, \tilde{C}L_{p_{a_m}}, \tilde{C}L_{p_{a_r}}\right) = \left(\bar{p}_{a_l}, \bar{p}_{a_m}, \bar{p}_{a_r}\right)$$

$$\left(L\tilde{C}L_{p_{a_l}}, L\tilde{C}L_{p_{a_m}}, L\tilde{C}L_{p_{a_r}}\right) = \left(\bar{p}_{a_l} - 3\sqrt{\frac{\bar{p}_{a_l}(1-\bar{p}_{a_l})}{n}}, \bar{p}_{a_m} - 3\sqrt{\frac{\bar{p}_{a_m}(1-\bar{p}_{a_m})}{n}}, \bar{p}_{a_r} - 3\sqrt{\frac{\bar{p}_{a_r}(1-\bar{p}_{a_r})}{n}}\right)$$

The fuzzy control limits are defined for a fuzzy rule $\tilde{p}$-control chart for a TFN case. The proposed standard deviation ($\tilde{\sigma}_{i.pF-C_p},\ i = l,m,r$) for fuzzy $\tilde{p}$-control chart with the help of process capability $C_p = \frac{USL_{i.pF-C_p} - LSL_{i.pF-C_p}}{6\sigma}, i = l,m,r$ using a JAVA script (Radhakrishnan and Balamurugan, 2011) is to calculate by the specified tolerance level from the relation $\frac{\sum_{j=1}^{m} D_{i_j}}{m},\ i = l,m,r\ \ and\ \ j = 1,2,...m.$

Therefore the resultant of proposed fuzzy control limits for fraction defectives using process capability is given below:

$$\left(U\tilde{C}L_{p_{a_l}-C_p}, U\tilde{C}L_{p_{a_m}-C_p}, U\tilde{C}L_{p_{a_r}-C_p}\right) = \begin{bmatrix} \bar{p}_{a_l} + \left(\frac{3}{\sqrt{n}} \times \tilde{\sigma}_{l.pF-C_p}\right), \\ \bar{p}_{a_m} + \left(\frac{3}{\sqrt{n}} \times \tilde{\sigma}_{m.pF-C_p}\right), \bar{p}_{a_r} + \left(\frac{3}{\sqrt{n}} \times \tilde{\sigma}_{r.pF-C_p}\right) \end{bmatrix}$$

$$\left(\tilde{C}L_{p_{a_l}-C_p}, \tilde{C}L_{p_{a_m}-C_p}, \tilde{C}L_{p_{a_r}-C_p}\right) = \left(\bar{p}_{a_l}, \bar{p}_{a_m}, \bar{p}_{a_r}\right)$$

$$\left(L\tilde{C}L_{p_{a_l}-C_p}, L\tilde{C}L_{p_{a_m}-C_p}, L\tilde{C}L_{p_{a_r}-C_p}\right) = \begin{bmatrix} \bar{p}_{a_l} - \left(\frac{3}{\sqrt{n}} \times \tilde{\sigma}_{l.pF-C_p}\right), \\ \bar{p}_{a_m} - \left(\frac{3}{\sqrt{n}} \times \tilde{\sigma}_{m.pF-C_p}\right), \bar{p}_{a_r} - \left(\frac{3}{\sqrt{n}} \times \tilde{\sigma}_{r.pF-C_p}\right) \end{bmatrix}$$

The α-cut control limits are also fuzzy sets which could be showed by triangular fuzzy number and the value of α-cut is determined based on the tightness of inspection, we can use a value near 1 for α. The fuzzy $\tilde{p}$-control limits using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}_{p_{a_l}}, U\tilde{C}L_{p_{a_m}}, U\tilde{C}L^{\alpha}_{p_{a_r}}\right) = \begin{pmatrix} \bar{p}^{\alpha}_{a_l} + 3\sqrt{\dfrac{\bar{p}^{\alpha}_{a_l}(1-\bar{p}^{\alpha}_{a_l})}{n}}, \\ \bar{p}_{a_m} + 3\sqrt{\dfrac{\bar{p}_{a_m}(1-\bar{p}_{a_m})}{n}}, \bar{p}^{\alpha}_{a_r} + 3\sqrt{\dfrac{\bar{p}^{\alpha}_{a_r}(1-\bar{p}^{\alpha}_{a_r})}{n}} \end{pmatrix}$$

