IJCRT.ORG ISSN : 2320-2882
INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

LATTICES IN A GAMMA SEMIRING

TILAK RAJ SHARMA

Department of Mathematics, Himachal Pradesh University,

Regional Centre, Khaniyara, Dharamshala, Himachal Pradesh (India)-176218

Abstract: - In this paper, the efforts are made to characterize some results on lattices of $\Gamma-$semirings. Since a complemented element plays an important role in the study of lattices, so we give characterization of some results on lattices, which are analogous to the corresponding results in semirings [2-3].

Keywords: $\Gamma-$semiring, simple, additive idempotent and multiplicative $\Gamma-$idempotent $\Gamma-$ semiring, Complemented elements in a $\Gamma-$ semiring, Lattices.

1. INTRODUCTION

From an algebraic point of view, semirings provide the most natural common generalization of the theories of rings and most of the techniques used in analyzing semirings are taken from ring theory and group theory. The set of nonnegative integers $\mathbb{N}$ with usual addition and multiplication provides a natural example of a semiring. There are many other examples of semiring such as the positive cone of a totally ordered ring. For a given positive integer n, the set of all $n \times n$ matrices over a semiring R forms a semiring with usual matrix addition and multiplication over R. But the situations for the set of all non-positive integers and for the set of all $m \times n$ matrices over a semiring R are different. They do not form semirings with the above operations, because multiplication in the above sense are no longer binary compositions. This notion provides a new kind of algebraic structure what is known as a $\Gamma-$ semiring.

The concept of $\Gamma-$semiring was introduced by M. M. K. Rao in 1995 [6] as a generalization of semiring as well as $\Gamma-$ ring (it may be recalled here that the notion of **"Γ"** was first introduced in algebra by N. Nobusawa in 1964). Later it was found that $\Gamma-$ semiring also provides an algebraic home to the negative cones of totally ordered rings and the sets of rectangular matrices over a semiring. For further study of semirings, $\Gamma-$semirings and their generalization, one may referred to [2-3][4-10].

In this paper, the efforts are made to characterize some results on lattices of $\Gamma-$ semirings. Furthermore, since a complemented element plays an important role in the study of lattices, so we give characterization of some results on lattices, which are analogous to the corresponding results in semirings [2-3]. Finally, we define harmony difference of elements of $\Gamma R^{\perp}$(set of all complemented elements) and proved that if $\mathrm{d} : \Gamma R^{\perp} \times \Gamma R^{\perp} \to \Gamma R^{\perp}$ be defined by $\mathrm{d} : (\mathrm{x}, \mathrm{y}) \to x \nabla \mathrm{y}$. Then d is metric on $\Gamma R^{\perp}$ with values in R.

2. PRELIMINARIES

The following definitions with examples and results are felt to be an inseparable part of this paper.

Definition 2.1: Let R and Γ be two additive commutative semigroups. Then R is called a $\Gamma-$ semiring if there exists a mapping $R \times \Gamma \times R \to R$ denoted by $x\alpha y$ for all $x, y \in R$ and $\alpha \in \Gamma$ satistifying the following conditions:

(i) $x\alpha(y+z) = (x\alpha y) + (x\alpha z)$.

(ii) $(y+z)\alpha x = (y\alpha x) + (z\alpha x)$.

(iii) $x\,(\alpha + \beta)z = (x\alpha z) + (x\beta z)$.

(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$.

Definition 2.3: A $\Gamma-$ semiring R is said to have a zero element if $0\gamma x = 0 = x\gamma 0$ and $x + 0 = x = 0 + x$ for all $x \in R$ and $\gamma \in \Gamma$.

Definition 2.4: A non-empty subset T of a $\Gamma-$ semiring R is said to be a sub $\Gamma-$ semiring of R if $(T, +)$ is a subsemigroup of $(R, +)$ and $x\gamma y \in T$ for all $x, y \in T$ and $\gamma \in \Gamma$.

Definition 2.5: A $\Gamma-$ semiring is said to have identity element if $x\gamma 1 = x = 1\gamma x$ for all $x \in R$ and $\in \Gamma$.

Definition 2.6: A $\Gamma-$ semiring R is said to be a commutative if $x\gamma y = y\gamma x$ for all $x, y \in R$ and $\gamma \in \Gamma$.

Example 2.7: Let R be the set of all even positive integers and Γ be set of all positive integers divisible by 3. Then with usual addition and multiplication of integers, R is a commutative $\Gamma-$ semiring.