$$\left(\tilde{C}L^{\alpha}_{p_{a_l}}, \tilde{C}L_{p_{a_m}}, \tilde{C}L^{\alpha}_{p_{a_r}}\right) = \left(\bar{p}^{\alpha}_{a_l}, \bar{p}_{a_m}, \bar{p}^{\alpha}_{a_r}\right)$$

$$\left(L\tilde{C}L^{\alpha}_{p_{a_l}}, L\tilde{C}L_{p_{a_m}}, L\tilde{C}L^{\alpha}_{p_{a_r}}\right) = \begin{pmatrix} \bar{p}^{\alpha}_{a_l} - 3\sqrt{\dfrac{\bar{p}^{\alpha}_{a_l}(1-\bar{p}^{\alpha}_{a_l})}{n}}, \\ \bar{p}_{a_m} - 3\sqrt{\dfrac{\bar{p}_{a_m}(1-\bar{p}_{a_m})}{n}}, \bar{p}^{\alpha}_{a_r} - 3\sqrt{\dfrac{\bar{p}^{\alpha}_{a_r}(1-\bar{p}^{\alpha}_{a_r})}{n}} \end{pmatrix}$$

Where

$p^{\alpha}_{a_l} = p_{a_l} + \alpha\left(p_{a_m} - p_{a_l}\right)$ and $p^{\alpha}_{a_r} = p_{a_r} + \alpha\left(p_{a_r} - p_{a_m}\right)$

The proposed standard deviation ($\tilde{\sigma}^{\alpha}_{a_l.pF-C_p}$ and $\tilde{\sigma}^{\alpha}_{a_r.pF-C_p}$) for fuzzy $\tilde{p}$ -control chart with the help of process capability $C_p = \dfrac{USL^{\alpha}_{i.pF-C_p} - LSL^{\alpha}_{i.pF-C_p}}{6\sigma}$ $r = a_l$ and a_r, using α-cut method is to calculate by the specified tolerance level from the relation

$$\frac{\sum_{j=1}^{m} p_{a_{l\,j}}}{m} + \alpha\left(\frac{\sum_{j=1}^{m} p_{a_{m\,j}}}{m} - \frac{\sum_{j=1}^{m} p_{a_{l\,j}}}{m}\right) \text{ for } \tilde{\sigma}^{\alpha}_{a_l.pF-Cp}$$

$$\text{and } \frac{\sum_{j=1}^{m} p_{a_{r\,j}}}{m} + \alpha\left(\frac{\sum_{j=1}^{m} p_{a_{r\,j}}}{m} - \frac{\sum_{j=1}^{m} p_{a_{m\,j}}}{m}\right) \text{ for } \tilde{\sigma}^{\alpha}_{a_r.pF-Cp}, j = 1, 2, \ldots m.$$

The proposed fuzzy $\tilde{p}$ -control limits with process capability using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}_{p_{a_l}-C_p}, U\tilde{C}L_{p_{a_m}-C_p}, U\tilde{C}L^{\alpha}_{p_{a_r}-C_p}\right) = \begin{bmatrix} \bar{p}^{\alpha}_{a_l} + \left(\dfrac{3}{\sqrt{n}} \times \tilde{\sigma}^{\alpha}_{l.pF-C_p}\right), \\ \bar{p}^{\alpha}_{a_m} + \left(\dfrac{3}{\sqrt{n}} \times \tilde{\sigma}_{m.pF-C_p}\right), \bar{p}^{\alpha}_{a_r} + \left(\dfrac{3}{\sqrt{n}} \times \tilde{\sigma}^{\alpha}_{r.pF-C_p}\right) \end{bmatrix}$$

$$\left(\tilde{C}L^{\alpha}_{p_{a_l}-C_p}, \tilde{C}L_{p_{a_m}-C_p}, \tilde{C}L^{\alpha}_{p_{a_r}-C_p}\right) = \left(\bar{p}^{\alpha}_{a_l}, \bar{p}^{\alpha}_{a_m}, \bar{p}^{\alpha}_{a_r}\right)$$

$$\left(L\tilde{C}L^{\alpha}_{p_{a_l}-C_p}, L\tilde{C}L_{p_{a_m}-C_p}, L\tilde{C}L^{\alpha}_{p_{a_r}-C_p}\right) = \begin{bmatrix} \bar{p}^{\alpha}_{a_l} - \left(\dfrac{3}{\sqrt{n}} \times \tilde{\sigma}^{\alpha}_{l.pF-C_p}\right), \\ \bar{p}^{\alpha}_{a_m} - \left(\dfrac{3}{\sqrt{n}} \times \tilde{\sigma}_{m.pF-C_p}\right), \bar{p}^{\alpha}_{a_r} - \left(\dfrac{3}{\sqrt{n}} \times \tilde{\sigma}^{\alpha}_{r.pF-C_p}\right) \end{bmatrix}$$

In this research article, the α-level fuzzy midrange transformation technique is used for the construction of fuzzy attribute control charts with the help of process capability based on fuzzy trapezoidal number.

The control limits of α-level fuzzy midrange for α-cut fuzzy $\tilde{p}$ -control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{p.mid} = \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right) + \left[\frac{3}{\sqrt{n}}\sqrt{\frac{\left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right)\left(1 - \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right)\right)}{2}}\right]$$

$$C\tilde{L}^{\alpha}_{p.mid} = \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right)$$

$$L\tilde{C}L^{\alpha}{}_{p.mid} = \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right) - \left[\frac{3}{\sqrt{n}}\sqrt{\frac{\left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right)\left(1 - \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right)\right)}{2}}\right]$$

The definition of α-level fuzzy midrange of sample j for fuzzy $\tilde{p}$ -control chart s

$$S^{\alpha}_{j:c.mid} = \frac{\left(\bar{p}_{a_{l\,j}} + \bar{p}_{a_{r\,j}}\right) + \alpha\left[\left(\bar{p}_{a_{m\,j}} - \bar{p}_{a_{l\,j}}\right) - \left(\bar{p}_{a_{r\,j}} - \bar{p}_{a_{m\,j}}\right)\right]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ;\ L\tilde{C}L^{\alpha}{}_{\tilde{p}.mid} \le S^{\alpha}_{j:\tilde{p}.mid} \le U\tilde{C}L^{\alpha}{}_{\tilde{p}.mid} \\ \text{Out-of-control} & ;\ \text{Otherwise} \end{cases}$$

The proposed standard deviation ($\tilde{\sigma}^{\alpha}_{Mid.pF-C_p}$) for α-level fuzzy midrange at $\tilde{p}$ -control chart with the help of process capability $C_p = \frac{USL^{\alpha}_{Mid.pF-C_p} - LSL^{\alpha}_{Mid.pF-C_p}}{6\sigma}$ using α-cut method is to calculate by the specified tolerance level from the relation

$$\left\{\frac{\sum_{j=1}^{m}\bar{p}_{a_{l\,j}}}{m} + \alpha\left(\frac{\sum_{j=1}^{m}\bar{p}_{a_{m\,j}}}{m} - \frac{\sum_{j=1}^{m}\bar{p}_{a_{l\,j}}}{m}\right)\right\} + \left\{\frac{\sum_{j=1}^{m}\bar{p}_{a_{r\,j}}}{m} + \alpha\left(\frac{\sum_{j=1}^{m}\bar{p}_{a_{r\,j}}}{m} - \frac{\sum_{j=1}^{m}\bar{p}_{{}_{m\,j}}}{m}\right)\right\}$$

for $\tilde{\sigma}^{\alpha}_{Mid.pF-Cp}$, , $j = 1, 2, \ldots m.$

The proposed control limits using process capability of α-level fuzzy midrange for α-cut fuzzy $\tilde{p}$ -control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{p.mid:C_p} = \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right) + \left[\frac{3}{\sqrt{n}}\tilde{\sigma}^{\alpha}_{Mid.pF-C_p}\right]$$

$$C\tilde{L}^{\alpha}_{p.mid:C_p} = \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right)$$