Definition 2.8: An element x of a $\Gamma-$semiring R is said to be additive idempotent if and only if $x + x = x$. If every element of R is additive idempotent then R is called additive idempotent $\Gamma-$ semiring. It is denoted by $I^{+}(\Gamma R)$.

Definition 2.9: An element x of a $\Gamma-$ semiring R is said to be multiplicative $\Gamma-$idempotent if there exists $\gamma \in \Gamma$ such that $x = x\,\gamma x$. If every element of R is multiplicative $\Gamma-$idempotent then R is called multiplicative $\Gamma-$ idempotent $\Gamma-$ semiring. It is denoted by $I^{\times}(\Gamma R)$.

Definition 2.10: A $\Gamma-$ semiring R is said to be $\Gamma-$idempotent if it is both additive idempotent and multiplicative $\Gamma-$ idempotent.

We will denote the set of all $\Gamma-$idempotent elements of a $\Gamma-$semiring R by $I(\Gamma \mathrm{R})$.

Definition 2.11: A $\Gamma-$ semiring R with identity is simple if and only if $x + 1 = 1 = 1 + x$ for all $x \in R$.

Definition 2.12: A $\Gamma-$ semiring R is centreless if and only if $x + y = 0$ implies that $x = y = 0$.

Example 2.13: Let $R = \mathbb{Z}^{+}$ be a set of non negative integers and $\Gamma = \{1\}$. Define a mapping $R \times \Gamma \times R \rightarrow R$ by $x1y = xy$ for all $x, y \in R$. Then R is a centreless Γ −semiring..

Definition 2.14: The Centre of a $\Gamma -$ semiring R is a subset of R consisting of all elements x of R such that $x\gamma y = y\gamma x$ for all $y \in R$ and $\gamma \in \Gamma$. It is denoted by $C(R)$.

Definition 2.15: Let x, y be elements of a $\Gamma -$ semiring R, then x **is $\Gamma -$ interior** y denoted by $\boldsymbol{x\nabla y}$ if and only if there exists an element $z \epsilon R$ such that $x\gamma z = z\gamma x = 0$ and $z + y = 1$ $for\ all\ \gamma \in \Gamma$.

Definition 2.16: An element x is complemented if and only if $x\nabla$x. That is, there exists an element $y \in R$ such that $x\gamma y = y\gamma x = 0$ and $x + y = 1$, $for\ all\ \gamma \in \Gamma$. This element y of R is the complement of x in R. We will denote complement of x by $x^{\perp}$.

Clearly if x is complemented then so is $x^{\perp}$ and $x^{\perp\perp} = x$.

Lemma 2.17[9]: Let R be a $\Gamma -$ semiring. Then

(i) R is simple if and only if $x = x + x\gamma y$ for all $x, y \in R$ and $\gamma \in \Gamma$.

(ii) R is simple if and only if $x = x + y\gamma x$ for all $x, y \in R$ and $\gamma \in \Gamma$.

(iii) R is simple if and only if $x\gamma y = x\gamma y + (x\beta z)\gamma y$ for all $x, y, z \in R$ and $\beta, \gamma \in \Gamma$.

Theorem 2.18[9]: Let R be a $\Gamma -$ semiring. Then every additive idempotent $\Gamma -$ semiring has a simple sub $\Gamma -$ semiring.

Let us denote the set of all complemented elements of $\boldsymbol{R}$ **by** $\boldsymbol{\Gamma R^{\perp}}$. Clearly $\Gamma R^{\perp} \neq \phi$, since $0^{\perp}$=1. Indeed, if $x \in \Gamma R^{\perp}, \gamma \in \Gamma$. Then $x + x^{\perp} = 1$, $x\gamma x^{\perp} = x^{\perp}\gamma x = 0$, for all $\gamma \in \Gamma$. Therefore $x = x\gamma 1 = x\gamma(x + x^{\perp}) = x\gamma x + x\gamma x^{\perp} = x\gamma x$. Thus $\Gamma R^{\perp}$ is multiplicative $\Gamma -$ idempotent, that is, $\Gamma R^{\perp} \subseteq I^{\times}(\Gamma R)$. Again let $x \in \Gamma R^{\perp}$ and $\gamma \in \Gamma$. Set $x \circledast y = x + x^{\perp}\gamma y$, for all $\in \Gamma$. Then $x \circledast x^{\perp} = x + x^{\perp}\gamma x^{\perp} = x + x^{\perp} = 1$ for all $x \in \Gamma R^{\perp}$. Also, if $x + y = 1$ then $x^{\perp} = x^{\perp}\gamma 1 = x^{\perp}\gamma(x + y) = x^{\perp}\gamma x + x^{\perp}\gamma y = x^{\perp}\gamma y$. Thus $x \circledast y = x + x^{\perp}\gamma y = x + x^{\perp} = 1$.

Lemma 2.19[10]: Let R be a $\Gamma -$ semiring such that $x, y \in \Gamma R^{\perp}$. Then x$\gamma y$, $x \circledast y \in \Gamma R^{\perp}$.

Lemma 2.20[10]: Let R be a $\Gamma -$ semiring and $x, y \in \Gamma R^{\perp}$. Then $x\gamma y = y\gamma x, for\ all\ \gamma \in \Gamma$.

Lemma 2.21[10]: Let R be a centreless $\Gamma -$ semiring. If $x, y \in \Gamma R^{\perp}$ and $\beta, \gamma \in \Gamma$. Then $(x\gamma y)\beta x^{\perp} = (x^{\perp}\gamma y)\beta x = 0$.

Theorem 2.22[10]: Let R be a centreless $\Gamma -$ semiring. Then $\Gamma R^{\perp}$ is $\Gamma -$ idempotent, commutative, simple $\Gamma -$ semiring with the mapping $\circledast : \Gamma R^{\perp} \times \Gamma \times \Gamma R^{\perp} \rightarrow \Gamma R^{\perp}$ defined by $x * \gamma * y = x\gamma y$, for all $x, y \in \Gamma R^{\perp}, \gamma \in \Gamma$.

Proposition 2.23[10] Let R be a centreless Γ −semiring. Then $\Gamma R^{\perp}$is a sub Γ −semiring of R if and only if $x + y$, $x\gamma y \in \Gamma R^{\perp}, x, y \in \Gamma R^{\perp}$.

Remark 2.24: Throughout this paper, R will denote a $\Gamma -$ semiring with **zero element** $\mathbf{'0'}$ and **identity element** $\mathbf{'1'}$ unless otherwise stated.

3. LATTICES IN A GAMMA SEMIRING

Complemented elements play an important role in study of lattices. Another major source of inspiration for the theory of $\Gamma-$ semirings is lattice theory.

Definition 3.1: Let a and b be two elements in a partially ordered set $(A, \leq)$. An element c is said to be an upper bound of a and b if $a \leq c$ and $b \leq c$, and an element c is said to be a least upper bound of a and b if c is an upper bound of a and b, and there is no other upper bound d of a and b such that $d \leq c$. Similarly, an element c is said to be a lower bound of a and b if $c \leq a$ and $c \leq b$, and an element c is said to be a greatest lower bound of a and b if c is a lower bound of a and b, and if there is no other lower bound d of a and b such that $c \leq d$.

Remark 3.2: A lattice is a partially ordered set in which every two elements have a unique least upper bound and a unique greatest lower bound. Let $(A, \leq)$ be a lattice. We define an algebraic system $(A, \vee, \wedge)$, where $\vee$ and $\wedge$ are two binary operations on A such that for a and b in A, $a \vee b$ is equal to the least upper bound of a and b and $a \wedge b$ is equal to the greatest lower bound of a and b.

Definition 3.3: A lattice is said to be a distributive lattice if the meet operation distributes over the join operation and the join operation distributes over the meet operation. That is, for any a, b and c

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \text{ and } a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

Definition 3.4: An element a in a lattice $(A, \leq)$is called a universal lower bound if for every element $b \in A$, we have $a \leq b$. An element a in a lattice $(A, \leq)$ is called a universal upper bound if for every element $b \in A$, we have $b \leq a$.

We shall use $'\mathbf{0}'$ to denote the universal lower bound and $'\mathbf{1}'$ to denote the universal upper bound of a lattice (if such bounds exist).

Definition 3.5: Let $(A, \leq)$ be a lattice with universal lower bound and upper bounds $'0'$ and $'1'$ respectively. For an element a in A, an element b is said to be a complement of a if $a \vee b = 1$ and $a \wedge b = 0$.

Definition 3.6: A lattice is said to be a complemented lattice if every element in the lattice has a complement. (Clearly, a complemented lattice must have universal lower and upper bounds).