$$L\tilde{C}L^{\alpha}{}_{p.mid:C_p} = \left(\frac{\bar{p}^{\alpha}_{a_l} + \bar{p}^{\alpha}_{a_r}}{2}\right) - \left[\frac{3}{\sqrt{n}}\tilde{\sigma}^{\alpha}_{Mid.pF-C_p}\right]$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ;\ L\tilde{C}L^{\alpha}{}_{p.mid:C_p} \le S^{\alpha}_{j:p.mid} \le U\tilde{C}L^{\alpha}{}_{p.mid:C_p} \\ \text{Out-of-control} & ;\ \text{Otherwise} \end{cases}$$

b. Illustration

The example provided by Mahajan (2005, Page No. 272) is considered here. The following data are the inspection results of magnets for nineteen observations.

Table 1: Results of inspection of magnets

Week	Number of magnets Inspected	Numbers of defective magnets	Fraction defective
1	724	48	0.066
2	763	83	0.109
3	748	70	0.094
4	748	85	0.114
5	724	45	0.062
6	727	56	0.077
7	726	48	0.066
8	719	67	0.093
9	759	37	0.049
10	745	52	0.070
11	736	47	0.064
12	739	50	0.068
13	723	47	0.065
14	748	57	0.076
15	770	51	0.066
16	756	71	0.094
17	719	53	0.074
18	757	34	0.045
19	760	29	0.038
Total	14091	1030	

The average sample size $= \frac{14091}{19} = 741.63 \approx 742$ say

Table 2: Triangular fuzzy numbers for fuzzy $\tilde{p}$ -control chart

Week	Fraction defective	a_l	a_m	a_r
1	0.0663	0.0643	0.0663	0.0700
2	0.1088	0.1060	0.1088	0.1130
3	0.0936	0.0916	0.0936	0.0980
4	0.1136	0.1130	0.1136	0.1180
5	0.0622	0.0602	0.0622	0.0650
6	0.0770	0.0760	0.0770	0.0790
7	0.0661	0.0630	0.0661	0.0700
8	0.0932	0.0900	0.0932	0.0940
9	0.0487	0.0460	0.0487	0.0520
10	0.0698	0.0678	0.0698	0.0718
11	0.0639	0.0610	0.0639	0.0650
12	0.0677	0.0670	0.0677	0.0697
13	0.0650	0.0620	0.0650	0.0680
14	0.0762	0.0742	0.0762	0.0782
15	0.0662	0.0630	0.0662	0.0670
16	0.0939	0.0910	0.0939	0.0950
17	0.0737	0.0717	0.0737	0.0757
18	0.0449	0.0410	0.0449	0.0480
19	0.0382	0.0362	0.0382	0.0402

$\bar{p}_{a_l} = 0.0708, \bar{p}_{a_m} = 0.0731$ and $\bar{p}_{a_r} = 0.0757$

The fuzzy center line and the fuzzy upper and fuzzy lower limits of the fuzzy rule $\tilde{p}$ -control chart are given as follows:

$$\left(U\tilde{C}L_{p_{a_l}}, U\tilde{C}L_{p_{a_m}}, U\tilde{C}L_{p_{a_r}}\right) = \left(\begin{array}{l} 0.0708 + \frac{3}{\sqrt{742}}\sqrt{0.0708(1-0.0708)}, \\ 0.0731 + \frac{3}{\sqrt{742}}\sqrt{0.0731(1-0.0731)}, \\ 0.0757 + \frac{3}{\sqrt{742}}\sqrt{0.0757(1-0.0757)} \end{array}\right)$$
$$= (0.0990, 0.1018, 0.1048)$$

$$\left(\tilde{C}L_{p_{a_l}}, \tilde{C}L_{p_{a_m}}, \tilde{C}L_{p_{a_r}}\right) = (0.0708, 0.0731, 0.0757)$$

$$\left(L\tilde{C}L_{p_{a_l}}, L\tilde{C}L_{p_{a_m}}, L\tilde{C}L_{p_{a_r}}\right) = \left(\begin{array}{l} 0.0708 - \frac{3}{\sqrt{742}}\sqrt{0.0708(1-0.0708)}, \\ 0.0731 - \frac{3}{\sqrt{742}}\sqrt{0.0731(1-0.0731)}, \\ 0.0757 - \frac{3}{\sqrt{742}}\sqrt{0.0757(1-0.0757)} \end{array}\right)$$
$$= (0.0425, 0.0444, 0.0465)$$