Definition 3.7: A complemented and distributive lattice is called Boolean lattice. A Boolean lattice $(A, \leq)$defines an algebraic system $(A, \vee, \wedge, \perp)$ is known as Boolean algebra, where $\vee, \wedge$ and $\perp$are the join, meet and the complementation operations respectively.

Definition 3.8: A $\Gamma-$semiring R is lattice ordered if and only if it also has the structure of a lattice such that for all $x, y \in R$ and $\gamma \in \Gamma$

(i) $x + y = x \vee y$

(ii) $x \gamma y = x \wedge y$, where partial order is one induced by the lattice structure on R .

Theorem 3.9: Let R be a $\Gamma-$ semiring. Then R is a bounded distributive lattice having unique minimal element 0 and unique maximal element 1 if and only if R is commutative, $\Gamma-$idempotent and simple $\Gamma-$semiring.

Proof: Let R be a bounded distributive lattice having unique minimal element 0 and unique maximal element1. Then R becomes a commutative, $\Gamma-$idempotent and simple $\Gamma-$semiring by defining $x+y=x\vee y$ and $x\gamma y=x\wedge y$ for all, $x,y\in R$.

Conversely, Let R be commutative, $\Gamma-$ idempotent and simple $\Gamma-$ semiring. Then define a relation '$\leq$' on R by $x\leq y$ if $x+y=y$ and $x\gamma y=x$.

(i) $x\leq x$ as $x+x=x$ and $x\gamma y=x$.

(ii) If $x\leq y$ and $y\leq x$ then $x+y=y$, $y+x=x$ and $x\gamma y=x$, $y\gamma x=y$. Thus $x=y$.

(iii) If $x\leq y$ and $y\leq z$ then $x+y=y$, $y+z=z$ and $x\gamma y=x$, $y\gamma z=y$.

Thus $x+z=x+(y+z)=(x+y)+z=y+z=z$ and $x\gamma z=(x\gamma y)\gamma z=x\gamma(y\gamma z)=x\gamma y=x$. This implies that $x\leq z$. Hence $(R,\leq)$ is a partially ordered set. Define the operation $\vee$ and $\wedge$ on R by $x+y=x\vee y$ and $x\gamma y=x\wedge y$ for all $x,y\in R,\gamma\in\Gamma$. Then it is easy to see that R is a bounded distributive lattice having unique minimal element 0 and unique maximal element 1.

Another well-known characterization of bounded distributive lattices is the following.

Proposition 3.10: Let R be a $\Gamma-$ semiring. R is a bounded distributive lattice having unique minimal element 0 and unique maximal element 1 if and only if it is commutative, $\Gamma-$ idempotent $\Gamma-$semiring and $x\wedge(x\vee y)=x=x\vee(x\wedge y)$ for all $x.y\in R$.

Proof: Let R be a bounded distributive lattice having unique minimal element 0 and unique maximal element 1. Let $x,y,z\in R$. Since $x\vee(x\wedge y)$ is the join of x and $x\wedge y$ we have $x\leq x\vee(x\wedge y)$. Again, $x\leq x$ and$(x\wedge y)\leq x$, we have $x\vee(x\wedge y)\leq x\vee x=x$. Therefore $x\vee(x\wedge y)=x$. By principal of duality, $x\wedge(x\vee y)=x$. Now the result follows from Theorem 3.9. Conversely, let R be a commutative, $\Gamma-$ idempotent $\Gamma-$ semiring and $x\wedge(x\vee y)=x=x\vee(x\wedge y)$ for all $,y\in R$. In the light of theorem 3.9, it is sufficient to show that R is simple. Putting $x=1$ in $x\wedge(x\vee y)=x=x\vee(x\wedge y)$ as $x+y=x\vee y$, $x\gamma y=x\wedge y$, we get $1\gamma(1+y)=1$ for all $y\in R$. That is $1+y=1$ for all $y\in R$. Hence R is simple.

Theorem 3.11: Let R be a $\Gamma-$ semiring. A commutative $\Gamma-$ semiring is a bounded distributive lattice if and only if it is simple multiplicative $\Gamma-$ idempotent $\Gamma-$ semiring.

Proof: This is a direct consequence of Lemma 2.17, Proposition 3.10. and Theorem 3.9.

Lemma 3.12: Let R be a simple $\Gamma-$ semiring then $(I^{\times}(\Gamma R),+)$ is a sub monoid of (R, +) and $I^{\times}(\Gamma R)\cap$ C(R) is a bounded distributive lattice.