The resultant of proposed fuzzy control limits for $\tilde{p}$ using process capability is given below:

$$\left(U\tilde{C}L_{p_{a_l}-C_p}, U\tilde{C}L_{p_{a_m}-C_p}, U\tilde{C}L_{p_{a_r}-C_p}\right) = \left[\begin{array}{l} 0.0708 + \left(\frac{3}{\sqrt{742}} \times 0.01083\right), \\ 0.0731 + \left(\frac{3}{\sqrt{742}} \times 0.01048\right), \\ 0.0757 + \left(\frac{3}{\sqrt{742}} \times 0.01052\right) \end{array}\right]$$
$$= (0.0720, 0.0743, 0.0768)$$

$$\left(\tilde{C}L_{p_{a_l}-C_p}, \tilde{C}L_{p_{a_m}-C_p}, \tilde{C}L_{p_{a_r}-C_p}\right) = (0.0708, 0.0731, 0.0757)$$

$$\left(L\tilde{C}L_{p_{a_l}-C_p}, L\tilde{C}L_{p_{a_m}-C_p}, L\tilde{C}L_{p_{a_r}-C_p}\right) = \left[\begin{array}{l} 0.0708 - \left(\frac{3}{\sqrt{742}} \times 0.01083\right), \\ 0.0731 - \left(\frac{3}{\sqrt{742}} \times 0.01048\right), \\ 0.0757 - \left(\frac{3}{\sqrt{742}} \times 0.01052\right) \end{array}\right]$$
$$= (0.0696, 0.0720, 0.0745)$$

The fuzzy $\tilde{p}$ -control limits using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}{}_{p_{a_l}}, U\tilde{C}L_{p_{a_m}}, U\tilde{C}L^{\alpha}{}_{p_{a_r}}\right) = \begin{pmatrix} 0.0723 + \frac{3}{\sqrt{742}}\sqrt{0.0723(1-0.0723)}, \\ 0.0731 + \frac{3}{\sqrt{742}}\sqrt{0.0731(1-0.0731)}, \\ 0.0773 + \frac{3}{\sqrt{742}}\sqrt{0.0773(1-0.0773)} \end{pmatrix}$$

$$= (0.1008, 0.1018, 0.1067)$$

$$\left(\tilde{C}L^{\alpha}{}_{p_{a_l}}, \tilde{C}L_{p_{a_m}}, \tilde{C}L^{\alpha}{}_{p_{a_r}}\right) = (0.0723, 0.0731, 0.0773)$$

$$\left(L\tilde{C}L^{\alpha}{}_{p_{a_l}}, L\tilde{C}L_{p_{a_m}}, L\tilde{C}L^{\alpha}{}_{p_{a_r}}\right) = \begin{pmatrix} 0.0723 - \frac{3}{\sqrt{742}}\sqrt{0.0723(1-0.0723)}, \\ 0.0731 - \frac{3}{\sqrt{742}}\sqrt{0.0731(1-0.0731)}, \\ 0.0773 - \frac{3}{\sqrt{742}}\sqrt{0.0773(1-0.0773)} \end{pmatrix}$$

$$= (0.0438, 0.0444, 0.0479)$$

The proposed fuzzy $\tilde{p}$-control limits with process capability using α-cut method for triangular numbers as follows:

$$\left(U\tilde{C}L^{\alpha}{}_{p_{a_l}-C_p}, U\tilde{C}L_{p_{a_m}-C_p}, U\tilde{C}L^{\alpha}{}_{p_{a_r}-C_p}\right) = \begin{bmatrix} 0.0723 + \left(\frac{3}{\sqrt{742}} \times 0.01060\right), \\ 0.0731 + \left(\frac{3}{\sqrt{742}} \times 0.01048\right), \\ 0.0773 + \left(\frac{3}{\sqrt{742}} \times 0.01055\right) \end{bmatrix}$$

$$= (0.0735, 0.0743, 0.0785)$$

$$\left(\tilde{C}L^{\alpha}{}_{p_{a_l}-C_p}, \tilde{C}L_{p_{a_m}-C_p}, \tilde{C}L^{\alpha}{}_{p_{a_r}-C_p}\right) = (0.0723, 0.0731, 0.0773)$$

$$\left(L\tilde{C}L^{\alpha}{}_{p_{a_l}-C_p}, L\tilde{C}L_{p_{a_m}-C_p}, L\tilde{C}L^{\alpha}{}_{p_{a_r}-C_p}\right) = \begin{bmatrix} 0.0723 - \left(\frac{3}{\sqrt{742}} \times 0.01060\right), \\ 0.0731 - \left(\frac{3}{\sqrt{742}} \times 0.01048\right), \\ 0.0773 - \left(\frac{3}{\sqrt{742}} \times 0.01055\right) \end{bmatrix}$$

$$= (0.0711, 0.0720, 0.0762)$$

The control limits of α-level fuzzy midrange for α-cut fuzzy $\tilde{p}$-control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{p.mid} = \left(\frac{0.0723+0.0773}{2}\right) + \left[\frac{3}{\sqrt{742}}\sqrt{\frac{\left(\frac{0.0723+0.0773}{2}\right)\left(1-\frac{0.0723+0.0773}{2}\right)}{2}}\right] = 0.1038$$

$$C\tilde{L}^{\alpha}_{p.mid} = \left(\frac{0.0723+0.0773}{2}\right) = 0.0748$$

$$L\tilde{C}L^{\alpha}{}_{p.mid} = \left(\frac{0.0723+0.0773}{2}\right) - \left[\frac{3}{\sqrt{742}}\sqrt{\frac{\left(\frac{0.0723+0.0773}{2}\right)\left(1-\frac{0.0723+0.0773}{2}\right)}{2}}\right] = 0.0458$$

The proposed control limits using process capability of α-level fuzzy midrange for α-cut fuzzy $\tilde{p}$ -control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}{}_{p.mid:C_p} = \left(\frac{0.0723+0.0773}{2}\right) + \left[\frac{3}{\sqrt{742}} \times 0.01058\right] = 0.0760$$

$$C\tilde{L}^{\alpha}_{p.mid:C_p} = \left(\frac{0.0723+0.0773}{2}\right) = 0.0748$$

$$L\tilde{C}L^{\alpha}{}_{p.mid:C_p} = \left(\frac{0.0723+0.0773}{2}\right) - \left[\frac{3}{\sqrt{742}} \times 0.01058\right] = 0.0736$$

in control Out-of- control

Table 3: Fuzzy $\tilde{p}$ -control chart limits and process condition

Sample	$S^{\alpha}_{j:c.mid}$	Process condition	
		$\tilde{p}.mid$	$\tilde{p}.mid : C_p$
1	0.0666	in control	Out-of- control
2	0.1090	Out-of- control	Out-of- control
3	0.0940	in control	Out-of- control
4	0.1143	Out-of- control	Out-of- control
5	0.0623	in control	Out-of- control
6	0.0772	in control	Out-of- control
7	0.0663	in control	Out-of- control
8	0.0928	in control	Out-of- control
9	0.0488	in control	Out-of- control
10	0.0698	in control	Out-of- control
11	0.0636	in control	Out-of- control
12	0.0679	in control	Out-of- control
13	0.0650	in control	Out-of- control
14	0.0762	in control	Out-of- control
15	0.0658	in control	Out-of- control
16	0.0936	in control	Out-of- control
17	0.0737	in control	in control
18	0.0448	Out-of- control	Out-of- control
19	0.0382	Out-of- control	Out-of- control

3. Conclusion

One of the most important SPC tools is attribute control chart that monitors quality characteristics. Some causes such as mental inspection, incomplete data and human judgments in quality characteristic that lead to exist some level of vagueness and uncertainty in attribute control chart, in these situations it is better to apply fuzzy set theory for control charts. Thus, in this paper, we developed a fuzzy p-chart using process capability to monitor attribute quality characteristic. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system. It is recommended to use proposed fuzzy $\tilde{p}$ -control chart as an alternative to Shewhart control chart.

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