Proof: Let $x,y\in I^{\times}(\Gamma R)$. Then $x\gamma x=x$ *and* $y\gamma y=y$. Therefore by Lemma 2.17 we have$(x+y)\gamma(x+y)$ $=(x+y)\gamma x+(x+y)\gamma y=x\gamma x+y\gamma x+x\gamma y+y\gamma y=x+y\gamma x+x\gamma y+y=x+y$. So, $x+y\in I^{\times}(\Gamma R)$, that is, $I^{\times}(\Gamma R)$ is closed under addition. Since it contains '0' so $I^{\times}(\Gamma R)$is a sub monoid of (R, +). Further, let $x,y\in C(R)$ then $x+y\in C(R)$. Therefore $x+y\in I^{\times}(\Gamma R)\cap C(R)$. Since $0,1\in I^{\times}(\Gamma R)\cap C(R)$

so $I^{\times}(\Gamma R)\cap C(R) \neq \phi$. Let $x, y \in I^{\times}(\Gamma R)\cap C(R)$ then surely $x\gamma y \in I^{\times}(\Gamma R)\cap C(R)$. Thus $I^{\times}(\Gamma R)\cap C(R)$ is a sub Γ-semiring of R which is also simple, since $x + 1 = 1$ for all $x \in I^{\times}(\Gamma R)\cap C(R)$. Now the result follows from Theorem 3.11.

Lemma 3.13: Let R be a Γ −semiring. Then every additive idempotent $\Gamma -$ semiring has a sub $\Gamma -$ semiring which is bounded distributive lattice.

Proof: Result immediate follows from Theorem 2.18 and Lemma 3.12.

Theorem 3.14: Let R be a centreless $\Gamma -$ semiring then $\Gamma R^{\perp}$ is a $\Gamma -$ Boolean algebra.

Proof: By Theorem 3.9, a commutative, $\Gamma -$ idempotent and simple $\Gamma -$ semiring is a bounded distributive lattice having unique minimal element 0 and unique maximal element 1. Thus, by Theorem 2.22., we find that $\Gamma R^{\perp}$ is such a lattice, which is complemented and so is a Γ −Boolean algebra.

Theorem 3.15: Let R be a centreless $\Gamma -$ semiring then the relation '$\leqslant$' on R defined by $x \leqslant y$ if and only if there exist an element $t \in \Gamma R^{\perp}$ such that $x = t\gamma y, for\ all\ \gamma \in \Gamma$ is a partial order relation on R.

Proof: Since $x = 1\gamma x\ , for\ all\ \gamma \in \Gamma$ so $x \leqslant x$ for all $x \in R$. Assume that $x \leqslant y$ and $y \leqslant x$ then there exist $p, q \in \Gamma R^{\perp},\ \beta, \gamma \in \Gamma$ such that $x = p\gamma y,\ y = q\beta x$. This implies that $p\alpha x = p\alpha(p\gamma y) = (p\alpha p)\gamma y = p\gamma y = x, for\ all\ \alpha \in \Gamma$ and so by the Lemma 2.20, $x = p\gamma y = p\gamma(q\beta x) = p\gamma(x\beta q) = (p\gamma x)\beta q = x\beta q = q\beta x = y$. Again, if $x \leqslant y$ and$y \leqslant z$ then there exist $p, q \in \Gamma R^{\perp},\ \beta, \gamma \in \Gamma$ such that $x = p\gamma y, y = q\beta z$. Thus $x = p\gamma y = p\gamma(q\beta z) = (p\gamma q)\beta z,\ p\gamma q \in \Gamma R^{\perp}$ (c.f. Lemma 2.19). This implies that $x \leqslant z$.

For a $\Gamma -$ semiring R we define the harmony difference of elements of $\Gamma R^{\perp}$ by $x\nabla \mathrm{y} = \mathrm{x}\gamma y^{\perp} + x^{\perp}\gamma y$. In particular if R is a centreless $\Gamma -$ semiring satisfying the condition that $\Gamma R^{\perp}$ is a sub $\Gamma -$ semiring of R (c.f. proposition 2.23) then by Theorem 3.14, $\Gamma R^{\perp}$ is a Γ −boolean algebra and this is just the harmony difference in the usual sense.

Now, we have the following theorem:

Theorem 3.16: Let R be a centreless $\Gamma -$ semiring. If $\mathrm{d} : \Gamma R^{\perp} \times \Gamma R^{\perp} \rightarrow \Gamma R^{\perp}$ be defined by $\mathrm{d} : (\mathrm{x}, \mathrm{y}) \rightarrow x\nabla \mathrm{y}$. Then d is metric on $\Gamma R^{\perp}$ with values in R.

Proof: If R is arbitrary centreless $\Gamma -$ semiring then it is clear that $\mathrm{d}\,(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{y}, \mathrm{x}) \geq 0$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{R}$ and $\mathrm{d}(\mathrm{x}, \mathrm{x}) = 0$, for all $\mathrm{x} \in \mathrm{R}$. Conversely, assume that $\mathrm{d}\,(\mathrm{x}, \mathrm{y}) = 0$. Since R is centreless $\Gamma -$ semiring, so $\mathrm{x}\gamma y^{\perp} = 0 = x^{\perp}\gamma y$ and $x = x\gamma 1 = x\gamma(y^{\perp} + y) = \mathrm{x}\gamma y^{\perp} + \mathrm{x}\gamma y = xy$, proving that $\mathrm{x} \leq \mathrm{y}$. Similarly, $\mathrm{y} \leq \mathrm{x}$ and so $x = y$. If $x, y, z \in R$ then $(x\nabla \mathrm{z})\gamma(x\nabla \mathrm{y} + y\nabla \mathrm{z}) = (\mathrm{x}\gamma z^{\perp} + x^{\perp}\gamma z)(\mathrm{x}\gamma y^{\perp} + x^{\perp}\gamma y + y\gamma y z^{\perp} + y^{\perp}\gamma z) = \mathrm{x}\gamma y^{\perp}\gamma z^{\perp} + x\gamma y z^{\perp} + x^{\perp}\gamma y\gamma z + x^{\perp}\gamma y^{\perp}\gamma z = x(y^{\perp} + y)z^{\perp} + x^{\perp}(y + y^{\perp})\gamma z = x\gamma z^{\perp} + x^{\perp}\gamma z = x\nabla \mathrm{z}$ (c.f. Lemma 2.19, 2.20 and 2.21) and so $\mathrm{d}\,(\mathrm{x}, \mathrm{z}) = x\nabla \mathrm{z} \leq x\nabla \mathrm{y} = y\nabla \mathrm{z} = \mathrm{d}\,(\mathrm{x}, \mathrm{y}) + \mathrm{d}\,(\mathrm{y}, \mathrm{z})$. Thus, d is metric on $\Gamma R^{\perp}$ with values in R. Note that if $x \in \Gamma R^{\perp}$ then $\mathrm{d}\,(\mathrm{x}, 0) = x\gamma 1 + x^{\perp}\gamma 0 = x$. Also $\mathrm{d}\,(\mathrm{x}, x^{\perp}) = x\gamma x + x^{\perp}\gamma x^{\perp} = x + x^{\perp} = 1$ for all $\mathrm{x} \in \Gamma R^{\perp}$.

REFERENCES

[1] Birkoff, G., Lattice Theory, Providence RI: American Mathematical Society, 25th edition, 1969.

[2] Golan, J. S., Semirings and their applications, Kulwer Academic Publishers, 1999.

[3] Golan, J. S., The Theory of semirings with applications in Mathematics and theorectical computer science . Pitman Monograph and Surveys in pure and applied Mathematics, 1992.

[4] Hedayati, H., Shum, K.P., “An Introduction to Γ −semirings”, International Journal Of Algebra, Vol. 5, no. 15, 2011 ,709-726.

[5] Johnstone, Peter T., Stone spaces, Cambridge University Press, Cambridge, 1982.

[6] Rao, M. M. Krishana., Gamma Semiring-I; Southeast Asian Bull. of Math., Vol. 19 (1995), 49-54.

[7] Saha, B.C., Sardar, S.K., On Nobusawa Gamma semirings; Universitatee Din Bacau Studii Si Cercetari Stiinfiee Seria Mathematica, 18 (2008), 283-306

[8] Sardar, S.K., Dasgupta, U. On Primitive Gamma Semirings, Novi Sad J. Math, Vol. 34, No. 1, 2004, 1-12.

[9] Sharma, Tilak Raj, Gupta, Shweta. Some Conditions on $\Gamma -$ Semirings, JCISS VOL.41 (2016), No. 1-2 pages 79-87.

[10] Sharma, Tilak Raj, Gupta, Shweta. Complementation of $\Gamma -$ Semirings, (submitted)